The BROWN-ELDREDGE ARITHMETICS

BOOK THREE

WITH ANSWERS

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88

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THE
BROWN-ELDREDGE
ARITHMETICS

BOOK THREE

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PREFACE

THIS series of arithmetics was prepared to meet the requirements of progressive school men and women who want practical, usable textbooks, free from fads and sufficient to their purpose. Each book of the series clearly provides for the mathematics required by the pupils of a given age. The material is well organized and adequately motivated throughout.

In the fullest sense of the words, these books are not the outcome of one set of experiments or the presentation of another; neither are they a compilation nor the result of hasty effort. They are the product of years of classroom experience, coupled with the observation of scores of capable classroom teachers with whom the authors have been in daily contact. Familiarity with educational experiments and investigations has enabled the authors to use the results of such research whenever those could be utilized in the selection, gradation, and arrangement of material so as to achieve greater economy in the learning process.

Throughout the three books of the series there is a maximum of practice and a minimum of theory. Principles have been simply and naturally developed; technical explanations have been omitted. At every step emphasis is placed upon the development of a high degree of accuracy and reasonable speed in computation. Drill exercises have been designed with this end in view; they provide for individual differences of pupils and assure a certain degree of attainment to those of less than average ability.

Problems which are genuinely practical grip the attention of the pupil because they are based upon his experiences. Large numbers of such problems, well chosen and well graded, go far toward lessening the teacher's labor because they assure the joyful coöperation of the learner. Numerous reviews, comprehensive and properly placed, insure a mastery of content and serve as a gauge of progress. Carefully planned tests help pupil and teacher alike to note the growth of accomplishment.

Lack of space prevents individual acknowledgment to the many superintendents, supervisors, teachers, parents, and experimenters who have so generously contributed valuable suggestions and carefully tested the materials during the preparation of these books. To all these the authors are deeply indebted.

This is the third book of a three-book series; it is planned to provide for work through the seventh and eighth grades of the average school. Unless the course of study provides otherwise many classes will complete this book as far as page 132 by the close of the seventh school year, from page 133 to the conclusion of the book by the close of the eighth school year.

February
1925

THE AUTHORS

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Answers

THE
BROWN-ELDREDGE ARITHMETICS

BOOK THREE

TO THE PUPIL

You have learned to add, subtract, multiply, and divide, using integers and common and decimal fractions. You have studied the tables of measures commonly used in the United States, you have learned how to find areas and volumes, and you know something about percentage.

You are now to study a very interesting part of arithmetic, one that is of great practical value. You will be interested to learn more of the uses of arithmetic in everyday life. You will see how extensively arithmetic is used in the home, the store, the shop, the office, the bank, about the mine, and on the farm. The knowledge that you acquire from this book will be of value to you throughout your life.

In each of the vocations and professions some knowledge of arithmetic is necessary, and one who has not acquired a high degree of accuracy and a reasonable degree of speed in computation is usually at a great disadvantage.

If wisely used, the drill exercises in this book will be of value to you because they will enable you to increase your speed and accuracy. Strive to excel your own best record. Realize that you are working for yourself, and that the knowledge and skill which you acquire will be of great value to you in the years to come. Begin the study of arithmetic in the upper grades with a determination to succeed, and your efforts will be richly rewarded.

READING AND WRITING LARGE NUMBERS

When reading the daily papers and magazines you will often find it necessary to read large numbers. You should now be able to read and write such numbers. The problems on pages 2 and 3 will test your ability to do this. You will recall that, for convenience in reading, large numbers are pointed off into periods as shown in the following example:

14,297,954,860

This number is read *fourteen billion, two hundred ninety-seven million, nine hundred fifty-four thousand, eight hundred sixty.*

The period to the left of billions is called *trillions.* It is used when expressing great magnitudes, like the distance to one of the stars. For example, the distance from the earth to the Pole Star is more than 250,000,000,000,000 miles. This number is read *two hundred fifty trillion miles.*

Exercise 1

Read the following:

1. The land area of the United States in the year 1920 was 3,026,789 square miles, and the population was 105,710,620.

2. On September 30, 1923, the public debt of the United States amounted to $22,125,614,247.31.

3. The average distance from the earth to the sun is 91,430,220 miles, and the average distance of the planet Mars from the sun is 139,312,226 miles.

4. It is estimated that during a recent year 734,900,000 gallons of water were used daily in New York City.

5. The following figures indicate the amounts of the products mentioned that were raised in the United States during a recent year: Barley, 194,000,000 bushels. Potatoes, 438,000,000 bushels. Corn, 2,875,000,000 bushels. Rye, 79,600,000 bushels. Oats, 1,255,000,000 bushels. Wheat, 818,000,000 bushels.

Write the following numbers in figures:

6. The population of the world was recently estimated at one billion, seven hundred two million, two hundred thirty thousand.

7. For a recent year the average daily attendance in the public schools of the United States was reported as twenty-one million, seven hundred thirty-two thousand, three hundred forty.

8. During a recent year the United States produced thirteen million, four hundred thirty-nine thousand, six hundred three bales of raw cotton, and exported three billion, one hundred seventy-nine million, three hundred thirteen thousand, three hundred thirty-six pounds.

9. During a recent year the raw silk imported into the United States was valued at one hundred eighty-one million, eight hundred eighty-two thousand, six hundred fifteen dollars.

10. Point off and read each of the following:

	a	*b*	*c*
1.	17298	3876927	84973867
2.	34072	4803760	39487362
3.	873947	3947623	92892897
4.	$10000000	$107846080	$1480000000

11. Write in words the numbers used in the following statement: In 1920, the population of Cincinnati was 401,247; of Buffalo, 506,775; of Scranton, 137,783; of Omaha, 191,601; of Grand Rapids, 137,634; of Atlanta, 200,616; of the United States, 105,710,620.

12. Find the area of your state in square miles, and the population at the last census. Write both in figures.

13. Find the cost of the schools of your city or district for the last year. Write the cost in words.

14. Find four large numbers in a newspaper or magazine, and tell the class what each represents.

15. Write on the blackboard five numbers of at least six figures each. Read each.

The Use of *Round* Numbers

In many cases where the use of exact numbers is not necessary, it is customary to express large amounts in *round* numbers. Round numbers may be expressed to the nearest hundred, thousand, ten thousand, and so on, as required.

For example: Instead of saying that the value of the dairy products of Wisconsin during a recent year was $180,306,599, we may state this amount to the nearest *hundred thousand* and call it $180,300,000.

Exercise 2

1. The distance from the planet Jupiter to the sun is 475,693,149 miles. State this distance to the nearest 1,000,000 miles.

2. Express each of the following numbers to the nearest 1000:

State	Population in 1920	State	Population in 1920
Alabama	2,348,174	Massachusetts	3,852,356
Colorado	939,629	Missouri	3,404,055
Connecticut	1,380,631	Ohio	5,759,394
Florida	968,470	Pennsylvania	8,720,017
Georgia	2,895,832	Texas	4,663,228
Illinois	6,485,280	Utah	449,396
Iowa	2,404,021	Washington	1,356,621

3. Express each of the following to the nearest $1000:

Year	Expenditures of the United States
1919	$35,152,211,464
1920	23,596,911,806
1921	14,113,978,775

4. During a recent year the United States produced 7,953,641 bales of raw cotton. Of this amount 3,339,113,489 pounds were exported. Express these amounts to the nearest 1000.

5. During a certain year the world's production of sugar was 36,827,854,240 pounds. Express this amount to the nearest 100,000 pounds.

MAKING AND READING A GRAPH

Walter's record on a test, in which he tried to do 10 addition examples within a certain time limit, was as follows: On the first trial he got 6 correct answers out of 10. On the second trial he got 8 correct answers out of 10. On the third trial he got 7, on the fourth trial 8, and on the fifth trial 9.

Walter's record is shown by the diagram in Fig. 1. Such a diagram is called a *graph*. By means of this graph you can at once note the progress that Walter made throughout the series of five tests.

By observing this graph, can you tell how many examples Walter solved correctly on his third trial? On his second trial? On his fourth trial? Can you show how this graph would have appeared if Walter's record on the five tests had been 5, 7, 5, 8, 9?

As shown by this graph, the figures at the bottom indicate the number of trials. Those along the left-hand side indicate the number of examples worked correctly.

You will find it interesting and profitable to make a graph of your record on each of the *Drill Exercises* on pages 7 to 16, inclusive. These graphs should be kept so that you may note the progress you make from time to time throughout the year.

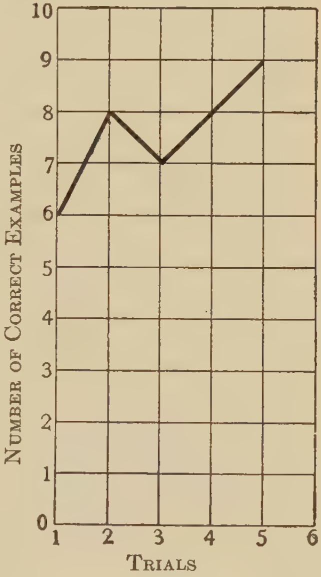


Fig. 1

It is evident that certain number relations may easily be seen when the number facts are expressed as a graph. Graphs of various kinds are now extensively used in many practical ways. You can find examples of different kinds of graphs in almost any newspaper or magazine and in many books.

Note to Teacher: If desired, this work may be omitted at this time, or taught and used for subsequent drill, as the needs of the class demand.

Fig. 2 is a graph of Helen's record on each trial. How many trials did she make?

Use this kind of graph to show your own records on *Drill Exercises 3* and *4*.

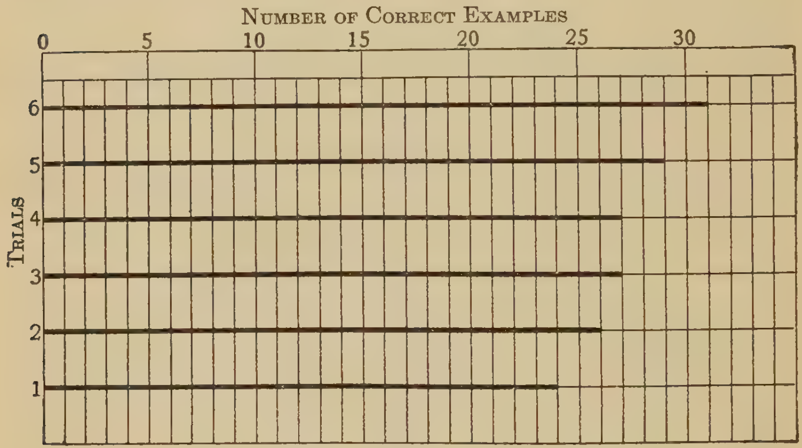


Fig. 2

How many examples did Helen work correctly on the first trial? On the fifth trial? On the third trial? On the sixth trial?

On which trial did Helen work the greatest number correctly? On which trial did she work the smallest number correctly?

How many more examples did Helen work correctly on the fifth trial than on the first trial?

Show on the illustration (Fig. 2) where the line would end if Helen had worked 26 examples correctly on the fifth trial. Where would the line end if she had worked 32 examples correctly on the fourth trial?

Which of the two graphs shown do you think most clearly shows the record of examples correctly worked? Explain why.

Place your pencil on Fig. 2 to show where each of the six lines would end if Helen had correctly worked the following number of examples: 21, 24, 28, 26, 27, 30.

DRILL EXERCISES

The following drill exercises should be used at frequent intervals throughout the year, for practice on the more common combinations that you have already studied. Some pupils of your grade can correctly give the answers to all the examples in each of the exercises from 1 to 5 inclusive, in two minutes. See how many you can finish in that time. If you cannot come up to that record, practice until you are able to do so.

Drill Exercise 1: Addition

1	4	5	6	4	7	5	9	8	7	6
7	3	0	5	2	8	7	4	6	9	9
5	6	3	1	3	2	3	1	3	0	5

5	3	8	7	3	2	3	1	2	4	3
9	8	8	9	6	4	9	7	0	7	8
4	7	5	6	2	5	7	5	9	4	6

5	9	3	1	4	6	5	8	7	8	3
5	4	0	9	3	8	4	3	9	5	8
7	7	8	6	7	9	7	4	3	9	0

Drill Exercise 2: Subtraction

39	91	46	75	29	43	24	74	36	45
8	6	3	7	7	8	5	8	7	9

72	87	96	73	14	65	78	51	37	76
5	4	5	6	7	2	5	6	8	8

27	39	41	32	48	92	65	88	33	47
3	8	4	7	6	8	7	7	5	3

16	17	16	36	18	15	13	19	14	15
9	8	7	8	9	8	6	5	9	6

Drill Exercise 3: Multiplication

74	37	36	48	27	46	52	74	38	54
5	4	3	2	9	6	7	8	9	3
23	25	42	35	79	56	88	73	29	87
6	8	9	7	3	5	4	6	2	3

Drill Exercise 4: Division

$4\overline{)32}$	$9\overline{)54}$	$7\overline{)84}$	$8\overline{)56}$	$9\overline{)72}$	$8\overline{)32}$	$6\overline{)48}$	$6\overline{)66}$
$6\overline{)36}$	$3\overline{)18}$	$3\overline{)36}$	$3\overline{)33}$	$7\overline{)21}$	$5\overline{)40}$	$7\overline{)28}$	$3\overline{)15}$
$8\overline{)40}$	$4\overline{)36}$	$5\overline{)45}$	$7\overline{)63}$	$9\overline{)81}$	$8\overline{)72}$	$3\overline{)30}$	$5\overline{)35}$
$5\overline{)50}$	$8\overline{)96}$	$8\overline{)80}$	$2\overline{)22}$	$5\overline{)55}$	$7\overline{)77}$	$7\overline{)42}$	$4\overline{)44}$
$5\overline{)20}$	$9\overline{)90}$	$7\overline{)56}$	$4\overline{)8}$	$5\overline{)25}$	$6\overline{)54}$	$4\overline{)28}$	$8\overline{)64}$
$3\overline{)27}$	$7\overline{)77}$	$4\overline{)24}$	$9\overline{)99}$	$7\overline{)49}$	$9\overline{)45}$	$9\overline{)63}$	$8\overline{)48}$

Drill Exercise 5: Addition

47	74	49	35	24	40	38
13	27	53	70	36	54	24
82	63	81	11	83	62	17
34	92	80	76	79	37	92
27	73	41	43	92	46	47
81	65	73	82	81	63	38

Drill Exercise 6: Multiplication

342	473	207	746	832	745
5	6	8	7	9	2
342	776	483	924	708	943
3	5	7	6	8	9

Drill Exercise 7: Addition

3	4	9	2	8	8	4	9
8	9	6	9	8	4	1	6
9	3	3	5	3	0	0	5
7	1	8	3	4	9	3	3
6	7	7	8	6	2	9	8
8	5	4	7	2	7	6	2

5	4	7	3	5	7	6	3
5	6	5	9	4	7	9	1
3	3	3	7	7	4	3	0
0	8	6	8	8	8	8	9
1	7	4	6	3	3	4	7
2	9	2	1	6	9	5	8

Drill Exercise 8: Subtraction

37	73	45	94	56	96	24
19	26	28	67	48	47	18

36	88	50	72	64	80	31
18	39	14	28	36	27	23

93	46	29	60	77	97	86
67	13	12	24	28	68	77

Drill Exercise 9: Addition

37	73	70	29	83	99	42
84	96	84	76	74	81	73
21	14	35	92	31	76	38
34	87	96	37	97	65	86

80	36	73	91	30	81	36
41	70	86	72	72	47	42
72	91	40	35	99	68	37
98	47	68	83	43	76	98

Drill Exercise 10: Multiplication

324	746	864	481
15	23	27	36

Drill Exercise 11: Subtraction

483	785	407	873	736	542
269	392	269	545	289	387
872	796	423	400	805	340
39	78	96	72	78	165

Drill Exercise 12: Division

32)768	74)6142	24)360
27)2241	56)4256	81)7047

Drill Exercise 13: Subtraction

764	964	586	932	821	948
398	787	148	477	646	379
807	378	403	500	487	876
469	219	268	326	279	298

Drill Exercise 14: Multiplication

736	356	785	227	496	748	507
497	243	142	236	357	469	369

Drill Exercise 15: Addition

384	487	473	384	829	924	876
732	326	739	742	735	378	739
563	974	807	906	314	692	504
827	832	667	311	709	739	837

Drill Exercise 16: Difficult Combinations

This list contains some difficult addition combinations. Find as many correct results as you can in fifteen seconds. Make six trials at different times, and graph the records of your trials.

3	8	7	8	5	7	9	5	4
8	9	8	4	7	9	9	3	8

3	4	6	7	3	5	6	5	6	8
5	9	8	7	7	9	5	8	7	8

Drill Exercise 17: Addition

Find the correct sum for as many of these examples as you can in four minutes.

427	398	329	742	695	209
836	724	637	306	708	394
370	680	842	792	467	726
542	392	739	844	382	385
721	786	548	397	564	279
837	349	673	186	923	547
396	192	924	409	865	983
850	748	300	573	318	746

Drill Exercise 18: Addition

Find the correct result for as many of these examples in addition as you can finish in three minutes.

3	4	3	9	5	7	3	7	9	3	4
7	7	6	6	4	8	6	5	7	7	7
8	2	5	5	2	9	4	8	6	9	9

3	4	7	8	3	4	5	9	1	9	3
2	6	8	4	5	8	6	2	3	8	5
5	9	3	7	9	3	9	7	8	6	7
8	5	4	6	8	7	9	6	5	7	8

Drill Exercise 19: Subtraction

Find the correct result for as many of these examples in subtraction as you can finish in one minute.

14	12	10	14	11	10	13	12	11	15
9	5	8	5	4	1	5	7	8	9
11	18	13	10	13	13	16	12	10	14
5	9	6	7	8	7	9	4	3	7
11	12	15	10	15	10	13	10	12	11
6	9	6	5	7	2	9	4	8	7
17	14	16	17	11	18	12	14	15	11
9	8	7	8	2	9	3	6	8	3
10	16	10	12	19	18	16	17	19	15
9	8	6	6	6	5	9	8	7	4

Drill Exercise 20: Addition

Find the correct result for as many of these examples in addition as you can finish in two minutes.

5	4	9	7	6	9	4	8	4	7	4	8
3	7	2	3	5	8	3	5	9	7	9	9
8	6	4	8	7	2	6	3	2	8	2	8
3	7	5	7	8	9	8	3	9	6	9	5
5	6	5	7	8	7	9	7	2	5	9	6
2	8	4	2	8	4	6	7	6	6	7	4

Drill Exercise 21: Subtraction

Find the correct result of as many of these subtraction examples as you can finish in two minutes.

743857	276830	837064	350046	792445
271463	129674	562873	297384	35716

Drill Exercise 22: Addition

Find the correct result for as many of these examples in addition as you can finish in five minutes. Make five trials at different times, and graph the records of your trials.

4	9	3	8	2	5	3	6	9	9
9	3	8	3	5	2	9	5	7	3
3	8	9	7	3	3	7	9	8	7
6	7	7	9	7	7	4	3	4	4
8	6	8	4	4	9	8	4	3	6
2	2	4	6	6	6	5	7	6	5
7	1	5	5	2	8	9	8	7	2
5	9	3	7	9	5	2	2	2	1
7	5	6	2	8	1	7	6	9	7
3	7	2	8	3	7	4	7	7	3
5	8	9	7	7	4	6	9	8	1
4	3	7	6	9	2	3	5	9	8

Drill Exercise 23: Division

Find the correct quotients, and remainders, if any, for as many of these examples as you can finish within the time limit stated.

GROUP I. (*Time after copying: 4 minutes.*)

$10366 \div 142$ $17658 \div 54$ $25172 \div 406$ $238640 \div 785$

GROUP II. (*Time after copying: 5 minutes.*)

$728592 \div 86$ $18330 \div 47$ $322344 \div 792$ $316968 \div 843$

Drill Exercise 24: Miscellaneous

Find the correct result for as many of these examples as you can finish in two minutes.

$\frac{1}{2} + \frac{1}{3}$

$\frac{7}{8} \div \frac{7}{12}$

$\frac{2}{3} \div \frac{3}{4}$

$\frac{1}{5} + \frac{1}{6}$

$24 \times \frac{2}{3}$

$\frac{3}{8}$ of 32

$\frac{5}{6} + \frac{1}{4}$

$\frac{7}{8} - \frac{2}{3}$

$\frac{3}{4} \div \frac{3}{16}$

$\frac{2}{3}$ of 42

$\frac{9}{16} \times \frac{1}{15}$

$12\frac{1}{2} \times \frac{3}{5}$

Drill Exercise 25: Addition

Find the correct result for as many of these examples in addition as you can finish in four minutes.

473	947	242	794	379	759
752	682	904	638	982	837
864	736	769	792	746	406
973	549	873	927	397	395
827	604	546	853	294	783
596	790	729	538	943	942
789	587	837	946	728	687
508	978	546	327	962	450
375	543	728	588	547	981

Drill Exercise 26: Multiplication

Find the correct result for as many of these multiplication examples as you can finish within the time limit stated.

Group I. (*Time after copying: 1 minute.*)

4796	3406	9027	9564	3047
5	4	6	7	8

Group II. (*Time after copying: 4 minutes.*)

1426	4972	3720	764
384	947	243	305

Group III. (*Time after copying: 5 minutes.*)

7486	4760	3649	2476	5275
305	720	706	430	3040

Group IV. (*Time after copying: 5 minutes.*)

708	490	965	438	987
436	906	734	870	652

Drill Exercise 27

Find the correct result for as many of these examples as you can finish in four minutes.

$\frac{1}{2}+\frac{2}{3}$	$\frac{2}{5}-\frac{2}{15}$	$\frac{5}{6}\times\frac{8}{15}$
$\frac{2}{3}\times\frac{3}{4}$	$\frac{3}{8}\div\frac{9}{16}$	$\frac{2}{3}\div\frac{3}{2}$
$\frac{5}{6}-\frac{1}{3}$	$\frac{2}{3}+\frac{3}{4}$	$\frac{4}{5}+\frac{2}{3}$
$\frac{2}{5}\div\frac{1}{6}$	$\frac{3}{5}\div\frac{9}{10}$	$\frac{7}{8}-\frac{3}{4}$

Drill Exercise 28: Addition

Keep your record of each trial on the following addition examples, and try from time to time to lower earlier records. Make a graph of your scores.

$436.82	$6350.47	$9054.36	$7096.81
751.60	2769.83	7563.87	1749.63
375.47	7046.70	5271.04	7248.79
362.89	9583.69	9386.75	2432.07
704.35	6798.03	7247.93	6387.54
287.16	7054.96	8762.74	9268.73
782.94	8788.75	9080.90	5746.05

Drill Exercise 29: Addition

Do not copy these addition examples, but work as many of them as you can in three minutes. Try this exercise daily, and graph your records for a week. Begin each day's work with a different example. Do not keep the answers of previous days.

3275	7183	9847	7947	8976	4592
4283	8074	3068	3682	7432	3618
7306	9367	9876	9574	8057	7543
9472	4792	5285	3865	7392	8574
8376	8075	4793	1794	8546	9265
5072	3692	6050	2837	3781	7104
3072	4208	7047	9470	2431	9046
9216	7326	3968	3682	9070	3857

Drill Exercise 30: Decimal Fractions

In the examples of this and the following exercises, the decimal points have not been placed in the results. Copy only the answers on a sheet of paper and place the decimal point properly in each.

$.14 \times 2.7 = 378$

$24 \times .032 = 768$

$.04 \times .05 = 20$

$.324 \times 6.2 = 20088$

$.02 \times .002 = 4$

$1.5 \times .015 = 225$

$400 \times .004 = 16$

$35 \times 35 = 1225$

$.035 \times .35 = 1225$

$2.6 \times 47.5 = 12350$

$.007 \times .007 = 49$

$.08 \times 800 = 64$

$9.5 \times .5 = 475$

$.082 \times 8.2 = 6724$

$.45 \times .45 = 2025$

$7 \times .43 = 301$

$.25 \times 25 = 625$

$250 \times 2.5 = 6250$

$.001 \times .001 = 1$

$.0014 \times 1.4 = 196$

Drill Exercise 31: Division of Decimals

(See instructions for *Drill Exercise 30*, above.)

$$\begin{array}{r} 246 \\ 42.3\overline{)1040.58} \end{array}$$

$$\begin{array}{r} 246 \\ 4.23\overline{)10405.8} \end{array}$$

$$\begin{array}{r} 246 \\ .423\overline{)1040.58} \end{array}$$

$$\begin{array}{r} 246 \\ 42.3\overline{)10.4058} \end{array}$$

$$\begin{array}{r} 246 \\ 4.23\overline{)10.4058} \end{array}$$

$$\begin{array}{r} 246 \\ 4.23\overline{)1.04058} \end{array}$$

$$\begin{array}{r} 246 \\ 423\overline{)104.058} \end{array}$$

$$\begin{array}{r} 246 \\ 423\overline{)10.4058} \end{array}$$

$$\begin{array}{r} 246 \\ 423\overline{).104058} \end{array}$$

$$\begin{array}{r} 246 \\ 42.3\overline{)104.058} \end{array}$$

Drill Exercise 32: Addition

It is sometimes desirable to make horizontal additions to save the time of copying figures. This and the exercises that follow will furnish practice in that kind of addition.

The table below shows the number of admissions to a circus for a period of five days. Find the total admissions of each kind for each day and for the week. Check the totals.

	Monday	Tuesday	Wednesday	Thursday	Friday	Totals
Men..............	1423	2108	2037	2690	2846	....
Women...........	708	1315	1409	1927	2109	....
Children.........	387	942	1165	2109	2364	....
Totals..........	....	....	....	....	....	....

Drill Exercise 33: Addition

The following table shows the school attendance of a certain city for one week. Find the total attendance of each grade for the week, the total of all grades for each day, and the grand total for the week. Check the totals.

	Monday	Tuesday	Wednesday	Thursday	Friday	Totals
1st Grade.......	414	321	433	417	412	....
2nd Grade.......	401	396	400	391	402	....
3rd Grade.......	382	387	374	375	370	....
4th Grade.......	431	337	330	343	342	....
5th Grade.......	308	311	302	316	309	....
6th Grade.......	290	278	293	281	287	....
7th Grade.......	203	209	200	220	216	....
8th Grade.......	186	180	191	174	179	....
Totals..........	....	....	....	....	....	....

Drill Exercise 34: Addition

The following table shows the figures from a family expense book for one year. Find the total of each item for the year, the total of all the items for each month, and the grand total for the year.

1927	Rent	Groceries	Milk	Meat	Laundry	Clothing	Light	Telephone	Incidentals	Totals
Jan....	$42	$27.80	$6.00	$11.40	$4.65	$50.60	$3.90	$1.50	$17.50	......
Feb....	42	26.35	6.20	11.85	4.90	22.35	4.60	1.50	18.75	......
Mar...	42	31.18	6.20	10.60	3.85	10.90	3.35	2.10	12.40	......
April..	42	24.07	5.80	11.45	5.40	38.60	3.20	1.50	26.35	......
May...	42	28.45	6.00	10.25	4.60	7.40	2.65	2.40	14.80	......
June...	42	24.13	5.40	9.60	3.95	18.20	2.15	1.60	21.15	......
July...	42	22.40	6.40	7.40	3.60	5.65	2.10	1.75	24.50	......
Aug...	42	24.85	6.00	7.25	4.20	17.20	2.30	1.50	15.70	......
Sept...	42	25.60	6.20	8.15	5.30	38.90	2.90	1.50	8.50	......
Oct....	42	28.35	6.60	8.65	3.45	62.40	2.95	2.00	13.90	......
Nov...	42	29.90	6.00	10.20	3.80	9.45	3.60	1.60	16.00	......
Dec...	42	33.85	6.10	11.35	4.10	16.30	3.80	1.70	17.40	......
Totals										

Drill Exercise 35: Addition

The seven clerks of Anderson & Company reported daily sales for a week as shown by the table below. Find each day's total sales, each clerk's total sales for the week, and the sales made by all the clerks in a week.

Clerk	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
J. B. Williams	$194.65	$208.34	$228.50	$164.80	$209.40	$356.40	......
R. M. Olson..	180.56	307.10	292.38	264.85	310.75	207.43	......
C. H. Barr...	218.95	204.35	384.75	319.05	196.84	109.48	......
F. M. Smith...	341.86	196.25	296.30	403.15	321.65	207.52	......
E. R. Evans..	138.47	203.52	341.35	218.73	243.84	196.48	......
M. V. Locke..	257.19	362.78	199.21	416.30	251.74	165.95	......
S. T. Hinds...	112.21	213.56	101.75	163.43	98.20	211.11	......
Totals.......							

SAVING TIME WHEN MAKING COMPUTATIONS

If you were asked to find the cost of four pounds of butter at twenty-five cents a pound, it would be absurd for you to make the computation on paper, because you can give the answer immediately by computing "in your head," as we sometimes say. Some people call this kind of computation *mental arithmetic*. By using mental arithmetic many problems are daily solved without paper or pencil.

You are already familiar with some of the short methods of computation in common use. A multiplication table is a short method. A knowledge of the tables enables you to make multiplications more quickly than you could obtain the results by addition.

In the lessons that follow you are to learn some new short methods of computation, and also how to apply them practically. Learn these thoroughly, and accustom yourself to working with them whenever you can save time by doing so. You will find that some of these short methods have been placed here for review. Even so, study them carefully and learn them well, for they will be of use to you not only during the remainder of your school years, but throughout life.

Short Methods in Multiplication

I. To multiply by 10, move the decimal point one place to the right, annexing a zero if necessary.

For example: $10 \times 84 = 840$ $10 \times 3.64 = 36.4$

II. To multiply by 100, move the decimal point two places to the right, annexing two zeros if necessary.

For example: $100 \times 32 = 3200$ $100 \times 3.784 = 378.4$

III. To multiply by 1000, move the decimal point three places to the right, annexing three zeros if necessary.

For example: $1000 \times 74 = 74,000$ $1000 \times .3568 = 356.8$

IV. To multiply an integer by a number ending in zero, multiply the integer by the number or numbers to the left of the zero, and then annex a zero to this product.

For example: 325×40=? First multiply 325 by 4. This gives 1300. Annex a zero to this number and obtain 13,000, the result.

V. To multiply a number by 5, multiply by 10, then divide this result by 2.

For example: $5\times784=\frac{7840}{2}=3920$

VI. To multiply a number by 25, multiply by 100, then divide this result by 4.

For example: $25\times73=\frac{7300}{4}=1825$

VII. To multiply a number by 125, multiply by 1000, then divide this result by 8.

For example: $125\times67=\frac{67000}{8}=8375$

VIII. To multiply a number by 9, multiply by 10, then subtract the multiplicand.

For example: $9\times492=4920-492=4428$

IX. To multiply a number by 33⅓, multiply by 100, then divide this result by 3.

For example: $33\frac{1}{3}\times78=\frac{7800}{3}=2600$

X. To multiply a number by 66⅔, multiply by 100, then take ⅔ of this result.

For example: $66\frac{2}{3}\times36=3600\times\frac{2}{3}=2400$

Exercise 3

1. Without the use of paper or pencil, multiply each of the numbers in this exercise by 10 by 100, by 5, by 25, and by 125.

	a	*b*	*c*
1.	376	9656	.072
2.	480	14.024	968
3.	26.32	4.64	1.44
4.	484.72	900.8	36.8

2. Without the use of paper or pencil, multiply each of the numbers in this exercise by $33\frac{1}{3}$, by $66\frac{2}{3}$, and by 9.

1.	279	73.06	881
2.	9846	164.7	140.76
3.	16408	9.54	39.69

Solve each of the following orally:

3. Twenty-five dozen eggs at $.36 a dozen.

4. One hundred yards of cloth at $$.37\frac{1}{2}$ a yard.

5. Twenty-five golf clubs at $4.80 each.

6. Ten dozen oranges at $.47 a dozen.

7. Ten bunches of celery at $.18 a bunch.

8. Twenty-five pounds of rice at $.11 a pound.

9. One hundred twenty-five cords of wood at $7.20 a cord.

10. A boy's wages for 12 months at $$33\frac{1}{3}$ a month.

11. A boy's wages for 9 months at $$66\frac{2}{3}$ a month.

12. A man's wages for 6 days of 8 hours each at $$.66\frac{2}{3}$ an hour.

13. One hundred dozen eggs at $.34 a dozen.

14. Show why each of the short methods you have just used gives the correct result.

15. Give three examples of each of the ten short methods in multiplication stated on pages 19 and 20.

Short Methods in Multiplication (*Continued*)

XI. To multiply a number by $12\frac{1}{2}$, first multiply by 100, then divide this result by 8.

For example: $12\frac{1}{2} \times 48 = \frac{1}{8}$ of $4800 = 600$

Suggest a short method for multiplying a number by 50. By $16\frac{2}{3}$. By $37\frac{1}{2}$. By $62\frac{1}{2}$. By $87\frac{1}{2}$. Give three illustrations of each method you work out.

XII. To multiply a number by 250, first multiply by 1000, then divide this result by 4.

For example: $250 \times 328 = \frac{1}{4}$ of $328{,}000 = 82{,}000$

Suggest a short method for multiplying a number by 500. By $333\frac{1}{3}$. By $666\frac{2}{3}$. By 750. Give three illustrations of each method you work out.

Exercise 4

1. Multiply the numbers in column *a* by $12\frac{1}{2}$, those in column *b* by $16\frac{2}{3}$, those in column *c* by 250, those in column *d* by 25, and those in column *e* by $33\frac{1}{3}$.

	a	*b*	*c*	*d*	*e*
1.	464	360	78	484	729
2.	384	612	420	388	468
3.	9672	486	364	204	324
4.	4800	354	792	760	864
5.	32.08	270	4.4	79232	936
6.	4240	42.6	980	49.6	522

2. Find the cost of 24 yards of cloth at $\$.12\frac{1}{2}$ a yard.

3. Find the cost of 25 yards of cloth at $1.30 a yard. At $2.25 a yard. At $3.60 a yard. At $.72 a yard.

4. Find the cost of 10 yards of cloth at $.42 a yard. At $.62 a yard. At $1.80 a yard. At $3.20 a yard.

5. Find the cost of $12\frac{1}{2}$ yards of cloth at $.32 a yard. At $.48 a yard. At $1.20 a yard. At $2.40 a yard.

Find the cost of each of the following:

6. Twenty-five dozen pencils at $.32 a dozen.

7. Thirty-six chairs at $25 each.

8. Seventy-five dozen erasers at $.44 a dozen.

9. Sixteen and two-thirds dozen rulers at $.36 a dozen.

10. Twenty-four yards of cloth at $.62½ a yard.

11. One hundred twenty-five chickens at $.85 each.

12. Sixteen yards of cloth at $.37½ a yard.

Short Methods in Addition

The method explained below is convenient to use for adding two numbers.

For example: 46+32=? *Think:* 46+30=76. 76+2=78.

The example shown below indicates a simple way to add three-figure numbers that end in zero.

For example: $4.20+$3.90=? *Think:* $4.20+$3.00=$7.20. $7.20+$.90 =$8.10.

Exercise 5

1. Make the following additions without using pencil or paper.

	a	*b*	*c*	*d*
1.	36+27	324+246	$4.80+$2.60	$3.90+$4.60
2.	49+88	740+360	$5.30+$2.80	$2.30+$4.50
3.	32+74	880+130	$9.40+$3.25	$7.60+$5.40
4.	93+72	365+ 65	$7.15+$3.20	$9.10+$4.35

2. There are 362 girls and 354 boys in a school building. What is the total number of pupils?

3. Mrs. Evans spent the following amounts while shopping: $3.45 for dress goods, $2.20 for groceries, $1.60 for a book. How much did she spend altogether?

4. Four boys weigh as follows: 98 lb., 105 lb., 108 lb., and 113 lb. What is their total weight?

Short Methods in Division

I. To divide by 10, move the decimal point one place to the left, prefixing a zero if necessary.

For example: $794 \div 10 = 79.4$ $.378 \div 10 = .0378$

II. To divide by 100, move the decimal point two places to the left, prefixing two zeros if necessary.

For example: $3840 \div 100 = 38.4$ $.437 \div 100 = .00437$

III. To divide by 1000, move the decimal point three places to the left, prefixing three zeros if necessary.

For example: $42{,}760 \div 1000 = 42.76$ $37 \div 1000 = .037$

IV. To divide by 5, multiply by 2 and divide by 10.

For example: $785 \div 5$ $785 \times 2 = 1570$ $1570 \div 10 = 157$

V. To divide by 25, multiply by 4 and divide by 100.

For example: $475 \div 25$ $475 \times 4 = 1900$ $1900 \div 100 = 19$

VI. To divide by 125, multiply by 8, and divide by 1000.

For example: $1750 \div 125$ $1750 \times 8 = 14{,}000$ $14{,}000 \div 1000 = 14$

VII. To divide by $12\frac{1}{2}$, multiply by 8 and divide by 100.

For example: $225 \div 12\frac{1}{2}$ $225 \times 8 = 1800$ $1800 \div 100 = 18$

VIII. To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.

For example: $800 \div 33\frac{1}{3}$ $800 \times 3 = 2400$ $2400 \div 100 = 24$

IX. To divide by $16\frac{2}{3}$, multiply by 6 and divide by 100.

For example: $250 \div 16\frac{2}{3}$ $250 \times 6 = 1500$ $1500 \div 100 = 15$

X. To divide by $66\frac{2}{3}$, multiply by 3 and divide by 200.

For example: $1200 \div 66\frac{2}{3}$ $1200 \times 3 = 3600$ $3600 \div 200 = 18$

Exercise 6

1. Perform the following divisions without pencil or paper.

	a	*b*	*c*	*d*
1.	$48 \div 33\frac{1}{3}$	$850 \div 25$	$95 \div 25$	$475 \div 25$
2.	$72 \div 12\frac{1}{2}$	$340 \div 66\frac{2}{3}$	$38 \div 125$	$780 \div 12\frac{1}{2}$
3.	$375 \div 125$	$43 \div 16\frac{2}{3}$	$48 \div 12\frac{1}{2}$	$35 \div 16\frac{2}{3}$

2. At $.16$\frac{2}{3}$ a yard, how many yards of cloth can be bought for $1.50? For $8?

3. An agent sold a farm for $20,000, at the rate of $125 an acre. How many acres did he sell?

4. At $.25 a yard, how many yards of cloth can be bought for $7.25? For $37.50? For $18.75? For $72.25?

5. At $.33$\frac{1}{3}$ a dozen, how many eggs can be bought for $7? For $52? For $75?

6. At $.66$\frac{2}{3}$ a gross, how many gross of pens can be bought for $12? For $18?

7. At the price of $.12$\frac{1}{2}$ a yard, how many yards of gingham can you buy for $3? For $7? For $11? For $8?

8. Show why each of the ten short methods in division stated on page 24 gives the correct result.

9. Give three examples of each of the short methods of division.

10. Carl says that when he has to divide a number by 50, usually he first multiplies the number by 2 and then divides the result by 100. You show whether or not Carl's plan gives correct results. Explain your answer.

11. You know that $37\frac{1}{2}$ is $\frac{3}{8}$ of 100. Can you suggest a short method to use when you have to divide a number by $37\frac{1}{2}$?

12. A new clerk finds that calico sells at $8\frac{1}{3}$ cents a yard. He wants to work out a short method to use when finding how many yards of this calico are to be sold for a certain number of dollars. Can you suggest a method?

HOW TO SOLVE A PROBLEM

You have now studied addition, subtraction, multiplication, and division of integers, of common fractions, and of decimal fractions. You are familiar with the measurement tables in common use in this country, and you know some of the practical short methods of computation. You can compute with some degree of speed and accuracy. It is now desirable that you learn further how to apply this knowledge to the solution of problems that arise at home, on the farm, about the mine, in the shop, store, bank, and office. This is the true test of your knowledge of arithmetic.

It is important that you develop the ability to *analyze* problems. Those whose daily work requires the solution of many problems must always think out the analyses, even though they do not write them out. In the problems that follow throughout this book you are asked to think out the analysis of each, and often to write it out. Such work will give you needed practice that will help you to solve the problems that arise when you later enter upon your household duties, your trade, or your profession.

Other things besides analysis are necessary if you are to solve the problems you meet. Your computations must be rapid and accurate; otherwise your work is of no value. Last, but very important, your figures must be neatly made and legible. Figures made carelessly cause a loss of time and money to yourself and to your employer. You may be willing to stand that loss, but you may be sure you will not find an employer who is willing to do so.

Below you will find a suggested method for *attacking*, that is, for *analyzing* a problem. You should study this carefully, and learn how to apply the method to other problems. Although there is no one best form of analysis, the one given here is a very good guide as to what an analysis should be. Study the statement of each problem thoroughly, think the solution through carefully, and do as much of the work as you can without pencil or paper. At the beginning, it is profitable to indicate the solution of every problem before you make your computations.

TO SOLVE A PROBLEM IN ARITHMETIC

I. Read the problem carefully so that you may understand
  ***a*) the facts that are stated or suggested,**
  ***b*) what you are to find.**

II. Decide upon the operations that are to be used.

III. Make the necessary computations accurately.

IV. Check the result.

THE SOLUTION OF A PROBLEM

Find the cost of $4\frac{3}{4}$ yards of cloth at 56 cents a yard.

I. (*a*) The facts stated:	The number of yards of cloth bought. The price per yard.
(*b*) What you are to find:	The total cost of the cloth.
II. The operation to be used:	Multiplication.
III. Computation:	$4\frac{3}{4}=\frac{19}{4}$ $\frac{19}{4}\times\$.56=\$2.66$
IV. Check the result:	Do this by making the computation a second time.

When a dozen lemons sell for 54 cents, how many can be bought for 36 cents?

I. (*a*) The facts stated:	The selling price per dozen.
(*b*) What you are to find:	The number that can be bought for 36 cents.
II. The operation to be used:	Division.
III. Computation:	$\frac{1}{12}$ of $\$.54=\$.045$, the cost of 1 lemon. $\$.36\div\$.045=8$, the number bought.
IV. Check the result:	Do this by making the computation a second time.

NOTE TO PUPIL: In most business transactions it is customary to consider fractions of a cent amounting to $\frac{1}{2}$ or more as an extra cent. Thus, $\$.45\frac{1}{2}$ is considered as \$.46. If the fraction is less than $\frac{1}{2}$, it is usually discarded in the final result.

The Use of Cancellation

When the solution is indicated, time may frequently be saved by crossing out the same factor from both dividend and divisor. Crossing out a common factor is called *canceling.* You are already familiar with the principle of cancellation from your study of common fractions, in which you learned that both the numerator and the denominator of a fraction may be divided by the same number without changing the value of the fraction. The solution of the problem below illustrates the use of cancellation.

If 12 small radio sets cost a merchant $168, what will 8 sets cost him at the same rate?

Solution: $\dfrac{\overset{56}{\cancel{168}}\times\overset{2}{\cancel{8}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{12}}}=112$, the number of dollars that 8 sets cost.

The multiplication and division necessary to solve this problem may be shown as illustrated above. Since both 8 and 12 are divisible by 4, the 8 and the 12 are canceled when we divide by 4, thus giving 2 and 3. Since both 168 and 3 are divisible by 3, we divide both by 3, thus giving 56 and 1. 2×56 =112, the number of dollars that 8 sets cost.

Exercise 7

In solving the following problems, use the method of analysis suggested on page 27. Use cancellation whenever possible.

1. If a farm containing 240 acres is worth $32,400, how much should be paid for 75 acres at the same rate?

2. The daily noon temperatures for a week were: 58°, 49°, 54°, 61°, 53°, 56° and 54°. What was the average for the week?

3. A farmer traded 18 dozen eggs, at 28 cents a dozen, for sugar at 9 cents a pound. How many pounds of sugar did he receive?

4. In a laundry, 48 burners, each using 7 cubic feet of gas per hour, burn 7 hours a day for 6 days. What will be the gas bill at 90 cents a thousand cubic feet?

5. The circumference of a driving wheel of a locomotive is 22 feet. How many revolutions will this wheel make when the locomotive goes a mile?

6. How far will the locomotive mentioned in *Problem 5* travel when a driving wheel makes 1000 revolutions?

7. When sound travels 1090 feet in a second, how far will it travel in 14 seconds?

8. At the rate given in *Problem 7*, how long will it take the report of a rifle to travel 14,170 feet?

9. If it takes light about 498 seconds to come from the sun to the earth, at the rate of 186,300 miles a second, what is the distance from the earth to the sun?

10. In a recent year 13,439,603 bales of raw cotton were produced in the United States. If a bale of cotton weighs 500 pounds, what was the production in pounds?

11. The receipts of a store for a week were: Monday, $268.40, Tuesday, $313.92, Wednesday, $284.63, Thursday, $347.24, Friday, $318.69, Saturday, $423.72. What were the average daily receipts?

12. Harry's mother bought a table for $34. She paid $9 cash and the remainder at the rate of $2.50 a week. How long did it take her to pay the bill in full?

13. When soap sells at the rate of 4 bars for 25 cents, how many bars can be bought for $1?

14. If your pulse beats 72 times per minute, how many times does it beat in a day?

15. How many bushels of shelled corn are in a load containing 2352 pounds? (A bushel of shelled corn weighs 56 pounds.)

16. When children's stockings sell for $66\frac{2}{3}$ cents a pair, how many pairs can you buy for $4?

COMMON FRACTIONS

Although you have already studied common and decimal fractions, a brief review at this time will help you to increase your skill in using them, and it will also increase your ability to apply them to the solution of practical problems.

A *fraction* is one or more of the equal parts of a unit.

For example: $\frac{3}{4}$ represents 3 of the 4 equal parts of a unit.

The unit may be a single thing, as 1 apple, 1 yard, 1 pound, or it may be a group of things, as 20 cows, 100 people, 7 houses.

The two numbers used to express a fraction are called the *terms* of the fraction.

For example: 3 and 4 are the terms of the fraction $\frac{3}{4}$.

The term which states into how many equal parts the unit has been divided is called the *denominator*. Denominator means *namer*. When the fraction is written in this form, $\frac{3}{4}$, the denominator is the number written below the line.

For example: 4 is the denominator of the fraction $\frac{3}{4}$.

The term which states how many of these parts of the unit are taken to form the fraction is called the *numerator*. Numerator means *numberer*. When the fraction is written in this form, $\frac{3}{4}$, the numerator is the number written above the line.

For example: 3 is the numerator of the fraction $\frac{3}{4}$.

A fraction whose numerator is less than its denominator is called a *proper fraction*.

For example: $\frac{3}{4}$ and $\frac{2}{3}$ are proper fractions.

A fraction whose numerator is equal to or greater than its denominator is called an *improper fraction* because its value is 1 or more than 1.

For example: $\frac{7}{7}$, $\frac{9}{8}$, and $\frac{14}{3}$ are improper fractions.

Exercise 8

1. Draw a line and divide it into 12 equal parts. What is each part called? Point out $\frac{6}{12}$ of the line. $\frac{8}{12}$ of the line.

2. Draw a circle, then divide it into 8 equal parts. Point out $\frac{5}{8}$ of the circle. $\frac{3}{8}$ of the circle.

3. Draw a line and show $\frac{1}{2}$ of it. $\frac{3}{4}$ of it. $\frac{5}{8}$ of it.

4. Write five proper fractions and tell what the numerator and denominator of each indicates.

5. Which of the following fractions indicates the greatest, and which the least value: $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$? By properly marking off four lines of equal length, show that your answer is correct.

6. Arrange the following fractions in the order of their value, placing the largest first: $\frac{2}{5}$ $\frac{2}{3}$ $\frac{2}{15}$ $\frac{2}{8}$ $\frac{2}{6}$

7. Arrange the following fractions in the order of their value, placing the smallest first: $\frac{3}{12}$ $\frac{7}{12}$ $\frac{5}{12}$ $\frac{9}{12}$ $\frac{8}{12}$ $\frac{1}{12}$

8. If I write two fractions with equal numerators but unequal denominators, how can you tell at once which fraction has the greater value? Give examples.

9. If I write two fractions with equal denominators but unequal numerators, how can you tell at once which has the greater value? Give examples.

10. Write three fractions, each having 5 for a numerator, and each having a value less than $\frac{5}{8}$.

11. Write three fractions, each having 6 for a denominator, and each having a value greater than $\frac{2}{6}$.

12. Write three fractions, each of which is greater in value than $\frac{5}{12}$ and less in value than $\frac{5}{8}$.

13. Into how many equal parts is the line in Fig. 3 divided? What is each part called?

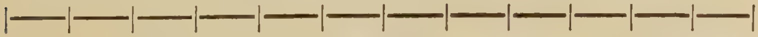


Fig. 3

How to Reduce a Fraction to Its Lowest Terms

Observe the line shown in Fig. 3, and compare $\frac{2}{3}$ of it with $\frac{8}{12}$ of it.

What is the result if you divide both terms of the fraction $\frac{8}{12}$ by 4?

Draw a line and divide it into 10 equal parts. Compare the length of $\frac{5}{10}$ of the line with $\frac{1}{2}$ of it.

What is the result if you divide both terms of the fraction $\frac{5}{10}$ by 5?

Draw two lines of equal length. Divide one of them into 4 equal parts, the other into 8 equal parts. How does the length of $\frac{3}{4}$ of one line compare with $\frac{6}{8}$ of the other?

What is the result if you multiply both terms of the fraction $\frac{3}{4}$ by 2?

The exercises above should call your attention to the following very important fact:

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

For example: Multiplying both terms of $\frac{4}{5}$ by 3, we get $\frac{12}{15}$, which is equal to $\frac{4}{5}$. Dividing both terms of $\frac{9}{12}$ by 3, we get $\frac{3}{4}$, which is equal to $\frac{9}{12}$.

When both the numerator and the denominator of a fraction are divided by the same number, the fraction is said to be *changed* or *reduced to lower terms.*

For example: Dividing both terms of $\frac{8}{12}$ by 4, we get $\frac{2}{3}$.

When both the numerator and the denominator of a fraction are multiplied by the same number, the fraction is said to be *changed* or *raised to higher terms.*

For example: Multiplying both terms of $\frac{2}{3}$ by 5, we get $\frac{10}{15}$.

A fraction may be reduced to its lowest terms by dividing both the numerator and the denominator by their greatest common divisor.

For example: $\frac{8}{12}$ may be reduced to its lowest terms by dividing both the numerator and the denominator by 4. $\frac{8}{12}=\frac{2}{3}$

Exercise 9

1. Tell which of the following fractions are not in lowest terms. Change them to lowest terms.

$\frac{3}{4}$ $\frac{2}{3}$ $\frac{5}{10}$ $\frac{7}{8}$ $\frac{12}{15}$ $\frac{8}{12}$ $\frac{9}{15}$ $\frac{6}{10}$ $\frac{16}{32}$

2. Change each of the following fractions to lowest terms.

$\frac{4}{16}$ $\frac{6}{32}$ $\frac{8}{24}$ $\frac{20}{25}$ $\frac{3}{15}$ $\frac{12}{16}$ $\frac{15}{25}$

3. Change each of the following fractions to 12ths. To 24ths.

$\frac{1}{3}$ $\frac{1}{6}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{5}{2}$ $\frac{3}{4}$ $\frac{7}{3}$ $\frac{1}{2}$ $\frac{5}{3}$

4. Change each of the following fractions to 20ths.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{3}{10}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{7}{10}$ $\frac{3}{5}$ $\frac{9}{10}$ $\frac{7}{5}$

5. Write six proper fractions in lowest terms.

6. Write six proper fractions which can be reduced to lower terms. Reduce each.

7. Write in lowest terms a fraction which equals each of the following:

$\frac{12}{16}$ $\frac{8}{24}$ $\frac{30}{50}$ $\frac{25}{75}$ $\frac{14}{70}$ $\frac{32}{64}$ $\frac{12}{32}$ $\frac{15}{12}$

8. How can a fraction be changed to lower terms?

9. When is a fraction in its lowest terms?

Mixed Numbers

A number that consists of an integer and a fraction is called a *mixed number.*

For example: $5\frac{1}{3}$ $7\frac{2}{5}$ $2\frac{3}{4}$

A mixed number is so called because it contains two kinds of units, integral units and fractional units.

A mixed number may be reduced to an improper fraction.

For example: $5\frac{1}{3}=\frac{15}{3}+\frac{1}{3}=\frac{16}{3}$ $7\frac{2}{5}=\frac{35}{5}+\frac{2}{5}+\frac{37}{5}$

A fraction may be regarded as an indicated division.

For example: $\frac{5}{2}$ may be regarded as indicating the division of 5 by 2.
$\frac{5}{2}=2\frac{1}{2}$

Exercise 10

1. Reduce the following to improper fractions.

	a	*b*	*c*	*d*	*e*
1.	$2\frac{1}{2}$	$3\frac{1}{3}$	$5\frac{1}{2}$	$4\frac{2}{3}$	$12\frac{1}{2}$
2.	$66\frac{2}{3}$	$62\frac{1}{2}$	$9\frac{1}{5}$	$5\frac{7}{8}$	$16\frac{2}{3}$
3.	$6\frac{1}{4}$	$37\frac{1}{2}$	$33\frac{1}{3}$	$7\frac{2}{5}$	$8\frac{3}{4}$

2. Reduce the following to integers or mixed numbers.

1.	$\frac{5}{3}$	$\frac{7}{2}$	$\frac{14}{3}$	$\frac{20}{5}$	$\frac{18}{8}$
2.	$\frac{24}{8}$	$\frac{91}{10}$	$\frac{72}{9}$	$\frac{49}{6}$	$\frac{37}{3}$
3.	$\frac{47}{5}$	$\frac{32}{10}$	$\frac{13}{5}$	$\frac{18}{15}$	$\frac{48}{12}$

3. Write a fraction greater in value than 2, but less than 3. Write another greater in value than 5, but less than 6.

4. Write five mixed numbers greater in value than 2 but less than 3.

5. How many integral units and how many fractional units are used to make up each of the following mixed numbers? $3\frac{2}{5}$ $4\frac{1}{2}$ $7\frac{3}{8}$

Similar Fractions

Fractions that have a common denominator are called *like fractions* or *similar fractions.*

For example: $\frac{1}{4}$ and $\frac{3}{4}$ are similar fractions.

The least number that will exactly contain two or more numbers is called the *least common multiple* of the numbers.

For example: 24 is the least common multiple of 6, 8, and 12.

You can make unlike fractions similar by finding the least common multiple of the denominators, then reducing each fraction to a fraction with this denominator.

For example: The least common multiple of the denominators of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ is 12. Reducing these fractions to twelfths, $\frac{2}{3}=\frac{8}{12}$, $\frac{3}{4}=\frac{9}{12}$, and $\frac{5}{6}=\frac{10}{12}$.

Exercise 11

1. Write five similar fractions.

2. Write three fractions and reduce them to similar fractions.

3. Write four fractions that are not similar.

4. Write two fractions that are not similar. Reduce them to similar fractions.

5. Is it possible to write two fractions that cannot be reduced to similar fractions? Why?

Addition of Fractions and Mixed Numbers

Since the size of the fractional unit is indicated by the denominator, and the number of such units is indicated by the numerator, two fractions may be added or subtracted by first expressing them with the same denominator, then adding or subtracting their numerators, as the case requires.

For example: $\frac{2}{3}+\frac{3}{4}=\frac{8}{12}+\frac{9}{12}=\frac{17}{12}=1\frac{5}{12}$

$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=\frac{6}{12}+\frac{8}{12}+\frac{9}{12}=\frac{23}{12}=1\frac{11}{12}$

Exercise 12

Make the additions indicated in the following examples, and reduce each result to its lowest terms.

	a	b	c	d
1.	$\frac{1}{2}+\frac{1}{3}$	$\frac{1}{9}+\frac{1}{10}$	$\frac{5}{12}+\frac{7}{10}$	$\frac{3}{5}+\frac{2}{3}+\frac{2}{15}$
2.	$\frac{1}{5}+\frac{1}{6}$	$\frac{1}{8}+\frac{1}{6}$	$\frac{5}{6}+\frac{3}{4}$	$\frac{4}{5}+\frac{3}{4}+\frac{3}{20}$
3.	$\frac{1}{5}+\frac{1}{8}$	$\frac{2}{3}+\frac{1}{5}$	$\frac{3}{16}+\frac{3}{8}$	$\frac{1}{4}+\frac{1}{3}+\frac{5}{6}$
4.	$\frac{1}{2}+\frac{1}{5}$	$\frac{3}{4}+\frac{1}{8}$	$\frac{4}{5}+\frac{5}{6}$	$\frac{7}{8}+\frac{3}{16}+\frac{7}{32}$
5.	$\frac{1}{6}+\frac{1}{4}$	$\frac{1}{4}+\frac{5}{8}$	$\frac{5}{8}+\frac{2}{3}$	$\frac{3}{4}+\frac{2}{3}+\frac{5}{12}$
6.	$\frac{1}{8}+\frac{1}{10}$	$\frac{2}{5}+\frac{3}{4}$	$\frac{3}{4}+\frac{5}{8}$	$\frac{5}{6}+\frac{1}{3}+\frac{3}{4}$
7.	$\frac{1}{4}+\frac{1}{6}$	$\frac{7}{8}+\frac{1}{6}$	$\frac{4}{5}+\frac{3}{4}$	$\frac{7}{8}+\frac{3}{4}+\frac{1}{2}$

Add the following mixed numbers:

	a	b	c	d	e
8.	$3\frac{2}{3}$ $4\frac{1}{4}$ $5\frac{1}{8}$ 12	10 $5\frac{3}{16}$ $4\frac{5}{8}$ $7\frac{1}{2}$	$5\frac{7}{10}$ $8\frac{2}{5}$ $4\frac{2}{15}$	$6\frac{3}{8}$ $9\frac{5}{16}$ $4\frac{3}{4}$	$4\frac{2}{3}$ $7\frac{1}{2}$ $6\frac{5}{6}$
9.	$3\frac{7}{8}$ 15 $4\frac{3}{4}$ $9\frac{1}{2}$ $8\frac{3}{8}$	$14\frac{2}{3}$ $27\frac{3}{5}$ $24\frac{7}{15}$ $6\frac{1}{5}$	$5\frac{3}{4}$ $9\frac{5}{6}$ $7\frac{2}{3}$ $8\frac{1}{12}$	$39\frac{5}{8}$ $17\frac{1}{2}$ $43\frac{3}{4}$ $17\frac{5}{6}$	$9\frac{3}{4}$ $5\frac{2}{3}$ $3\frac{5}{6}$ $4\frac{7}{12}$

10. Mary needs $\frac{3}{8}$ of a yard of organdie for the cuffs of a dress, and $1\frac{1}{2}$ yards for the neck. How much organdie should she buy?

11. Frank planted $\frac{1}{4}$ of an acre in potatoes, $\frac{3}{8}$ of an acre in strawberries, and $\frac{1}{2}$ of an acre in melons. How much land did he plant?

12. Helen's height is $58\frac{1}{2}$ inches. How tall will she be when her height has increased $3\frac{3}{8}$ inches?

13. Some boy scouts walked $7\frac{1}{2}$ miles one day, $8\frac{3}{4}$ miles the second day, $4\frac{1}{8}$ miles the third day, and $6\frac{1}{2}$ miles the fourth day. How far did they walk in the four days?

14. Harry worked $5\frac{3}{4}$ hours on Monday, $7\frac{1}{2}$ hours on Tuesday, $6\frac{2}{3}$ hours on Wednesday, and $7\frac{3}{4}$ hours on Thursday. How many hours did he work altogether?

15. A farmer hauled four loads of oats to market. The first load contained $78\frac{1}{4}$ bushels, the second $69\frac{3}{4}$ bushels, the third $74\frac{1}{2}$ bushels, and the fourth $81\frac{1}{8}$ bushels. How many bushels were in the four loads?

16. One side of a triangle is $7\frac{3}{4}$ inches, another side is $9\frac{3}{8}$ inches, and the third side is $8\frac{1}{2}$ inches. What is the sum of the sides?

17. Fred weighs $74\frac{1}{2}$ pounds, and his brother weighs $97\frac{3}{4}$ pounds. Is their combined weight as great as the weight of their father who weighs $171\frac{3}{4}$ pounds?

Subtraction of Fractions and Mixed Numbers

To subtract one fraction from another, it is usually necessary to reduce the two fractions to a common denominator. When that has been done, the numerators are subtracted to form the numerator of the result, and the difference is written over the common denominator.

For example: $\frac{3}{4}-\frac{1}{3}=?$

$$\frac{3}{4}-\frac{1}{3}=\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$$

Exercise 13

1. Write two similar fractions. Find their difference.

2. Write two fractions that are not similar, and find their difference.

3. Make the following subtractions and reduce the results to lowest terms.

	a	*b*	*c*	*d*	*e*
1.	$\frac{1}{2}-\frac{1}{3}$	$\frac{7}{8}-\frac{5}{6}$	$\frac{11}{15}-\frac{2}{3}$	$\frac{3}{4}-\frac{3}{5}$	$\frac{5}{6}-\frac{5}{12}$
2.	$\frac{1}{5}-\frac{1}{8}$	$\frac{3}{4}-\frac{2}{3}$	$\frac{5}{6}-\frac{2}{3}$	$\frac{7}{8}-\frac{4}{5}$	$\frac{4}{5}-\frac{2}{3}$
3.	$\frac{1}{6}-\frac{1}{10}$	$\frac{5}{8}-\frac{3}{5}$	$\frac{7}{6}-\frac{3}{4}$	$\frac{2}{3}-\frac{5}{8}$	$\frac{7}{16}-\frac{3}{8}$
4.	$\frac{1}{8}-\frac{1}{10}$	$\frac{7}{16}-\frac{3}{8}$	$\frac{7}{8}-\frac{2}{3}$	$\frac{3}{4}-\frac{7}{10}$	$\frac{5}{8}-\frac{3}{16}$
5.	$\frac{2}{3}-\frac{1}{6}$	$\frac{5}{6}-\frac{3}{4}$	$\frac{8}{7}-\frac{7}{8}$	$\frac{7}{16}-\frac{3}{8}$	$\frac{15}{16}-\frac{4}{5}$
6.	$\frac{3}{4}-\frac{2}{5}$	$\frac{2}{3}-\frac{5}{12}$	$\frac{3}{2}-\frac{2}{3}$	$\frac{4}{5}-\frac{3}{4}$	$\frac{4}{5}-\frac{3}{15}$

4. Subtract the following mixed numbers:

1.	$9\frac{3}{5}$ $\underline{4\frac{1}{3}}$	$16\frac{3}{4}$ $\underline{12\frac{1}{3}}$	$32\frac{5}{6}$ $\underline{18\frac{2}{3}}$	$47\frac{11}{16}$ $\underline{32\frac{5}{8}}$	$24\frac{3}{4}$ $\underline{18\frac{5}{6}}$
2.	$14\frac{1}{5}$ $\underline{9\frac{3}{5}}$	$28\frac{3}{4}$ $\underline{15\frac{2}{3}}$	$76\frac{1}{4}$ $\underline{24\frac{5}{12}}$	$36\frac{3}{8}$ $\underline{14\frac{5}{8}}$	$72\frac{7}{8}$ $\underline{29\frac{2}{3}}$
3.	42 $\underline{18\frac{2}{3}}$	74 $\underline{53\frac{5}{8}}$	$29\frac{1}{3}$ $\underline{9\frac{5}{6}}$	$43\frac{7}{8}$ $\underline{18\frac{9}{10}}$	$63\frac{2}{3}$ $\underline{46\frac{7}{8}}$

Multiplication of Fractions

Study each of the examples below, then state a rule for finding the product in any similar example.

A. $3\times\frac{2}{5}=\frac{6}{5}=1\frac{1}{5}$. How much is $4\times\frac{5}{7}$?

B. $\overset{2}{\cancel{4}}\times\frac{5}{\underset{3}{\cancel{6}}}=\frac{10}{3}=3\frac{1}{3}$. How much is $10\times\frac{7}{15}$?

C. $\frac{\cancel{3}}{\cancel{4}}\times\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}=\frac{2}{3}$. How much is $\frac{4}{5}\times\frac{15}{16}$?

D. $4\frac{1}{2}\times3\frac{1}{3}=\frac{\overset{3}{\cancel{9}}}{\cancel{2}}\times\frac{\overset{5}{\cancel{10}}}{\cancel{3}}=15$. How much is $3\frac{3}{4}\times6\frac{2}{3}$?

You will recall that $\frac{2}{3}$ of $\frac{5}{6}$ means the same as $\frac{2}{3}\times\frac{5}{6}$. In like manner, $\frac{5}{8}$ of $\frac{3}{15}$ means the same as $\frac{5}{8}\times\frac{3}{15}$.

Exercise 14

Perform the multiplications indicated below:

	a	b	c	d	e
1.	$5\times\frac{2}{3}$	$\frac{3}{4}$ of $\frac{7}{8}$	$\frac{2}{3}\times\frac{3}{4}\times\frac{5}{8}$	$\frac{7}{8}\times\frac{4}{15}$	$\frac{7}{12}\times\frac{3}{4}$
2.	$4\times\frac{5}{6}$	$\frac{2}{3}$ of $\frac{15}{16}$	$4\frac{1}{2}\times5\frac{1}{3}$	$\frac{5}{6}\times\frac{8}{15}$	$\frac{4}{5}\times\frac{7}{8}$
3.	$\frac{3}{5}\times10$	$\frac{4}{5}$ of $\frac{5}{8}$	$2\frac{1}{2}\times3\frac{1}{3}$	$\frac{4}{5}\times\frac{2}{3}$	$\frac{2}{3}$ of $\frac{5}{12}$
4.	$\frac{2}{3}\times\frac{3}{8}$	$\frac{3}{4}$ of $1\frac{2}{3}$	$\frac{20}{24}\times\frac{8}{12}\times\frac{3}{16}$	$\frac{7}{15}\times\frac{3}{4}$	$\frac{5}{6}$ of $\frac{8}{15}$
5.	$\frac{4}{5}\times\frac{5}{4}$	$\frac{5}{8}$ of $\frac{14}{15}$	$\frac{1}{2}$ of $4\frac{1}{2}$	$\frac{3}{8}$ of $4\frac{1}{2}$	$\frac{4}{5}$ of $2\frac{1}{2}$
6.	$\frac{5}{6}\times\frac{7}{10}$	$\frac{5}{6}$ of 66	$7\frac{1}{2}\times1\frac{3}{5}$	$\frac{3}{4}$ of $3\frac{1}{3}$	$\frac{7}{8}$ of $5\frac{1}{4}$
7.	$\frac{3}{8}\times\frac{2}{5}$	$\frac{1}{6}$ of $\frac{1}{5}$	$6\frac{2}{3}\times1\frac{4}{5}$	$\frac{7}{8}$ of $2\frac{1}{2}$	$\frac{3}{16}$ of $\frac{4}{5}$
8.	$\frac{4}{5}\times\frac{5}{6}$	$\frac{3}{4}$ of $\frac{7}{8}$	$3\frac{1}{2}\times\frac{6}{5}$	$\frac{5}{6}$ of $2\frac{2}{5}$	$\frac{3}{8}\times\frac{4}{5}$
9.	$\frac{3}{4}\times\frac{5}{8}$	$\frac{5}{6}$ of $\frac{4}{5}$	$5\frac{1}{4}\times1\frac{2}{3}$	$\frac{3}{5}$ of $\frac{15}{16}$	$\frac{7}{16}\times\frac{4}{3}$
10.	$\frac{1}{2}\times\frac{5}{6}$	$\frac{1}{5}$ of $\frac{3}{4}$	$3\frac{1}{3}\times1\frac{4}{5}$	$\frac{9}{16}$ of $2\frac{2}{3}$	$\frac{5}{4}\times\frac{4}{5}$
11.	$\frac{7}{8}\times\frac{2}{3}$	$\frac{2}{5}$ of $\frac{7}{8}$	$7\frac{1}{2}\times1\frac{1}{3}$	$\frac{7}{10}$ of 5	$\frac{7}{12}$ of $\frac{3}{5}$
12.	$\frac{3}{5}\times\frac{5}{6}$	$\frac{5}{6}$ of $\frac{4}{15}$	$8\frac{1}{3}\times\frac{2}{5}$	$\frac{7}{8}$ of $\frac{4}{5}$	$\frac{3}{16}$ of $\frac{4}{5}$

Division of Fractions

If you are asked to divide \$3 by \$5 you at once state the result as $\frac{3}{5}$. In like manner, if you divide 3 *eighths* by 5 *eighths*, the result is $\frac{3}{5}$.

From this it is plain that we may divide like numbers. We may, therefore, divide one fraction by another, if the fractions are alike, that is, if they have a common denominator. If the fractions are not alike, that is, if they have unlike denominators, we first reduce them to a common denominator, and then divide the numerators.

For example: $\frac{3}{8}\div\frac{5}{8}=\frac{3}{5}$
$\frac{2}{3}\div\frac{4}{5}=\frac{10}{15}\div\frac{12}{15}=\frac{10}{12}=\frac{5}{6}$
$\frac{3}{8}\div\frac{4}{5}=\frac{15}{40}\div\frac{32}{40}=\frac{15}{32}$

When dividing mixed numbers, first reduce the mixed numbers to improper fractions, then perform the division.

For example: $3\frac{2}{3}\div 2\frac{2}{5}=\frac{11}{3}\div\frac{12}{5}=\frac{55}{15}\div\frac{36}{15}=\frac{55}{36}=1\frac{19}{36}$

The short method of dividing one fraction by another shown below will be found quite as satisfactory as the one explained above, and much more practical.

For example: $\frac{2}{3}\div\frac{4}{5}=\frac{10}{15}\div\frac{12}{15}=\frac{10}{12}$. The same result may be obtained, however, as follows: $\frac{2}{3}\times\frac{5}{4}=\frac{10}{12}$.

It is plain from the above example that we may get the result of dividing one fraction by another by inverting (turning upside down) the divisor, and then multiplying the resulting fractions. From this principle we get the following rule:

To divide one fraction by another, invert the divisor and multiply.

The inverse of 4, (which really means $\frac{4}{1}$,) is $\frac{1}{4}$. What is the inverse of 6? Of 8? Of 10? Of 12? Of 9? Of 32?

The inverse of $\frac{2}{3}$ is $\frac{3}{2}$. What is the inverse of $\frac{5}{6}$? Of $\frac{7}{12}$? Of $\frac{9}{32}$? Of $\frac{16}{9}$? Of $3\frac{2}{3}$?

What is the inverse of each of the following? 7 15 $\frac{3}{4}$ $\frac{7}{8}$

Exercise 15

Make the following divisions:

	a	b	c	d	e
1.	$\frac{4}{5} \div \frac{2}{3}$	$8 \div \frac{1}{2}$	$87\frac{1}{2} \div 37\frac{1}{2}$	$\frac{7}{15} \div 4\frac{2}{3}$	$14\frac{2}{5} \div \frac{8}{15}$
2.	$\frac{6}{7} \div \frac{3}{4}$	$14 \div \frac{1}{3}$	$6\frac{1}{4} \div \frac{5}{16}$	$\frac{5}{8} \div 3\frac{3}{4}$	$9\frac{1}{2} \div 7\frac{3}{5}$
3.	$\frac{3}{4} \div \frac{6}{5}$	$6\frac{1}{2} \div 1\frac{2}{3}$	$9\frac{3}{5} \div 1\frac{1}{4}$	$3\frac{1}{5} \div 1\frac{3}{5}$	$\frac{7}{16} \div 2\frac{5}{8}$
4.	$\frac{7}{8} \div \frac{5}{6}$	$5\frac{1}{4} \div 3\frac{1}{2}$	$\frac{9}{16} \div \frac{5}{8}$	$2\frac{5}{8} \div 1\frac{3}{4}$	$2\frac{1}{4} \div \frac{3}{8}$
5.	$\frac{2}{3} \div \frac{7}{8}$	$8\frac{2}{5} \div \frac{8}{15}$	$\frac{5}{4} \div \frac{4}{5}$	$5\frac{1}{3} \div 2\frac{2}{5}$	$4\frac{1}{5} \div \frac{7}{15}$
6.	$\frac{4}{15} \div \frac{8}{9}$	$62\frac{1}{2} \div 12\frac{1}{2}$	$\frac{10}{7} \div \frac{7}{10}$	$9\frac{3}{4} \div 1\frac{5}{8}$	$33\frac{1}{3} \div 4\frac{1}{6}$

REVIEW: COMMON FRACTIONS

1. Make the computations called for in the following examples.

	a	*b*	*c*	*d*	*e*
1.	$\frac{4}{5} \times \frac{5}{4}$	$\frac{7}{9} \div \frac{14}{15}$	$\frac{1}{5} + \frac{1}{8}$	$\frac{5}{6} \times \frac{3}{4}$	$\frac{7}{8}$ of $\frac{5}{6}$
2.	$\frac{2}{3} + \frac{5}{4}$	$\frac{3}{5} + \frac{2}{3}$	$\frac{9}{16} \times \frac{16}{9}$	$\frac{7}{8}$ of $\frac{2}{3}$	$\frac{3}{4} - \frac{2}{3}$
3.	$\frac{3}{8} + \frac{2}{3}$	$\frac{7}{16} \div \frac{7}{16}$	$6\frac{1}{4} \times 12\frac{1}{2}$	$3\frac{3}{4} \times 3\frac{1}{5}$	$\frac{5}{16} - \frac{1}{5}$
4.	$\frac{7}{6} - \frac{4}{5}$	$9\frac{1}{3} \div 1\frac{1}{6}$	$37\frac{1}{2} \div 62\frac{1}{2}$	$7\frac{1}{8} \div \frac{3}{16}$	$\frac{4}{5} - \frac{3}{4}$
5.	$\frac{8}{9} \div \frac{4}{3}$	$8\frac{4}{5} \div \frac{11}{20}$	$62\frac{1}{2} \times 12\frac{1}{2}$	$7\frac{1}{2} \div 1\frac{1}{4}$	$9\frac{1}{2} \div 7\frac{3}{5}$
6.	$\frac{11}{12} - \frac{5}{8}$	$\frac{5}{6} + \frac{3}{4}$	$1\frac{7}{8} \div 1\frac{2}{3}$	$\frac{3}{5}$ of $\frac{15}{16}$	$\frac{5}{16}$ of $\frac{4}{5}$

2. A merchant cut $14\frac{3}{4}$ yd. from a bolt of cloth containing $38\frac{2}{3}$ yd. How many yards were left?

3. A boy who weighed $86\frac{3}{4}$ lb. gained $4\frac{1}{2}$ lb. in three months. How much did he then weigh?

4. A workman cut one length of $34\frac{5}{6}$ ft., and another length of $18\frac{1}{2}$ ft., from a wire $93\frac{2}{3}$ ft. long. How much was then left?

5. Helen carried from the grocery a basket containing $3\frac{1}{2}$ lb. of sugar, $2\frac{3}{4}$ lb. of rice, and $1\frac{1}{2}$ lb. of tomatoes. The basket weighed $1\frac{1}{8}$ lb. What was the total weight?

6. A room is $15\frac{1}{2}$ ft. long and $12\frac{3}{4}$ ft. wide. What is the distance around it?

7. A piece $22\frac{3}{4}$ ft. long was broken from a flag pole $74\frac{1}{2}$ ft. high. What was the length of the remaining piece?

8. How wide a piece of goods must a dressmaker use to make a ruffle $3\frac{1}{2}$ in. wide, if it is necessary to turn under $1\frac{3}{8}$ in.?

9. A $2\frac{3}{4}$-inch nail was driven through a board $1\frac{1}{2}$ in. thick. How far did the nail extend beyond the board?

10. A barrel of flour contains 196 lb. How many $3\frac{1}{2}$-pound sacks can be filled from it?

11. During a basketball game Sam threw 12 baskets out of 20 trials. What fractional part of his throws were successful?

12. If a farmer sows $1\frac{2}{3}$ bushels of seed to the acre, how many bushels are needed to sow a field of 10 acres?

13. A grocer bought sugar at the rate of $8.40 per hundred pounds, and sold it at $9\frac{1}{4}$ cents per pound. What was his gain on a sale of 20 pounds?

14. A piece of cloth $1\frac{2}{3}$ yards long sold for $.60. What was the rate per yard?

15. A boy ran 100 yards in $13\frac{1}{5}$ seconds. What was his rate per second?

16. The ice on a pond was $14\frac{1}{2}$ inches thick when the workers quit cutting one evening. The next morning it was $15\frac{7}{8}$ inches thick. How much did it freeze during the night?

17. Find the cost of $2\frac{7}{8}$ yards of silk at $2.40 per yard.

18. Find the cost of the following:

1. $4\frac{3}{4}$ yd. of gingham at $.32 per yd.
2. $3\frac{3}{4}$ yd. of silk at $1.80 per yd.
3. 16 yd. of cheesecloth at $\$.05\frac{1}{2}$ per yd.
4. $16\frac{1}{2}$ doz. eggs at $.34 per doz.
5. $7\frac{2}{3}$ lb. of butter at $.36 per lb.

19. What will $3\frac{3}{4}$ pounds of meat cost at $.32 a pound?

20. A man divided $\frac{3}{4}$ of a pound of shot equally among 4 boys. What part of a pound did each receive?

21. During a storm $56\frac{3}{4}$ feet of a tree was broken off. The part remaining measures $17\frac{1}{6}$ feet. How high was the tree?

22. The captain of a vessel told Marie that the upper deck is $36\frac{3}{4}$ feet above the water, and the top of the mast is $46\frac{1}{2}$ feet above the deck. How high is the top of the mast above the water?

23. The record time for the 100-yard dash is $9\frac{3}{5}$ seconds. A boy ran the distance in $12\frac{1}{2}$ seconds. What was the difference between his time and the record time?

24. From $3\frac{1}{2}$ bushels of seed a farmer raised a crop of $34\frac{1}{2}$ bushels. What was the average yield per bushel of seed?

25. When the price of sugar increases from $8.50 to $9.25 per hundred pounds, what is the increase in price per pound expressed as a fraction?

26. Which is cheaper and by how much, a remnant of cloth containing $1\frac{3}{4}$ yards bought for $.56, or $\frac{7}{8}$ of a yard of the same material bought at a cost of $.35?

27. Mrs. Williams paid $.75 for a roast weighing $3\frac{3}{4}$ pounds. What was the price per pound?

28. How much should be paid for a $11\frac{1}{2}$-pound turkey at $.28 a pound?

29. A room is $20\frac{3}{4}$ feet long and $18\frac{1}{2}$ feet wide. What is the distance around it?

30. Of a class of 36 pupils, 4 more than $\frac{2}{3}$ voted to have a picnic instead of a party. How many pupils voted to have the party?

31. Give three examples to prove each of the following statements:

- *1.* Multiplying both terms of a fraction by the same number does not change the value of the fraction.
- *2.* Dividing both terms of a fraction by the same number does not change the value of the fraction.
- *3.* If two fractions have the same numerator, the fraction with the smaller denominator has the larger value.
- *4.* If two fractions have the same denominator, the fraction with the larger numerator has the larger value.

DECIMAL FRACTIONS

Although you have studied decimal fractions and learned much about them, you will find it profitable to review what you have studied, learn some new facts, and get additional practice with decimals. Decimal fractions are now more extensively used than they were some years ago, and you will frequently find it necessary to use such fractions.

Reading and Writing Decimal Fractions

Read a decimal fraction as though it were a whole number, and give it the name of the last decimal order to the right.

For example: .25 is read 25 *hundredths.*
.345 is read 345 *thousandths.*
.3274 is read 3274 *ten-thousandths.*
.57486 is read 57,486 *hundred-thousandths.*
.293847 is read 293,847 *millionths.*

What decimal order is *millionths?* What order is *thousandths?* What order is *hundredths?*

Read the following decimals: .527 .35 .048 .75 .4278 .0048 .54632 .02305 .002478 .000376

In reading a number that consists of a whole number and a decimal fraction, that is, a *mixed decimal* like 4.6, the decimal point is read *and.*

For example: 4.6 is read *four and six tenths.* 3.1416 is read *three and one thousand four hundred sixteen ten-thousandths.*

Read the following mixed decimals: 5.14 81.375 6.0192 73.057365 83.91 19.03102 46.00278 156.738

When reading a decimal that contains a fraction, like $.042\frac{2}{3}$, the fraction is given the denomination of the decimal order at its left.

For example: $.042\frac{2}{3}$ is read *forty-two and two-thirds thousandths.*

Read the following: $7.42\frac{3}{5}$ $.078\frac{5}{6}$ $.2154\frac{8}{9}$

You will often find it necessary to express a decimal fraction of several orders correct to a certain number of orders. That is, .542 may be expressed to the nearest *hundredth* as .54. In the same way, .0376 may be expressed to the nearest *thousandth* as .038.

Read the following, expressing each to the nearest *hundredth:*

.317 .0834 .97836 .045 .0274 .76549

Exercise 16

Read the following:

	a	b	c	d
1.	.425	.748565	.0786	7.004
2.	.0374	.30024	.00038	24.24
3.	.79	.378	.0706	$2.2\frac{2}{3}$
4.	.00076	.45	.99	500.005
5.	.9	.405	.480065	3.1416
6.	14.5	.00048	39546.005	$.00\frac{2}{3}$
7.	4.386	38.0005	500.5	74.0015
8.	152.14	.408	.505	7.08
9.	$.17\frac{1}{3}$	400.008	78462.005	.005
10.	$5.17\frac{1}{3}$	$.0\frac{2}{3}$	16.048	.7346
11.	.00546	$175.5\frac{1}{3}$	$.66\frac{2}{3}$	19.019
12.	$.1\frac{1}{3}$	847.0042	$145.06\frac{1}{4}$	29.29

Write as decimals:

13. Four and seventeen *hundredths.*

14. Five hundred and fifteen *thousandths.*

15. Two thousand and two *thousandths.*

16. Seven and two-thirds *tenths.*

17. Five and sixteen and one-third *thousandths.*

18. Three thousand two hundred twenty-four *millionths.*

19. Fifteen hundred and fifteen *thousandths.*

20. Express to the nearest *hundredth:*

	a	*b*	*c*	*d*
1.	.4873	38.746	3.1416	4.073
2.	.0079	154.005	14.573 mi.	19.106
3.	52.195	38.276	984.7487 ft.	.0534
4.	7.086	42.753	985.027	.0065

21. Express to the nearest *thousandth:*

1.	24.1908	14.0608	194.25078	48.30762
2.	.7009	3.0476	.364785	3.1416
3.	19.0547	23.1572	.84093	.07068

22. Express to the nearest *tenth:*

1.	.94	18.0648	.50972	3.1416
2.	.793	342.043	14.01768	25.087
3.	.0843	75.629	.7847	92.509

23. Express to the nearest *ten-thousandth:*

1.	.07864	19.70265	5.08047	14.703692
2.	7.30847	324.00923	13.296809	5.04073

How to Add and Subtract Decimal Fractions

When you add decimal fractions, write the decimal points of the amounts to be added under one another, as shown in *Example 1* on the next page. After that, add the numbers in each decimal order, just as you add integers. The decimal point in the result is placed in the same column as the other decimal points.

To subtract one decimal from another, first write the numbers so that the decimal point of the number to be subtracted is exactly beneath that of the minuend. Then subtract just as you subtract integers, and place the decimal point of the result in the same column as the other decimal points.

Exercise 17

1. Add and check each of the following examples:

a	*b*	*c*	*d*
3.847	42.0306	0.843	3.026
0.732	0.184	7.09	4.71
27.406	7.0207	3.842	15.079
1.843	382.74	15.607	9.857

2. 7.342+8.904+3.7

3. 26+18.3+.2+3.406

4. 9.432+2.087+.14

5. 38.0746+0.37+19.3

6. 34.8−13.25

7. 0.846−0.257

8. 27−3.1416

9. 987.03−24.0786

10. A street car company owns three lines of track, containing the following numbers of miles: 42.8 mi., 15.75 mi., 27.42 mi. How many miles of track does the company own?

11. When starting on a trip, the speedometer of Mr. Brown's automobile registered 3472.8 mi. At the end of the trip it registered 4218.4 mi. How long was the trip?

12. On a trial flight an airplane flew 94.38 mi. the first hour, 88.15 mi. the second hour, 91.55 mi. the third hour, and 103.24 mi. the fourth hour. How far did it fly in the four hours?

13. A man had $883.60 in a bank. He drew out $247.50, and on the following day he deposited $378.90. How much did he then have in the bank?

14. A dealer received 5 carloads of coal weighing as follows: 36.5 tons, 38.25 tons, 39.125 tons, 37.45 tons, and 31.8 tons. What was the total weight?

How to Multiply Decimal Fractions

How much is .04+.04+.04? Then how much is 3×.04?

How much is .15+.15+.15+.15? Then how much is 4×.15?

How much is .006+.006+.006? Then how much is 3×.006?

If an automobile travels 0.6 mile in 1 minute, how far will it travel at the same rate in 3 minutes? In 5 minutes?

Study the following examples, then explain how to find the position of the decimal point in the product when a decimal is multiplied by a whole number, and when it is multiplied by another decimal.

$4\times.7=2.8$ $4\times.07=.28$ $.4\times.7=.28$
$4\times.007=.028$ $.04\times.007=.00028$ $.004\times.007=.000028$

How much is $.5\times.6$? How much is $2.5\times.6$? How much is $3\times.0021$? How much is $.002\times.03$? How much is $.0003\times.004$?

Make a rule for pointing off the number of decimal places in any product.

How many decimal places in the product of each of the following? Find each product.

$.3\times.04$ $.07\times.007$ $.002\times.03$
$05\times.3$ $6\times.04$ $.3\times.002$

Exercise 18

1. The figures in the following products are correct and in the proper order. The decimal points in the products are omitted. Place the decimal points correctly and read the products.

	a	*b*	*c*	*d*
1.	47.06 7 32942	.048 2.4 1152	3.248 .037 120176	32.87 .916 3010892
2.	.0048 .09 432	15.005 4.3 645215	3.1416 2.8 879648	23.075 .63 1453725
3.	43.07 2.08 895856	5.084 .6 30504	.0546 3.05 166530	72.403 5.8 4199374

2. The figures in the following products are correct. If the decimal point in each is not correctly placed, change it, and give your reasons for doing so.

	a	*b*	*c*	*d*
1.	42.3 .15 6.345	.048 .36 1.728	32.05 .243 7.78815	20.47 .025 .51175
2.	3.24 7.5 2.4300	.4873 2.045 9.965285	7.304 2.05 1497320	.0382 .452 172664

3. What does each 1 in the number .1111 represent? In the number 1.111? In the number 11.11? In the number 111.1? In the number 1111?

4. How many times is .4 as large as .04? How many times is .04 as large as .004?

5. What is the effect upon the value of a number when the decimal point is moved one place to the left? Two places? Three places? Give two examples of each.

6. What is the effect upon the value of a number when the decimal point is moved one place to the right? Two places? Three places? Give two examples of each.

7. How do .3 and .30 compare in value? .30 and .300?

8. What is the effect upon the value of a decimal when one or more zeros are annexed to it? Give three examples.

9. At sight, multiply each of the following numbers by 10. By 100. By 1000: .14 .0054 .8 .036 15.035 3.1416

10. Multiply each of the following numbers by 10. By 100 By 1000: .34 1.414 9 .6 36,000 3.04

11. Make the following multiplications:

	a	*b*	*c*
1.	47.86×.035	3.1416×.16	3.04×3.04
2.	.074×3.726	.625×4.38	5.0476×27.3

12. How far will an airplane travel in 3.6 hours at the rate of 82.75 miles an hour?

13. The speed of a ship is measured by *knots*. A knot is 1.1516 miles. What is the rate in miles of a steamer traveling 16 knots per hour? 18.5 knots per hour? 21.5 knots per hour?

14. If a bushel of potatoes contains 47.34 pounds of water, what is the weight of the water in a load containing 38.5 bushels?

15. A train ran 34.75 miles an hour for 2.4 hours. How many miles did it run?

16. A boy took 184 steps, each of which was 2.75 feet long. How far did the boy walk?

17. A field containing 17.375 acres yielded 22.4 bushels of wheat per acre. What was the total yield?

18. A pound of beef contains .166 pounds of fat. How much fat in a 4.5-pound roast?

19. If a gallon of milk weighs 8.6 pounds, what is the weight of the milk that may be put into a tank with a capacity of 114.5 gallons?

How to Divide a Decimal by an Integer

How much is \$6 ÷ 2? How much is 6 feet ÷ 2? How much is 6 *tenths* ÷ 2? How much is 0.6 ÷ 2?

Your answers show that a number representing dollars, feet, tenths, or anything else may be divided by an integer. It is plain, then, that a number representing *tenths*, *hundredths*, *thousandths*, or any other decimal order may be divided by an integer.

For example:

$$\begin{array}{r} 0.2 \\ 4\overline{)0.8} \end{array} \qquad \begin{array}{r} 0.002 \\ 4\overline{)0.008} \end{array} \qquad \begin{array}{r} 0.016 \\ 5\overline{)0.080} \end{array}$$

As shown by the third example above, zeros may be annexed to the right of the dividend whenever it is necessary to do so in order to complete the division.

Make a rule for fixing the decimal point in the quotient when a decimal fraction is divided by an integer.

When dividing a decimal fraction by an integer it is often necessary to use long division. In such cases the example may be solved as shown below. You should study this carefully.

$$\begin{array}{r} .384 \\ 27\overline{)10.368} \\ \underline{81} \\ 226 \\ \underline{216} \\ 108 \\ \underline{108} \end{array}$$

If the divisor is an integer, the first quotient figure is placed above the right-hand figure of that part of the dividend first used. Thus, the 3 of the quotient is here placed above the right-hand figure of the 103.

The decimal point in the quotient is placed directly above the decimal point in the divisor.

Zeros may be annexed to the right of the dividend if necessary.

Exercise 19

Divide and check. Annex zeros to the right of the dividend if necessary.

	a	b	c	d	e
1.	$5\overline{).35}$	$7\overline{)1.428}$	$8\overline{)3.0}$	$9\overline{).4257}$	$6\overline{)32.406}$
2.	$4\overline{)8.24}$	$5\overline{).6}$	$8\overline{)7.}$	$.5\overline{)1.475}$	$4\overline{).0820}$
3.	$5\overline{).2}$	$5\overline{)0.025}$	$4\overline{)3.}$	$7\overline{).0637}$	$8\overline{).0176}$
4.	$3\overline{).4287}$	$9\overline{)8.37}$	$3\overline{)52.02}$	$7\overline{)1.484}$	$6\overline{)3.048}$

When solving the following examples, place the decimal point before making the division.

	a	b	c
5.	$64.432 \div 16$	$1244.4 \div 17$	$19.437 \div 31$
6.	$7.686 \div 105$	$1.6578 \div 307$	$16.215 \div 23$
7.	$.1161 \div 27$	$.63 \div 15$	$.01554 \div 37$
8.	$264.312 \div 36$	$3.6851 \div 43$	$33.626 \div 43$
9.	$668.368 \div 74$	$743.986 \div 82$	$91.205 \div 85$
10.	$88.32 \div 23$	$.055188 \div 146$	$3313.44 \div 72$

How to Divide a Decimal by a Decimal

In making any division, the value of the quotient is not changed if the dividend and the divisor are multiplied by the same number.

For example: $42 \div 7 = 6$ This quotient is the same as $420 \div 70$.

$$\begin{array}{r} 6 \\ 7\overline{)42} \end{array} \qquad\qquad \begin{array}{r} 6 \\ 70\overline{)420} \end{array}$$

From this it is plain that when dividing 6 by .3, you can obtain the correct result by multiplying both dividend and divisor by a number that will make the divisor an integer. In this case that number is 10.

For example: $6 \div .3 = 60 \div 3 = 20$

Show this fact by means of two other examples.

It is plain, then, that you can most easily divide one decimal by another by making the divisor an integer. Always multiply the divisor by the least number that will make it an integer. That number may be 10, or 100, or 1000, according to the order of the decimal. The multiplication can be performed by moving the decimal point in the divisor the required number of places to the right. The decimal point in the dividend must also be moved the same number of places to the right.

For example: $.67\overline{).28341} = 67\overline{)28.341}$

$.0032\overline{)1.7472} = 32\overline{)17472}$

By what number was the dividend and divisor multiplied in each of the above examples? Why?

Exercise 20

1. By what number must the dividend and divisor in each of the following examples be multiplied in order to make the divisor an integer and keep the quotient the same?

	a	*b*	*c*
1.	$.48 \div .4$	$.0036 \div .006$	$.014 \div .2$
2.	$1.5 \div .03$	$8.432 \div .04$	$676.809 \div .0009$

2. Supply the missing number in each of the following examples.

	a	*b*
1.	$.32 \div .8 = ? \div 8$	$16.824 \div .08 = ? \div 8$
2.	$.015 \div .05 = ? \div 5$	$.032 \div .004 = ? \div 4$

Divide and check the following:

	a	b	c
3.	$.1081 \div 2.3$	$.06336 \div 1.32$	$378 \div 2.7$
4.	$113664 \div .024$	$.0025812 \div .054$	$1454.06 \div 50.14$
5.	$102.2998 \div 53.842$	$3.36766 \div .046$	$37.8125 \div 6.05$
6.	$1.36854 \div .18$	$.01625 \div .125$	$60.8832 \div 15.1$
7.	$110668 \div .146$	$49.5616 \div 7.04$	$.001216 \div .038$

How to Change a Common Fraction to a Decimal

You have learned that any common fraction may be regarded as an indicated division. Thus, $\frac{2}{5}$ may be regarded as $2 \div 5$, and $\frac{3}{7}$ may be regarded as $3 \div 7$. Since this is true, any common fraction may be reduced to a decimal fraction by performing the indicated division.

Reduce $\frac{3}{11}$ to a decimal fraction.

$\frac{3}{11} = 3 \div 11.$

$$\begin{array}{r} .272\tfrac{8}{11} \\ 11\overline{)3.000} \\ 2\,2 \\ \hline 80 \\ 77 \\ \hline 30 \\ 22 \\ \hline 8 \end{array}$$

In case there is a remainder after the division is carried as far as desired, the quotient may be expressed as a complex decimal. In this case, $0.272\frac{8}{11}$. If desired, the quotient may be expressed *approximately*, that is, as a decimal correct to any given number of places. Thus, $\frac{3}{11} = .273$, correct to three places, or .27, correct to two places.

The table at the top of the next page shows the decimal values of the more usual common fractions. It may be used now and later referred to. By memorizing this table you will be able to save much time when making computations.

$\frac{1}{2} = .50$
$\frac{1}{3} = .33\frac{1}{3}$
$\frac{1}{4} = .25$
$\frac{1}{5} = .20$
$\frac{1}{6} = .16\frac{2}{3}$
$\frac{1}{7} = .14\frac{2}{7}$
$\frac{1}{8} = .12\frac{1}{2}$
$\frac{1}{9} = .11\frac{1}{9}$
$\frac{1}{10} = .10$
$\frac{2}{3} = .66\frac{2}{3}$
$\frac{3}{8} = .37\frac{1}{2}$
$\frac{5}{8} = .62\frac{1}{2}$
$\frac{7}{8} = .87\frac{1}{2}$
$\frac{5}{6} = .83\frac{1}{3}$
$\frac{3}{4} = .75$

Exercise 21

1. Reduce each of the following to a decimal fraction.

	a	*b*	*c*	*d*	*e*	*f*
1.	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{25}$	$\frac{7}{8}$	$\frac{1}{50}$
2.	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{19}{8}$	$\frac{7}{20}$	$\frac{23}{50}$

2. Reduce each of the following to a complex decimal of three decimal places.

1.	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{11}$	$\frac{2}{3}$	$\frac{5}{12}$
2.	$\frac{11}{12}$	$\frac{7}{15}$	$\frac{3}{7}$	$\frac{5}{11}$	$\frac{3}{11}$	$\frac{11}{15}$

How to Find an Approximate Quotient

An *approximate* quotient is one that is nearly exact. It is sometimes desirable to find an approximate quotient, when using decimal fractions. The example solved below, together with the explanation, will make clear the meaning of an approximate quotient and show how one may be found.

```
        .1957
247)48.361
    24 7
    23 66
    22 23
     1 431
     1 235
       1960
       1729
        231
```

In this example the quotient has been carried to *ten-thousandths*. Since 7 *ten-thousandths* is more than $\frac{1}{2}$ of 1 *thousandth*, the approximate quotient, correct to the nearest thousandth, is .196.

If the next number after the last one required is less than 5, the last one required remains unchanged. If the next number after the last one required is 5 or more, the last required number is increased by 1.

The quotient correct to *tenths* is .2.
The quotient correct to *thousandths* is .196.

Exercise 22

Find the quotients in the following examples correct to the nearest thousandth:

	a	b	c
1.	48.371÷.25	.347÷1.26	.6843÷.074
2.	52.36÷.497	423.185÷3.7	2÷7
3.	784.29÷82	937.8÷4.5	29.748÷13.6
4.	92.056÷3.7	306.927÷.038	73.943÷.084

Find the quotients in the following examples correct to the nearest hundredth:

5.	7÷15	3.768÷2.7	.0725÷1.07
6.	8.47÷.026	3.1416÷1.3	437÷6.73
7.	9÷11	84.063÷924.5	3.048÷7.04
8.	3.4÷7.3	31.284÷.56	629.5÷8.23

9. The total weight of the 11 men on a college football team is 1894.2 pounds. What is the average weight to the nearest tenth of a pound?

10. Mr. Williams sold 27 lots for $26,555. What was the average selling price of each lot to the nearest cent?

11. The 47 cars on a freight train are loaded with coal. The total weight of the coal is 1641.24 tons. What is the average weight of the coal in each car? Find the result to the nearest tenth of a ton.

12. The total age of the 35 pupils in a seventh-grade class is 428.75 years. What is the average age to the nearest tenth of a year?

13. A racing automobile traveled 25 miles in 22.5 minutes. At this rate, how far would the automobile travel in 30 minutes? Find the answer to the nearest tenth of a mile.

REVIEW: DECIMAL FRACTIONS

1. The number of inches of rainfall in a city for each of the first five months of the year was as follows: 4.3, 4.7, 3.6, 5.2, 6.1. What was the average rainfall for the five months?

2. A gallon of water weighs 8.3 pounds. What is the weight of the water in a tank that contains 146.25 gallons?

3. A field of 18.5 acres yielded 451.7 bushels of wheat. What was the average yield per acre?

4. A cubic foot of water weighs 62.5 pounds, and a cubic foot of gold weighs 19.3 times as much. What would be the weight of a cubic foot of gold? Of a cubic inch of gold?

5. Find the cost of 348 pounds of pork at $9.25 per 100 pounds.

6. Find the cost of 4250 envelopes at $2.60 per 1000.

7. Find the cost of 7850 bricks at $11.40 per 1000.

8. Find the cost of 4850 calendars at $1.10 per 100.

9. Mrs. Johnson paid $3.50 for a turkey which weighed 12.5 pounds. What was the price per pound?

10. The bank deposits and withdrawals of a merchant during a week were as follows:

	DEPOSITS	WITHDRAWALS
Monday.......	$294.65	$184.30
Tuesday.......	347.85	....
Wednesday....	792.60	473.85
Thursday......	247.82	143.60
Friday........	427.18	231.42
Saturday......	527.86	321.43

What was the difference between the deposits and the withdrawals for the week?

11. Arrange the following fractions in order of size, placing the largest first: .09 .07984 .1024 .03786 .061

12. An automobile traveled a mile in 52 seconds. What was the rate per hour?

13. The area of the United States is 3,026,789 square miles. In 1920 the population was 105,710,620. What was the population per square mile at that time?

14. Mr. Adams paid $28.40 for an automobile tire. At the end of 6142 miles the tire was worthless. What was the cost per mile for this tire?

15. The area of Texas is 265,896 square miles. The area of Rhode Island is 1248 square miles. How many times is the area of Rhode Island contained in the area of Texas?

16. A basketball team played 9 games and won 7. Another team played 8 games and won 5. What fractional part of the games played did each win? Which team made the better record? (Express the fractions as decimals.)

17. A boy raised 62.5 bushels of corn on .75 acres. Find the yield per acre.

18. In 1920 the population of the United States, exclusive of Alaska, was 105,710,620. The rural population was 51,406,017. How many hundredths of the total population for that year was the rural population?

19. Mr. Adams took a motor trip of 144.9 mi. During the first hour he traveled 24.3 mi., during the second, 23.9 mi., during the third hour, 26.8 mi. At what average speed per hour was it necessary to travel to complete the trip in the next 3 hours?

20. A lot with sides that measure 86.5 ft., 72.2 ft., 84.9 ft., and 69.3 ft. was fenced with wire at $.32 a yard. What was the cost of the fencing?

21. George received a letter from his cousin William in London in which William said that he had accepted a position at 5 pounds a month. When the English pound is worth $4.3665, what does William's salary amount to in United States money?

22. Show by examples the effect of moving the decimal point one, two, or three places to the right. Show the effect of moving the decimal point one, two, or three places to the left.

SOME USES OF ARITHMETIC IN BUSINESS

As you know, arithmetic is extensively used in business because of the many computations that must be made. You may think that unless you go into business you will have but little use for arithmetic. This idea, however, is wrong. Even though you do not go into business, you will have relations with those who do. You will need arithmetic to carry on such relations, and even in so simple a matter as buying an article at the store you will need to know some arithmetic.

Because business houses make many transactions of the same kind, they simplify matters by using printed forms on which are recorded the facts necessary to each kind of transaction. These are called *commercial forms*. You are now to learn something about the more common commercial forms, how and why they are made as they are, what they mean, and how they are used.

Sales Slips, Bills, and Statements

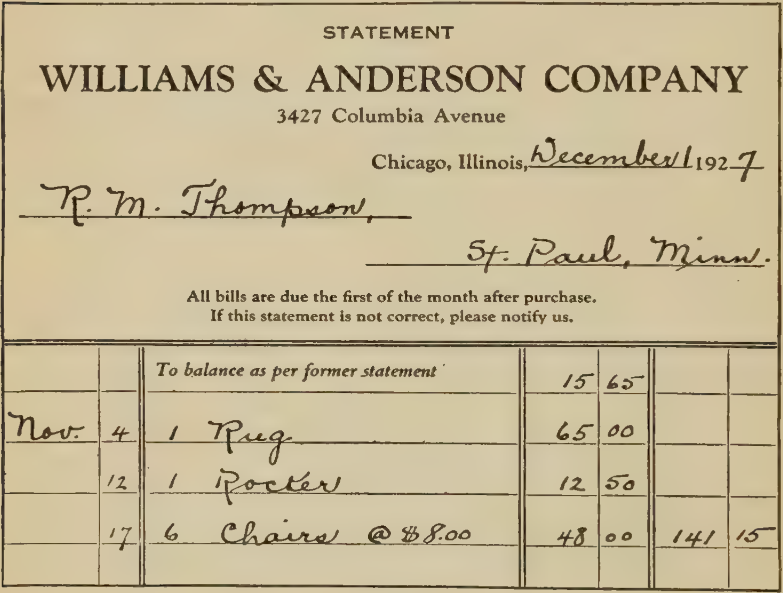
Usually, when you buy anything at the grocery or department store, and often when you make a purchase at some other kind of store, the clerk hands you a *sales slip*. Sometimes the sales slip is enclosed in the package that contains your purchase. Whether the sale is made for cash or *on account*, that is, *charged*, the sales slip is usually given to the customer.

A sales slip is a record of the articles bought, the price of each, and the total amount. When you have a charge account which you settle at the first of the month, you should always retain the sales slips so that you may know that you pay the correct amount at the time of settlement.

Some people call a sales slip a *bill*. This, however, is not quite correct. The bill is properly a sales slip that states the terms on which the sale is made. Bills are sent out generally by wholesale houses. Like the sales slip, the bill also contains an itemized statement of the articles bought, the price of each, and the total amount.

Can you now explain why you should retain your sales slips when you carry a charge account?

A *statement* is a record of the purchases made during the month. Most business houses send out statements on the first business day of each month. These are, of course, sent only to customers who have charge accounts. The statement does not always mention the items bought; it may give only the date and the amount of each sale, and the total for the month.



STATEMENT

WILLIAMS & ANDERSON COMPANY

3427 Columbia Avenue

Chicago, Illinois, December 1 1927

R. M. Thompson,

St. Paul, Minn.

All bills are due the first of the month after purchase.
If this statement is not correct, please notify us.

		To balance as per former statement	15	65		
Nov.	4	1 Rug	65	00		
	12	1 Rocker	12	50		
	17	6 Chairs @ $8.00	48	00	141	15

Fig. 4

Fig. 4 shows a common form of statement. Name the items purchased and give the cost of each. Are all charges correct? Is the total correct? Who made the purchases? When were they made? From whom were they made? If the statement contains an error, what should Mr. Thompson do?

Although statements are usually sent out on the first business day of the month, they may be paid any time during the month.

Although statements often differ in detail, yet all contain the following facts:

1. The name of the seller, the location of his business, and the date when the statement is issued.
2. The name and the address of the buyer.
3. The terms of the sale.
4. The date and the amount of each day's purchases. Some statements itemize the purchases of each day, but this is not common.
5. The total amount of the purchases for the month.

When handling sales slips, bills, or statements, the addition of all the items to find the total is called *footing* the bill.

Many stores include with the packages duplicate sales slips of each day's purchases. At the end of the month only the final sales slip containing the total to that date is sent to the customer. When the customer settles his account, all original sales slips are handed to him.

Get some sales slips, bills, and statements from your home or from your grocer or some other merchant, and compare them to see how they are alike and how they differ. Bring them to class and be prepared to explain them to your classmates.

Exercise 23

Use appropriate names for the dealer and the purchaser, and prepare a statement for each of the following problems. Rule your own forms.

1. June 4: 3 lb. of butter at 42 cents per lb., 10 lb. of sugar at $9\frac{1}{2}$ cents per lb., 1 lb. of coffee at 38 cents per lb. June 8: $\frac{1}{2}$ lb. of tea at 80 cents per lb., 3 bunches of radishes at 5 cents per bunch, 2 brooms at 70 cents each. June 19: 1 pk. of potatoes at 24 cents, 1 head of lettuce at 12 cents, $2\frac{1}{2}$ lb. of butter at 44 cents per lb. June 27: 1 pkg. of raisins at 14 cents, 2 bunches of onions at 10 cents each.

2. May 7: 1 pair roller skates, $1.50. May 17: 1 tennis net, $4.75, 1 knife, $.75. May 23: 1 flash light, $1.25.

3. June 7: 16 boxes of oranges at $4.10 each, 8 crates of grapefruit at $4.60 each, 8 barrels of sweet potatoes at $4.20 each.

4. July 15: $8\frac{1}{2}$ doz. shirts at $16.80 per doz., $5\frac{1}{2}$ doz. shirts at $24.90 per doz., $4\frac{2}{3}$ doz. shirts at $15.30 per doz.

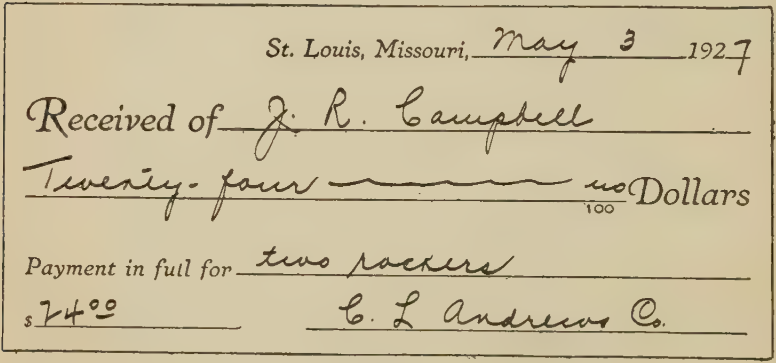
5. September 12: 48 bu. oats at $.42, 96 bu. corn at $.94, 36 bu. wheat at $1.16.

6. Rule a form like that shown in Fig. 4, and make out a bill for four items purchased at a drug store.

7. Make out a bill showing three purchases made at a dry-goods store during the month.

Getting a Receipt

Whenever you pay a bill you should have some proof that you have done so. Otherwise you may be asked to pay it again. The most common proof or evidence that a bill has been paid is a *receipt*. This is a form like that shown in Fig. 5.



St. Louis, Missouri, May 3 1927

Received of J. R. Campbell

Twenty-four ——— no/100 Dollars

Payment in full for two rockers

$24 00 C. L. Andrews Co.

Fig. 5

Who paid the bill? To whom was it paid? How much was paid? For what was the money paid? When was the bill paid? What information should you be able to obtain from a receipt? Do you think you should keep receipts? Why? Can you tell from Fig. 5 whether or not the person who paid the bill owes any more money for the rockers to the same company? How?

When a bill is paid by check, the canceled check itself, which is later returned by the bank, shows that the bill has been paid. Should you keep canceled checks? Why?

If possible, get some returned or *canceled* checks from some one you know, bring them to class, and be prepared to explain to your classmates how they serve as receipts.

Many people, when paying an account, return the bill or statement to the business house, together with a check, postal money order, postage stamps, or cash for paying it. In such cases the business house usually stamps "*Received Payment*" or "*Paid*" across the bill, signs it, and returns it to the payer as proof of payment. This is called a *receipted bill* or *receipted statement.*

Do you think it necessary for a business house to return a receipted statement to you if you pay your account by check or postal money order? Why? Should you ask for a receipted statement if you pay an account by sending postage stamps or cash? Why?

Exercise 24

1. George Martin paid you $8.50 for the rent of a boat for two months. Write a proper receipt.

2. Suppose that you sold garden produce to a neighbor at various times during the month of June, and that the total bill is $6.45. Make out the receipt to be given when the bill is paid on July 1.

3. Show how the bills in *Problems 1* and *2* on page 59 should be receipted when paid.

4. Have you ever paid a bill for yourself or for your parents and obtained a receipt?

5. Make out a statement for the following and show how it should be receipted when paid. Superior Furniture Company sold to Mr. R. W. Lewis the following: 2 rockers at $14.50 each; 1 mahogany stand at $17.25; 4 chairs at $8.60 each.

6. Use today's prices to write such a bill as a grocer might send

ACCOUNTS: HOW TO KEEP THEM

Bankers, business people, and other successful men and women frequently urge the wisdom of saving a part of one's earnings. There are now many opportunities that make saving profitable. As you have learned, the United States Government urges people to save, and furnishes several forms of practical saving devices.

Among the reasons for saving are these: First, saving money encourages *thrift*, which means a proper and effective use of money, time, effort, and opportunity. People who are really thrifty usually get along well in the world, and those who are not thrifty do not usually get along so well.

A second important reason why you should save is that the accumulated savings of many people produce vast sums of money that may be used for carrying on business. Accumulated savings are loaned to responsible business people who are then able to enlarge factory, store, and farm, and so give work to more people, and thus increase production. So you see, your savings, even though they be small, may really help to make the country prosperous.

How to Keep a Personal Account

The first step toward saving is keeping a personal account book like that shown in Fig. 6. Such an account is called a *cash account*. It shows what money you receive, and for what you receive it; it also shows what money you spend, and for what you spend it. Those who keep personal account books are usually encouraged to save more and to spend less.

In keeping a personal account book, all money received, that is all *cash receipts*, are written or *entered* on the left-hand page. This page is called the *debit side* of the account. All money paid out is entered on the right-hand page. This is called the *credit side* of the account. As shown in Fig. 6, *cash on hand* means the money in hand when the account book was started. At the end of any period, say of any month, the difference between the total receipts and the total expenses is called a *balance of cash*. This indicates the amount of money that the person keeping the account book should have in cash. This balance, or *balance on hand*, is always written at the top of the receipts page at the opening of a new period. Two horizontal lines should be drawn under the total on each page of the account book whenever a period is closed.

1927	Receipts		
Oct. 1	Cash on hand	4	18
" 5	Raking lawns		75
" 8	Delivering parcels		40
" 23	Sale of bicycle	18	50
		23	83
Nov. 1	Balance on hand	3	43

1927	Expenses		
Oct. 3	Cap		75
" 6	Football game		50
" 13	Magazines	2	50
" 18	Church		75
" 27	Book		90
" 27	Savings bank deposit	15	00
Oct. 31	Balance of cash	3	43
		23	83

Fig. 6

As shown by Fig. 6, how much were this boy's receipts for the month of October? How much cash did he have at the beginning of the month? How much were his expenses for the month? How much was the balance at the close of the month? How much cash did he have on hand on November 1? Do you think this boy was right in calling his savings-bank deposit an expense? Why?

Rule two forms like those above and list some receipts and expenses, using dates of the present month. Find the totals of receipts and of expenses, and find the balance on hand at the end of the month. If you are not earning any money, write items that might appear if you were doing so.

Exercise 25

Rule blanks as shown in Fig. 6, then make a cash account from each of the following. Balance the account in each case.

1. June 1, cash on hand, \$5.42. *Receipts:* June 6, care of lawn, \$.85; June 13, delivering groceries, \$1.25; June 18, sale of lettuce, \$.40; June 24, birthday gift, \$3.50. *Expenses:* June 4, tennis shoes, \$1.50, book, \$1.25; June 15, circus, \$.50; June 27, necktie, \$.50; June 28, show, \$.15, deposited in savings bank, \$5.00.

2. December 1, cash on hand, \$8.38. *Receipts:* December 1, allowance for the month, \$2.50; December 7, sale of dolls' dresses, \$1.25; December 14, errands, \$.35; December 25, Christmas gifts, \$7.80. *Expenses:* December 3, candy, \$.10; December 4, handkerchiefs, \$.50; December 11, sweater, \$6.50; December 17, Christmas gifts, \$3.65.

3. September 1, cash on hand, \$14.82. *Receipts:* September 4, magazine subscriptions, \$14.50; September 9, errands, \$.65; September 15, sale of books, \$1.90; September 27, errands, \$.45; September 28, sale of old paper, \$.34. *Expenses:* September 5, money order to magazine publishers, \$10.50; September 8, picture show, \$.20; September 12, handkerchiefs, \$.50; September 19, candy, \$.10; September 23, deposited in savings account, \$15.25.

4. November 1, cash on hand, \$17.93. *Receipts:* November 2, allowance for month, \$12; November 4, sale of fancy work, \$4.50; November 17, errands for father, \$.60; November 27, birthday presents, \$3.60. *Expenses:* November 6, 2 books at \$1.25 each; November 16, birthday gift, \$.75; November 21, charity, \$1.00; November 29, shoes, \$5.75; November 30, deposited in savings account, \$6.50.

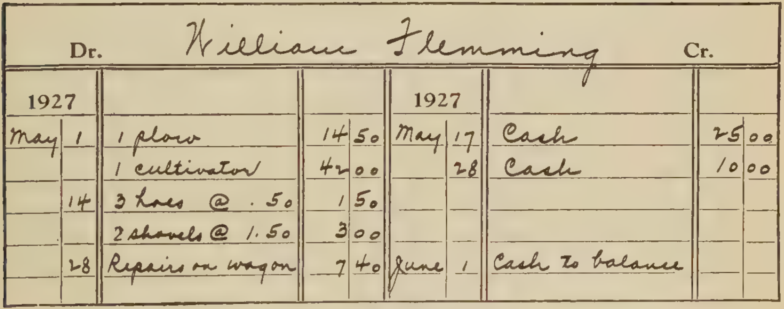
5. Keep a cash account for yourself next month. Save a part of your earnings every month, and saving will soon become a habit.

6. Do you know any boy or girl who is required to keep an account of receipts and expenses? Do you think the requirement a good one? Why?

A Customer's Account

Business houses that sell goods to customers *on credit*, that is, *on account*, find it necessary to keep accounts of these transactions. As there are usually a great many such accounts, besides other kinds of accounts, trained experts called bookkeepers are often placed in charge of the work.

Customers' accounts are usually kept in a book, one page of which is assigned to each customer. Here the customer is charged for the goods he buys, or for the services rendered to him, and he is credited with the payments he makes, the goods he returns, and any services he may in turn render to the business house. As shown by Fig. 7, both the *debit* and the *credit* items are kept on a single page.



Dr. William Flemming Cr.

1927					1927				
May	1	1 plow	14	50	May	17	Cash	25	00
		1 cultivator	42	00		28	Cash	10	00
	14	3 hoes @ .50	1	50					
		2 shovels @ 1.50	3	00					
	28	Repairs on wagon	7	40	June	1	Cash to balance		

Fig. 7

Which side of Fig. 7 shows the debit accounts? How is this indicated? How is the credit side indicated? Whose account is this? What purchases were made, and on what dates? What payments were made, and on what dates? Copy Fig. 7 and insert the proper figures at the right of the entry, *Cash to balance.*

Accounts like the one shown on this page make it possible for the dealer to learn the various items sold to each customer and to keep in mind the payments that the customer makes. Such accounts also enable the dealer to know who his best customers are, how promptly they pay their bills, and how much credit he should allow them.

Exercise 26

Put each of the following in account form. Balance.

1. Sold to Sam Evans: June 1, 3 shirts at $1.50 each, 1 hat at $2.50, 2 neckties at $.75 each, 4 collars at $.20 each. Mr. Evans paid the account in full on June 30.

2. Sold to Mrs. Henry Cummins: July 1, 4 yd. of muslin at $.28 a yd., 6 handkerchiefs at $.20 each; July 23, 4½ yd. of lace at $.40 a yd., 3½ yd. of silk at $2.60 a yd. Mrs. Cummins paid the account in full on July 31.

3. Sold to Mrs. James Atwood: December 5, 3 books at $1.25 each; December 9, 1 book at $1.50, 2 boxes stationery at $.75 each; December 22, 1 fountain pen at $4.50, 1 paper weight at $0.40. Mrs. Atwood paid the account in full on December 31.

4. Sold to Henry Simmers: March 2, 1 knife at $1.50, 1 saw at $2.80; March 11, 1 hatchet at $1.25, 1 pair hinges at $.50; March 28, 1 lawn mower at $12.50. Mr. Simmers paid the account in full on March 28.

5. Sold to Carl Olson: April 3, 1 farm wagon, $60, 1 plow, $14.25; April 27, 1 cultivator, $38.50, 4 hoes at $.50 each. Mr. Olson made a cash payment of $45 on April 20, and on April 28 he delivered to the dealer from whom the purchases were made 2 cords of wood at $8.50 a cord.

6. Sold to Mr. F. M. Lewis: May 6, 1 fishing rod, $4.75, 1 reel, $5.25, 2 fishlines at $1.50 each, 1 stringer, $.25; May 23, 1 fishing net, $2.25. Mr. Lewis paid $10 on the account on May 23. What was the balance due June 1?

7. On June 3 Mrs. Harry Collins owed a certain merchant $12.34. During the month of June she made the following purchases from the merchant: June 5, 4½ yards of lace at $.48 a yard, 1¾ yards of ribbon at $.60 a yard; June 14, 6 handkerchiefs at two for $.35, a pair of gloves at $1.75; June 23, 3½ yards of muslin at $.42 a yard. What was the balance due on her account on July 1?

PERCENTAGE

The Meaning of *Per Cent*

You have learned that *per cent*, usually expressed by the sign % instead of by the words, is only another name for *hundredths*. When we say that 45% of the pupils in a school attended a football game, we mean that 45 pupils out of every 100 attended the game. That is, 45% may be expressed as $\frac{45}{100}$, or as .45. In like manner, 7% may be expressed as $\frac{7}{100}$, or as .07, 19% as $\frac{19}{100}$, or as .19, $12\frac{1}{2}$% as $\frac{12\frac{1}{2}}{100}$, or as $.12\frac{1}{2}$.

From this the meaning of the following table becomes clear:

1% means .01
5% means .05
10% means .1
$12\frac{1}{2}$% means $.12\frac{1}{2}$

100% means 1
150% means 1.5
200% means 2
875% means 8.75

Exercise 27

Read the following as per cents:

	a	b	c	d	e	f	g	h
1.	$\frac{5}{100}$	$\frac{9}{100}$	$\frac{35}{100}$	$\frac{70}{100}$	$\frac{100}{100}$	$\frac{125}{100}$	$\frac{5\frac{1}{2}}{100}$	$\frac{12\frac{1}{2}}{100}$
2.	.08	.10	.40	1.00	3.00	4.25	1.75	2.00
3.	$.12\frac{1}{2}$	$.62\frac{1}{2}$	$.37\frac{1}{2}$	$.33\frac{1}{3}$	$.06\frac{1}{4}$	$.87\frac{1}{2}$	$.09\frac{1}{2}$	$.06\frac{2}{3}$

Write each of the following as a decimal fraction:

4.	7%	18%	15%	20%	50%	75%	$16\frac{1}{2}$%	$3\frac{1}{4}$%
5.	$2\frac{1}{2}$%	$33\frac{1}{3}$%	$37\frac{1}{2}$%	96%	$62\frac{1}{2}$%	$6\frac{1}{4}$%	82%	$87\frac{1}{2}$%

State the meaning of each of the following:

6. A boy gained 10% in weight in one year.

7. A team won 75% of all games played.

8. The weight of a piece of ice decreased 25% in one day.

9. A man sold a lot for 50% more than he paid for it.

10. A calf gained 100% in weight.

How to Express a Common Fraction as a Per Cent

You have already learned from page 52 how to reduce any common fraction to a decimal fraction. Now, since a decimal fraction is only another way of expressing per cent, it is plain that any common fraction may be expressed as a per cent.

For example: $\frac{1}{2}=.50$ or 50%. $\frac{7}{8}=.87\frac{1}{2}$ or $87\frac{1}{2}\%$.

Since it is often necessary to express fractions as per cents when making computations, you should memorize the following more common equivalents, and so save time when you later come to solve problems where these are used.

$\frac{1}{2}=50\%$
$\frac{1}{3}=33\frac{1}{3}\%$
$\frac{1}{4}=25\%$
$\frac{1}{5}=20\%$

$\frac{1}{6}=16\frac{2}{3}\%$
$\frac{1}{8}=12\frac{1}{2}\%$
$\frac{1}{10}=10\%$
$\frac{1}{12}=8\frac{1}{3}\%$

$\frac{2}{3}=66\frac{2}{3}\%$
$\frac{3}{8}=37\frac{1}{2}\%$
$\frac{5}{8}=62\frac{1}{2}\%$
$\frac{7}{8}=87\frac{1}{2}\%$

For example: Since $50\%=\frac{1}{2}$, you can find 50% of any number by taking $\frac{1}{2}$ of it. Since $8\frac{1}{3}\%=\frac{1}{12}$, to find $8\frac{1}{3}\%$ of any number you may take $\frac{1}{12}$ of it.

Exercise 28

1. What per cent of a gallon is a quart?

2. What per cent of a foot is an inch?

3. What per cent of a yard is a foot?

4. What per cent of a quart is a pint? What per cent of a gallon is a pint?

5. What per cent of a dollar is a dime? What per cent of a dollar is a quarter? What per cent of a dollar is a dollar?

6. What per cent of ten cents is five cents? What per cent of a dollar is ten cents?

7. What per cent of fifty cents is twenty-five cents? What per cent of a dollar is twenty-five cents? What per cent of five dollars is twenty-five cents?

Express the following fractions as per cents:

	a	b	c	d	e
8.	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
9.	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{1}{50}$	$\frac{1}{3}$	$\frac{2}{3}$
10.	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{1}{10}$	$\frac{3}{5}$	$\frac{1}{16}$

Write the answers to as many of the following problems as you can in three minutes:

	a	b	c
11.	$87\frac{1}{2}\%$ of 240 gal.	$12\frac{1}{2}\%$ of 4000	$87\frac{1}{2}\%$ of 560
12.	50% of \$36	$16\frac{2}{3}\%$ of \$180	50% of \$35
13.	20% of 450 mi.	$66\frac{2}{3}\%$ of 42 ft.	$33\frac{1}{3}\%$ of 6 ft.
14.	$8\frac{1}{3}\%$ of 480 lb.	25% of 280 men	10% of 340 acres
15.	$37\frac{1}{2}\%$ of 96¢	10% of \$380	25% of 52 in.
16.	50% of 42 in.	$33\frac{1}{3}\%$ of 90 acres	$12\frac{1}{2}\%$ of 48 mi.

Find the following:

17.	7% of 45	$4\frac{3}{4}\%$ of \$760	$5\frac{1}{2}\%$ of \$480
18.	24% of 450	32% of 380	45% of 640 mi.
19.	16% of 25	95% of 4850	8% of 45.2 lb.
20.	$5\frac{1}{2}\%$ of 60	$4\frac{1}{2}\%$ of \$450	7.5% of 84 ft.

Find the following, using common fractions instead of per cents:

21.	$33\frac{1}{3}\%$ of \$480	$62\frac{1}{2}\%$ of \$848	$16\frac{2}{3}\%$ of \$744
22.	$66\frac{2}{3}\%$ of 369 yd.	$12\frac{1}{2}\%$ of 64 lb.	75% of 3844 tons
23.	$166\frac{2}{3}\%$ of 360 acres	25% of \$96	$12\frac{1}{2}\%$ of 2000 lb.
24.	$12\frac{1}{2}\%$ of 64 lb.	50% of 640 acres	75% of 640 acres
25.	$37\frac{1}{2}\%$ of \$3600	$66\frac{2}{3}\%$ of \$240	50% of \$124
26.	$33\frac{1}{3}\%$ of 5280 ft.	$12\frac{1}{2}\%$ of 5280 ft.	$37\frac{1}{2}\%$ of 896 mi.

Exercise 29

1. In a school of 450 pupils, 8% are in the seventh grade. How many pupils are in that grade?

2. George weighs 85 lb., and Henry weighs 20% more than George. How much does Henry weigh?

3. A farmer who owned 240 acres increased his farm 25%. How many acres did he then own?

4. A boy earned $416 in a year, and saved 45% of it. How much did he save?

5. My gas bill for the month is $2.40. If I pay the bill promptly, I will get a reduction of 5%. How much will I save by prompt payment?

6. A dealer buys shoes at $3.80 a pair, and sells them at $33\frac{1}{3}$% more than the cost. What is the selling price?

7. Each day a dairy cow gives 30 lb. of milk that contains 3.4% butter fat. What is the butter fat worth per week at $.38 a pound?

8. Henry had $45 in a savings bank paying 4% interest. How much did Henry's money earn in a year?

9. Katharine says that 5% of 48 is more than $4\frac{1}{2}$% of 56. Helen says that it is not. Show who is right.

10. A baseball team won 75% of the 48 games played. How many games did the team win?

11. John owns two 100-dollar bonds of the Fourth Liberty Loan. From these bonds he receives $4\frac{1}{4}$% each year for the use of his money. How much will John receive from the bonds in 5 years?

12. Sam's father bought an automobile for $1400. After using it for a year, he sold it for 25% less than it cost him. What was the selling price?

13. Harry weighed 75 lb. in June. He gained 4% in weight during the summer vacation. How much did he then weigh?

14. An agent's sales amounted to $460 in one day. He received $2\frac{1}{2}\%$ of this amount as his pay. How much did he receive?

15. A real estate dealer sold my house for $4800. I paid him $1\frac{1}{2}\%$ for making the sale. How much did I pay him?

16. A butcher states that in dressing a calf for the market, 40% of the weight is lost. What should be the dressed weight of a calf whose live weight is 120 lb.?

17. Of the 300 pupils in a school, 46% are boys. How many girls are in the school?

18. An agent paid $1260 for an automobile, and sold it for $33\frac{1}{3}\%$ more than the cost. What was the selling price?

19. During a certain sale a merchant advertised a reduction of $16\frac{2}{3}\%$ from the marked price of all goods. What should be the selling prices of articles marked as follows: Suit, $36; hat, $2.40; tie, $1.50; sweater, $6.00?

20. A man paid a yearly rent of $420. The landlord raised his rent 15%. What rent did the man then pay?

21. A family with a yearly income of $2400 pays 24% of this amount for food, 18% for rent, 21% for clothing, 5% for magazines, books, and recreation, and saves 18%. The remainder is spent for miscellaneous items. Find how much is spent for each purpose.

22. During one year a cow gave 10,620 lb. of milk containing 4.1% of butter fat. What was the value of the butter fat at $.32 a pound?

23. The water in a certain variety of potatoes makes $78\frac{2}{10}\%$ of the weight. What is the weight of the water in a load containing 45 bushels of potatoes? (1 bu. of potatoes weighs 60 lb.)

24. Mary's mother put 450 eggs in incubators. Of these 80% hatched. How many chickens had she?

25. The population of a certain city in 1920 was 48,680. Since that date the population has increased 15%. What is the population now?

26. The green fodder from a certain corn field weighed 120,000 lb. When dried, the fodder had lost $62\frac{1}{2}\%$ of its weight. How much did it weigh when dry?

27. A farmer was offered 95 cents per 100 lb. for his crop of 14,840 lb. of potatoes at digging time. He stored the potatoes, and sold them in March at $1.10 per 100 lb. How much did he gain or lose by storing them, if they lost 10% in weight while in storage?

28. At a sale, Mrs. Adams bought a table at a reduction of 30%. The table was marked $35. What was the sale price?

29. A man raised 480 bu. of potatoes in one year. He increased the yield 25% the next year. How many bushels did he raise the second year?

30. On tag day Susan sold 60 tags. Katharine sold 55% as many as Susan. How many did both sell?

31. A class cleared $48.80 by giving a Christmas entertainment. The class voted to give 75% of the amount to buy Christmas baskets for the needy. How much did the class give for Christmas baskets?

To Find What Per Cent One Number Is of Another

In your study of fractions you learned how to answer such questions as the following:

What part of 10¢ is 5¢? The answer is $\frac{5}{10}$ or $\frac{1}{2}$.
What part of $40 is $4? The answer is $\frac{4}{40}$ or $\frac{1}{10}$.

It is plain that 5¢ is 50% of 10¢, because 5¢ is $\frac{1}{2}$ of 10¢, and $\frac{1}{2}=50\%$. In like manner, $4 is 10% of $40, because $4 is $\frac{1}{10}$ of $40, and $\frac{1}{10}=10\%$.

If, now, you are asked to find what per cent 7 is of 13, you proceed in the same way.

For example: $7\div13=\frac{7}{13}$. $\frac{7}{13}=.53\frac{11}{13}=53\frac{11}{13}\%$.

In order to find 4% of 150 you proceed as follows:

$.04\times150=6.00$

In order to find what per cent 6 is of 150 you proceed as follows:

A required per cent $\times 150 = 6$
Therefore, the required per cent $= 6 \div 150 = \frac{6}{150} = \frac{1}{25} = 4\%$.

A team played 36 games and won 27. What per cent of the games played did the team win?

A required per cent $\times 36 = 27$.
Therefore, the required per cent $= 27 \div 36 = \frac{27}{36} = \frac{3}{4} = 75\%$.

A boy earned \$24 during the month of December, and saved \$15. What per cent of his earnings did he save?

A required per cent $\times \$24 = \$15$.
Therefore, the required per cent $= \$15 \div \$24 = \frac{15}{24} = \frac{5}{8} = 62\frac{1}{2}\%$.

Exercise 30

1. What per cent of 8 is 4?

2. What per cent of 25 is 5?

3. What per cent of 50 is 20?

4. What per cent of 60 is 40?

5. What per cent of 18 is 24?

6. What per cent of 10 is 8?

7. What per cent of \$36 is \$18?

8. What per cent of 24 is 6?

9. If you sleep 8 hours out of the 24, what per cent of the time do you sleep?

10. A newsboy bought 60 papers and sold 54 of them. What per cent of his papers did he sell?

11. In an arithmetic test, Robert correctly solved 9 of the 10 problems. What per cent of the problems did he solve correctly?

12. A boy weighed 75 pounds at the beginning of the year, and gained 15 pounds during the year. What per cent did he gain in weight?

13. The single-trip fare from my home to a nearby city is \$.35. What per cent do I save if I buy a 12-ride ticket for \$3.78?

14. A firm which owed \$58,360 failed, and paid \$46,688 to its creditors. What per cent of its total debts did it pay? How much did a man receive, if the firm owed him \$4500?

15. A dealer buys butter at \$.32 a pound, and sells it at \$.40 a pound. What per cent of the cost does he gain?

16. By better methods of cultivation, the yield of a field of corn was increased from 40 bu. to 60 bu. per acre. What was the per cent of increase?

17. When a man's wages are increased from \$3.50 to \$4.00 a day, what is the per cent of increase?

18. When a man's wages are decreased from \$4.00 to \$3.50 per day, what is the per cent of decrease?

19. A baseball team in a big league won 77 out of 140 games. What per cent of games did the team win?

20. Roy deposited \$80 in a savings bank, and at the end of a year he received \$3.20 for the use of his money. What per cent was paid for the use of the money?

21. In one test, Susan correctly solved 9 of the 12 problems; in another test she correctly solved 12 out of 15. What per cent of the problems did she solve correctly in each test?

22. In a high school of 500 students, 90 are seniors. What per cent of the students are seniors?

23. What per cent of the pupils in your class are boys?

24. In a spelling test Marjorie correctly spelled 28 of the 30 words. What per cent of the words did she spell correctly?

25. In 1910 the population of Alabama was 2,138,093. In 1920 the population was 2,348,174. What was the per cent of increase during that period?

26. What was the per cent of increase in the population of the capital of your state from 1910 to 1920?

27. A business man invested \$30,000, and received an income of \$2100 in one year. What per cent of the investment did he receive?

28. A earns \$2700 a year and saves \$540. B earns \$2100 a year and saves \$462. Show which man saves the larger per cent of his salary.

29. During one year Lillian increased in height from 60 inches to 62 inches. What per cent did she increase in height?

30. At one time there were 13 states in the Union. There are now 48. What has been the per cent of increase in the number of states?

31. Walter had been earning 45 cents an hour. He accepted a place that paid him 50 cents an hour. What was the per cent of increase in his hourly wages?

32. A live hog weighed 320 lb. When dressed it weighed 250.8 lb. What per cent of its weight was lost in butchering?

33. George's father bought an automobile in March for $1250, and sold it a year later for $1050. What per cent of its cost did the car lose in a year?

34. A merchant bought a radio set for $20, and sold it for $28. What per cent of the cost did he gain?

35. A farmer put 1400 bushels of corn in a crib in the fall. At the time of selling it had shrunk to 1320 bushels. What per cent did the corn shrink?

36. What per cent of the 24 hours are you in school? What per cent of the 24 hours do you usually sleep?

37. An automobile increased its speed from 15 to 42 miles an hour. What was the per cent of increase in speed?

38. A 12-pound ham weighed only 9.6 pounds after it was cooked. What per cent of the weight was lost in cooking?

39. The population of several states for 1910 and for 1920 is given below. What was the per cent of increase in population in each?

STATE	POPULATION IN 1910	POPULATION IN 1920
California..........	2,377,549	3,426,861
Illinois...........	5,638,591	6,485,280
Ohio.............	4,767,121	5,759,394
Texas............	3,896,542	4,663,228

To Find the Number of Which a Given Number Is a Given Per Cent

If you are told that 20 equals 5 per cent of another number, then plainly 1 per cent of the "other number" will equal $\frac{1}{5}$ of 20, or 4. Then, since the "other number" equals 100 per cent of itself, it will equal 100×4, or 400.

If 12% of a number is 36, what is the number?

Solution

12% of the number = 36
1% of the number = $\frac{1}{12}$ of 36, or 3
100% of the number = 100×3 = 300

The 105 boys in a school are 42% of all the pupils. How many pupils are in the school?

Solution

42% of the number of pupils = 105
1% of the number of pupils = $\frac{1}{42}$ of 105, or 2.5
100% of the number of pupils = 100×2.5 = 250

Exercise 31

Find a number of which:

	a	b	c
1.	20 is 4%	20 is 200%	36 is 9%
2.	12 is 3%	24 is 300%	72 is 50%
3.	18 is 6%	90 is 2%	7 is 100%
4.	200 is 40%	14 is 7%	13 is 25%
5.	35 is 7%	45 is 20%	240 is 300%

6. Mary said, "I am thinking of a number, 4% of which is 12." Find the number if you can.

7. If 14% of a number is 28, what is the number?

8. Mr. Warren told his son that he sold 120 acres, which was 30% of his farm. How many acres were in the farm before he made the sale?

9. Sam said that his baseball team won 9 games, which was 60% of all games played. How many games did his team play?

10. At a sale, Carl's mother bought a lamp for \$20.20. This was $16\frac{2}{3}$% less than the price asked before the sale. What was the price of the lamp before the sale?

11. A farmer sold his crop of 414.9 bushels of corn in February. How many bushels did he have in October, if the corn shrunk 7.8% from October to February?

12. An ice dealer must provide 3150 tons of natural ice for the season. How many tons must he put in his icehouse, if he allows for a shrinkage of 30%?

13. During his seventh school year Alfred increased his lung capacity 6%. At the end of the year his lung capacity was 169.6 cubic inches. What was it at the beginning of the year?

14. While standing outdoors without protection, a binder decreased 15.5% in value. It was then sold for \$135.20. What was the original value?

15. A certain kind of cloth loses 2% of its length when it is sponged. How long a piece of this cloth must a tailor cut before sponging to have 9.8 yards after sponging?

16. A certain firm failed in business and was able to pay only 72% of what it owed. Carson & Company received \$4140 as payment of their account. What was the total amount of their account?

17. Sweaters somewhat damaged by smoke and water were sold for \$4.56 each at a fire sale. The reduction in price amounted to 24%. What was the price of a sweater before the fire?

18. A house was sold at auction for \$4420. The selling price was 32% less than the owner originally paid for the house. How much did the owner pay?

19. Used sewing machines were sold at \$27 each. This was 25% less than the price of the new machines. What was the price of the new machines?

REVIEW: PERCENTAGE

1. A man now receives $3.50 a day. How much will he receive a day if his wages are increased 12%?

2. When the price of sugar increased from 8¢ to 9¢ a pound, what was the per cent of increase?

3. When the price of sugar decreased from 9¢ to 8¢ a pound, what was the per cent of decrease?

4. Out of 9 problems, Walter solved 7 correctly. Cora solved 9 out of 11 correctly. Who solved the larger per cent of problems correctly?

5. Mr. Clark invested $4800 and received a profit of $288 on his investment. What was the per cent of profit?

6. In a school of 350 pupils, 14% are in the first grade. How many pupils are in the first grade?

7. A farmer delivered to a creamery 480 lb. of milk that tested 3.6% butter fat. He was paid $.32 a pound for butter fat. How much money did he receive?

8. An automobile going at the rate of 18 miles an hour decreased its speed to 10 miles an hour. What was the per cent of decrease?

9. At your last birthday, what per cent was your age of what it was 6 years before?

10. Henry bought a dog for $3 and sold it for $10. What per cent of the cost did he gain?

11. What per cent of a gallon is a quart? What per cent of a quart is a gallon?

12. What per cent of a pound is an ounce? What per cent of an ounce is a pound?

13. The distance from New York to San Francisco via the Panama Canal is 5305 miles. This distance is 40.47% less than the distance via the Strait of Magellan. What is the distance via the Strait of Magellan?

14. The record for the 100-yard dash is $9\frac{3}{5}$ seconds. It was formerly $9\frac{4}{5}$ seconds. By what per cent was the record reduced?

15. A poultry dealer finds that chickens lose 27% in weight when dressed. A hotel manager orders 511 lb. of dressed chickens. How many pounds of live chickens will be required to fill the order?

16. A white potato weighing 20 oz. contains 12.5 oz. of water. What is the per cent of water?

17. A newspaper states that its circulation is now 49,973, an increase of 18% over that of the previous year. What was the circulation last year?

18. After the selling price of a certain shoe increased 15%, I paid $6.90 a pair for them. What was the selling price before the increase?

19. Arthur told Clarence that after gaining 5% in weight during the summer vacation he weighed 89.25 lb. What was Arthur's weight at the close of school?

20. One automobile has a 50-horsepower engine, another has a 40-horsepower engine. By what per cent is the first more powerful than the second?

21. In a year a cow gave 14,840 lb. of milk that contained 593.6 lb. of butter fat. What was the per cent of butter fat?

22. A butcher found that a 220-pound hog lost 28% of its weight in being dressed. What did it weigh after being dressed?

23. A dressed steer weighed 775 lb. after losing 38% of its weight in being dressed. What was the live weight of the steer?

24. Ask your mother or the grocer to tell you the prices of five common articles of food one month ago. Find out the prices of these articles today, then find the per cent of increase or decrease in the price of each. Find the average per cent of increase or decrease for the entire list. If you can find out the prices of six months ago or of a year ago, compare them with those of today.

25. A city that was asked to raise $4300 for the Red Cross subscribed $4800. By what per cent did the city exceed the request?

26. After receiving an increase of 15% in wages, Mr. Andrews received $5.29 a day. What were his daily wages before he received the increase?

27. The area of Texas is 265,896 sq. mi., and the area of Connecticut is 4965 sq. mi. What per cent of the area of Connecticut is the area of Texas? What per cent of the area of Texas is the area of Connecticut?

28. A poultry man found that on the average 84% of the eggs placed in an incubator hatch. If he wishes to obtain 546 chicks, how many eggs should be placed in the incubator?

29. It is estimated that, for every $10 spent by the average wage earner's family, $3.78 is for food, $1.60 for clothing, $1.49 for rent, $.60 for light and fuel, $.44 for furnishings, and $2.09 for miscellaneous items. Find the per cent of wages spent for each purpose.

30. A man with a yearly income of $2000 made the following budget: Food 35%, clothing 11%, rent 25%, light and heat 8%, savings 10%, emergency 5%, miscellaneous 6%. What was the amount of each item in his budget?

31. A yearly income of $4500 was budgeted as follows: Food 22%, clothing 10%, rent, heat, and light 25%, savings 20%, improvement 5%, emergency 8%, miscellaneous 10%. What was the amount of each item in the budget?

32. What was the per cent of increase in population in each of the following cities from 1910 to 1920?

CITIES	POPULATION IN 1910	POPULATION IN 1920
Dallas	92,104	158,976
Detroit	465,766	993,678
Mobile	51,521	60,777
Omaha	124,096	191,601
Seattle	237,194	315,312

33. The wholesale price of sugar, $8.60 per hundred pounds, is raised $.40. If the grocer increases the price 1 cent a pound, what per cent of the increase in the wholesale price per pound is his increase in price per pound?

34. The following table shows the record of games won and lost by some of the teams in a certain school. What per cent of the games played did each team win?

Team	Games Won	Games Lost
Giants	11	14
Lions	12	13
Midgets	16	9
Tigers	18	7

35. Make percentage problems based on three experiences that required a knowledge of percentage.

SOME USES OF PERCENTAGE IN BUSINESS BUYING AND SELLING

Gain and Loss

By the *cost* of an article, a merchant usually means more than just the money he pays for the article itself. You must remember that in addition to what he pays for the goods he expects to sell, every merchant has many other expenses. These expenses include the rent paid for a store, or other place to do business, the freight and insurance on shipments, wages paid to sales people and others, insurance on the goods and equipment in the store, and many other expenses. To the merchant all these items are *costs*, just as the price he pays for his goods is a cost. Miscellaneous expenses of doing business are called *overhead expenses*. When a merchant speaks of the *cost* of an article, and when the term *cost* is used in the problems that follow, unless otherwise stated, you are to understand that the cost includes the original cost of the article itself, plus the overhead expenses.

Although the per cent of gain or of loss is sometimes based on the cost, many business men compute the per cent of gain or of loss on the selling price as a basis. You should be able to make the computation by using either the cost or the selling price as a basis.

For example: A merchant bought a suit for \$16 and sold it for \$20. What per cent of the cost did he gain? What per cent of the selling price?

SOLUTION

Since he gained \$4 on a cost of \$16, the gain, based on the *cost* was $\frac{4}{16}$, or 25%.

Since he gained \$4 on a selling price of \$20, the gain based on the *selling price* was $\frac{4}{20}$, or 20%.

A gain of $\frac{1}{4}$, or 25%, on the cost is the same as a gain of $\frac{1}{5}$ or 20% on the selling price.

That the above statement is always true may be seen from the following:

A gain of 25% of the cost = $\frac{1}{4}$ of the cost.
Therefore: $\frac{5}{4}$ of the cost = the selling price.
$\frac{1}{4}$ of the cost = $\frac{1}{5}$ of the selling price.
$\frac{4}{4}$ of the cost = $\frac{4}{5}$ of the selling price.

Since the entire cost = $\frac{4}{5}$ of the selling price, therefore $\frac{1}{5}$, or 20%, of the selling price is gained.

George bought a bicycle for \$30 and sold it for \$36. What was the per cent of gain on the cost? On the selling price?

SOLUTION

Since the gain was \$6 on a cost of \$30, the gain based on the *cost* was $\frac{6}{30}$, or 20%.

Since the gain was \$6 on a selling price of \$36, the gain based on the *selling price* was $\frac{6}{36}$ or $16\frac{2}{3}$%.

A gain of $\frac{1}{5}$, or 20%, of the cost is the same as a gain of $\frac{1}{6}$, or $16\frac{2}{3}$%, of the selling price.

Exercise 32

1. Find the per cent of gain based on the cost and also on the selling price.

	a		*b*	
	COST	GAIN	COST	GAIN
1.	4¢	1¢	4¢	1¢
2.	\$3.00	\$1.00	\$10	\$9
3.	\$ .80	.20	\$300	\$200

2. Find the gain or loss and the selling price.

	a		*b*	
	Cost	Rate of Gain	Cost	Rate of Loss
1.	20¢	10%	20¢	10%
2.	$1.80	16⅔%	$300	33⅓%
3.	$380	33⅓%	$4.80	16⅔%
4.	$1.50	20%	8¢	12½%

3. A merchant bought hats for $3 each, and sold them for $4 each. What per cent of the selling price did he gain?

4. Walter bought a watch for $5, and sold it for $4. What per cent of the cost did he lose?

5. Mr. Howard bought a house and lot for $6000, and sold them at a gain of $1200. What per cent of the cost did he gain?

6. A merchant buys shirts at $18 a dozen What must be the selling price of each in order that he may gain 20% of the cost?

7. A dealer buys 650 tons of coal at $7.50 a ton, including freight, and sells the coal at $8.40 a ton. How much does he gain? What per cent of the cost is the gain?

8. A dealer bought shoes at $5 a pair, and sold them at $6 a pair. What per cent of the cost did he gain? What per cent of the selling price?

9. Mary's mother made a cake at a cost of $.70. She sold it for $1. What per cent of the selling price did she gain?

10. A chair that cost $7.50 was sold for $9. What per cent of the selling price was gained?

11. Mr. Andrews bought a lot for $1500, and sold it for $1200. What per cent of the cost did he lose?

12. When a dealer buys eggs at $.35 a dozen and sells them at $.40 a dozen, what per cent of the selling price does he gain?

13. What per cent of the selling price is gained when the gain is 10% of the cost?

14. A dealer reduced the price of a book from $1.25 to $.75. What was the per cent of reduction?

15. A dealer paid $45 for a desk which he sold for $60. What per cent of the cost did he gain? What per cent of the selling price?

16. A merchant marked some cloth at $1.75 a yard, and then sold it at 20% less than the marked price. If the cloth cost him $1.20 a yard, what per cent of the cost did he gain? What per cent of the selling price?

17. A grocer bought potatoes at $.90 per cwt., and sold them at $.75 a bushel. What per cent of the cost did he gain? (1 bu. of potatoes weighs 60 lb.)

18. A hardware dealer paid $1.20 for a hammer. His operating expenses are 20% of the cost of his goods. He sells the hammer to gain 10% on cost and operating expenses. What is the selling price?

19. A dealer bought a carload of hogs weighing 15,200 lb. at the market price of $9.50 per hundred. He sold them at $10.25. What per cent of the cost did he gain? How much did he gain?

20. A grocer's sales for the year were $85,500. He lost 2% of this amount in bad accounts. The cost of the goods sold was $60,800, and the cost of conducting the business was $15,200. What was the net gain for the year?

21. A dealer buys a 15-pound ham at $.18 a pound. It loses 32% of its weight in baking. At what price per pound, to the nearest cent, must it be sold, to yield a gain of 30% on the cost?

22. A grocer bought sugar at the rate of $8.75 per cwt., and sold it at 9¾¢ a pound. What per cent of the cost did he gain?

23. A dealer paid $40 a dozen for sweaters, and sold them at $5 each. What was the per cent of gain on the selling price?

24. A merchant sold pencils at 5¢ each, and by doing so gained 20% of the cost. What was the cost of the pencils by the dozen? What was the cost per pencil?

25. A baseball team played 128 games and won 84. What per cent of the games were won?

26. A cookbook states that $3\frac{1}{2}$ teaspoonfuls of salt are equal to a tablespoonful. What per cent of a tablespoonful of salt is a teaspoonful?

27. When Henry increased his monthly savings from $4.50 to $6, by what per cent did his monthly savings increase?

28. An automobile was going at the rate of 14 miles an hour. The driver increased the speed 250%. At what rate was the automobile then going?

29. A merchant bought knives at $10.80 a dozen. At what price must he sell each knife in order to gain $33\frac{1}{3}\%$ of the cost?

Discount

Retailers sometimes hold special sales of certain goods at reduced prices. These reductions, or *trade discounts*, as they are called, are intended to induce people to buy now instead of later. The merchant is willing to take smaller profits, or none at all, in order to dispose of certain goods. In this manner he gets rid of unseasonable merchandise, avoids the payment of insurance and storage on a part of his stock, makes a more rapid turnover of his stock, and obtains cash with which to make more profitable investments.

Since reduction sales are so common you should understand discount.

A knowledge of discount is also valuable when buying from catalogues. These usually quote *list prices* from which certain discounts are allowed. Wholesalers who sell by catalogue use this plan. They often grant greater discounts to those who buy goods in large amounts than to those who buy only small quantities.

In order to encourage the prompt payment of bills, many firms offer a *cash discount* to their customers. One discount is usually given if goods are paid for immediately, a smaller discount if they are paid for within ten days, and a still smaller discount if the payment is made within thirty days.

A cash discount of 10 per cent is usually referred to as "one-tenth off"; one of 33⅓ per cent as "one-third off." The amount of a bill after deducting the discount is called the *net price* or the *net amount.*

For example: A retailer bought goods listed at $1450 with a discount of 18%. Find the *discount* and the *net amount* of his bill.

List price $1450
.18 rate of discount
11600
1450
$261.00 discount

$1450 list price
261 discount
$1189 net price

Exercise 33

1. A dealer buys an article listed at $30, and is allowed a discount of 10%. What is the net price of the article?

2. A tennis racket listed at $5 is sold to a retailer at a discount of 20%. What does the retailer pay for the racket?

3. A furniture dealer advertises a sale allowing ⅓ off. What should be the selling price of a chair which before the sale was marked $9.90?

4. One merchant offers a discount of 20% on silk marked $3.50 a yard. Another offers the same pattern and quality at $4.20 a yard, less a discount of 30%. Which is the better purchase?

5. A merchant bought a lawn mower for $7.50, and marked it to sell at $9.50. Late in the season he sold it at a discount of 20%. How much did he gain or lose?

6. At an August sale a furniture dealer offered a discount of 20%. At that rate, what would be the sale price on each of the following articles? Bed $15; chair $6.90; table $24.60; rocker $12.60; couch $34.50; bookcase $44.75.

7. Henry's father told him that he might not spend more than $1.50 for a baseball glove. The catalogue price of the glove Henry wanted was $1.80, with a discount of 25%. Postage was 10 cents extra. Could Henry buy this glove?

8. Find the net price paid for goods listed as follows:

	a	*b*	*c*
1.	\$485 less 10%	\$94.50 less 60%	\$250 less $2\frac{1}{2}\%$
2.	\$24.50 less 20%	\$4398 less 10%	\$864 less $12\frac{1}{2}\%$
3.	\$4760 less 25%	\$648.60 less $16\frac{2}{3}\%$	\$.18 less $33\frac{1}{3}\%$
4.	\$8.40 less 5%	\$3.30 less 20%	\$4.80, with $\frac{1}{4}$ off

9. A merchant offered the following articles at a special sale. Find the rate of discount on each.

	Regular Price	Sale Price
1. Girls' Dresses	\$ 9.50	\$ 7.60
2. Women's Dresses....	\$16.50	\$11.00
3. Women's Low Shoes.	\$ 6.50	\$ 3.90
4. Women's Gloves....	\$ 2.40	\$ 1.92

10. At an "After-Christmas Sale" Charles bought a radio set for \$16.40. The price before Christmas was \$24.60. What was the rate of discount?

11. A customer bought \$148.50 worth of furniture at a discount of 24% from the marked price. The clerk said that the net price would be \$122.86. The customer said that the clerk had made an error. What was the amount of the error, if one was made?

12. A merchant bought a dozen electric fans at the list price of \$9.60 each, less a discount of 20%. What did he pay for the fans?

13. A merchant pays \$.60 a gross for pens, less a discount of $16\frac{2}{3}\%$. He sells them at \$.01 each. How much does he receive for a gross? What per cent of the cost does he gain?

14. The Superior Hammer Company bought \$8460 worth of raw materials. The terms were: 60 days, net; 4% off, 30 days; 10% off, 10 days. Find the amount of the bill if it is paid after 8 days. After 28 days.

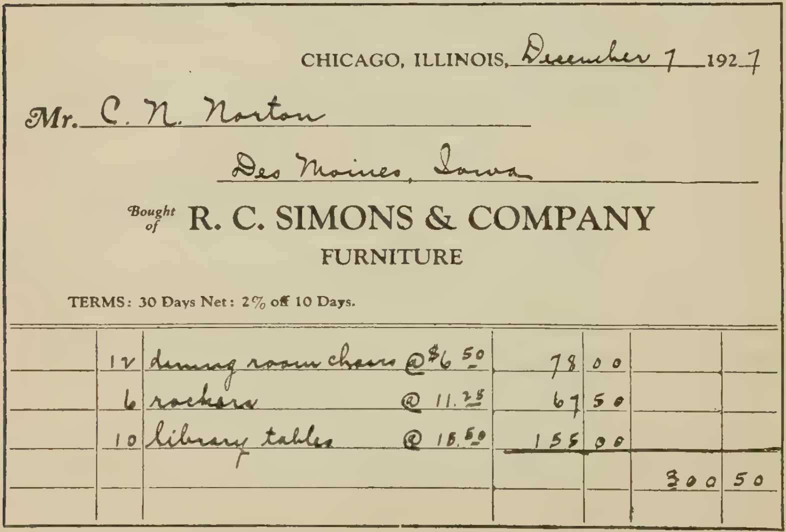
15. Find newspaper advertisements of sales offering special discounts. From these make some problems for the class to solve. Be sure you can solve your own problems.

The Invoice

The bill that a wholesaler makes out for the goods which he sells is usually called an *invoice*. An invoice should state the name of the seller and of the buyer, and the place of business of each. It should give the date and the terms of the sale. It should give a detailed list of the articles sold, including the amount and the price of each item, and it should indicate the total.

Unless otherwise specified, shipment is made *f. o. b.* shipping point. Sometimes transportation charges are paid by the seller, sometimes by the buyer.

The buyer should inspect a shipment immediately and check it with the invoice to see that both are correct. If the invoice is correct, it should be paid according to the terms of sale. If paid by check the canceled check should be kept as a receipt.



CHICAGO, ILLINOIS, December 7 1927

Mr. C. N. Norton

Des Moines, Iowa

Bought of **R. C. SIMONS & COMPANY**

FURNITURE

TERMS: 30 Days Net: 2% off 10 Days.

	12	dining room chairs @ $6.50	78	00		
	6	rockers @ 11.25	67	50		
	10	library tables @ 15.50	155	00		
					300	50

Fig. 8

Note to Pupil: The letters *f. o. b.* mean *free on board*, and indicate the point from which the customer pays the shipping charges. *For example:* f. o. b. Chicago.

According to Fig. 8, who is the seller? Who is the buyer? What is the amount of the invoice? Is there a freight charge shown on this invoice? How much will Mr. Norton pay if he pays within ten days? How much will he pay if he pays within thirty days? When must this invoice be paid?

Exercise 34

Rule forms like the one shown in Fig. 8, and make out invoices for the following:

1. Three doz. pairs cotton blankets @ \$1.65 per pair; 9 doz. pairs wool blankets @ \$4.90 per pair; 4½ doz. pairs extra heavy wool blankets @ \$6.40 per pair.

2. Eight doz. pairs men's buckskin gloves @ \$11.80 per doz.; 4½ doz. pairs men's kid gloves @ \$18.20 per doz.

3. Six and a half doz. linen handkerchiefs @ \$7.20 per doz.; 15 doz. Swiss embroidered handkerchiefs @ \$4.80 per doz.; 9¾ doz. initial handkerchiefs @ \$8.40 per doz.

4. Find the net amount of each of the above invoices, if the purchaser takes advantage of a 2% cash discount.

Discount Series

Merchants, especially wholesale dealers, often classify their customers according to size of orders and promptness with which bills are paid. Some customers are allowed one discount, others two, and still others three discounts. If more than one discount is given, the discounts are referred to as a *discount series*. A cash discount on the whole bill may also be granted.

In computing the net amount of the bill when several discounts are allowed, the first discount is based on the amount of the original bill, the second discount is then based on the amount of the original bill after deducting the first discount. If a third discount is granted, it is based on the amount of the bill after deducting the second discount, and so on. More than three discounts are not usually given.

For example: What is the net amount of a bill of goods listed at **$360,** and sold at discounts of 20% and 10%?

SOLUTION

$360	original bill	$288	basis for 2nd discount
.20		.10	
$72.00	1st discount	$28.80	2nd discount
$360		$288.00	
72		28.80	
$288	basis for 2nd discount	$259.20	net amount of bill

In practice, business men often find the single rate of discount which equals two separate discounts. In the *Solution* given above, the discounts are 20% and 10%. A discount of 20% leaves 80% of the list price to be paid. A discount of 10% leaves 90% of the second price to be paid. 90% of 80% = 72%. It is plain, then, that since discounts of 20% and 10% leave 72% of the list price to be paid, these two discounts equal a single discount of 28%.

The discount that equals a series of discounts is always *less* than the sum of the discounts in the series.

Exercise 35

1. What is the net price of a bill of goods listed at $480, with discounts of 10% and 20%?

2. A dealer allowed discounts of 25% and 15%. A clerk sold goods listed at $144, and gave a single discount of 40%. How much was the clerk's error.

3. An implement dealer made the following purchases, and took advantage of a 2% cash discount: 8 plows @ $38.75 each, less 20%; 7 hay rakes @ $24 each, less 15%; 5 haystackers @ $42.50 each, less 28%. What was the net amount of the bill?

4. On a certain kind and quality of goods, one dealer offers a purchaser discounts of 20% and 10%. Another dealer offers him 25% and 5%. Which is the better offer?

5. Goods listed at $1800 are bought at discounts of 25% and 10%, and sold at the list price. What per cent of the cost is gained?

6. Goods marked $80 are bought at discounts of 10% and 10%, and sold at the list price. What per cent of the cost is gained?

7. Find the net price of goods marked $100, with the following discounts:

	a	*b*
1.	25%, 10%, and 5%	10%, 25%, and 5%
2.	25%, 5%, and 10%	5%, 10%, and 25%
3.	10%, 5%, and 25%	5%, 25%, and 10%

8. What do you learn by comparing your results in *Problem 7?*

9. Many firms have tables showing a single discount equivalent to a series of discounts. Prepare a table showing the single discount equivalent to each of the following: 20% and 30%; 10% and 10%; 10% and 2%; 20% and 5%.

10. A merchant bought $2500 worth of goods, and received discounts of 20% and 10%. He paid freight charges of $12.90. What was the cost of the goods?

11. If you were buying an article, would you prefer discounts of 20% and 10%, or of 25% and 5%?

12. Explain why discounts of 10% and 5% are not the same as a discount of 15%.

13. A gasoline engine listed at $220 was sold at discounts of 20% and 5%. What was the selling price?

14. A dealer bought 1800 feet of garden hose at 25 cents a foot less discounts of 20% and 10%. What was the total amount of the bill, if the freight charges were $4.60?

15. Give some reasons why discounts are offered.

16. Why can a dealer afford to offer a small discount for cash?

17. Explain why wholesalers usually give a discount to their customers.

Commission

Merchants and distributors often appoint persons to sell goods on *commission*, that is, for a per cent of the money obtained from sales. Such persons are called *agents*. There is a certain advantage in this plan for both the merchant, or *principal*, and the agent. The agent receives pay in proportion as he sells goods; the principal pays only for the service actually rendered. The money retained by the agent or paid to him is called the *commission*. That paid to the principal is called the *net proceeds*.

Others besides salespeople work on the commission basis. Architects and engineers draw plans and make estimates for a per cent of the cost of the completed work. Lawyers often collect the unpaid accounts of merchants for a per cent of the amount collected. A certain class of merchants, known as *commission merchants*, accept farm produce sent by shippers, then sell it to retailers and others for a per cent of the sale price.

For example: During vacation Henry sold 75 sets of books at \$12 each. His agreement with the publishers was that he should keep $33\frac{1}{3}\%$ of the selling price for his services. What was Henry's commission? What were the net proceeds of the sales?

75 sets of books at \$12 each $= 75 \times \$12 = \$900$.
$33\frac{1}{3}\%$ of $\$900 = \frac{1}{3}$ of $\$900 = \$300$, Henry's commission.
$\$900 - \$300 = \$600$, net proceeds of sales returned to publishers.

Exercise 36

1. Florence sold books on a commission of 15%. Her sales during the month of June amounted to \$420. What was her commission?

2. Mr. Harter paid an architect 2% commission for drawing the plans for a house that cost \$6500. What was the commission?

3. Sam's father clerks in a store. He receives \$15 a week, and a commission of 2% on all the sales he makes. Last week his sales were \$655. How much did he earn?

4. My agent in Chicago sold a shipment of wheat for \$3280. The commission was $1\frac{1}{4}\%$, and the freight was \$142. What amount was due me?

5. Find the commission and the net proceeds of each of the following:

	a		*b*	
	Sale	Rate of Commission	Sale	Rate of Commission
1.	$250	4%	$45	2 %
2.	$1785	10%	$6450	5 %
3.	$1280	2%	$1656	33⅓%

6. A lawyer collected 60% of a debt of $950 for a merchant, and charged a commission of 5% of the sum collected. What amount did the merchant receive?

7. A real estate agent sold a 160-acre farm for $130 an acre. He received a commission of 2%. What was the amount of the commission?

8. A commission merchant sold 125 barrels of apples, at $3.40 a barrel, on a commission of 10%. The freight was $48.30 and the drayage $8.25. Find the commission and the net proceeds.

9. A man was offered a salary of $2800 a year, or 3% on all sales if he received no salary. He chose the first plan and sold $68,000 worth of goods during the year. How much would he have gained or lost by the second plan?

10. A fruitgrower shipped 360 boxes of apples to a commission merchant. The merchant sold the apples at $1.40 a box. The freight, drayage, and storage amounted to $165.45. The commission was 3%. What were the net proceeds?

11. On my order, a commission merchant bought 24,000 pounds of potatoes at $1.10 per hundred pounds. His commission was 2%. What did the potatoes cost me per bushel?

12. Katharine is agent for two magazines. The subscription price of one is $5.00 a year, of the other $2.50 a year. She secured 28 subscriptions for the first and 46 for the second, on a commission of 20%. How much did she earn?

13. A commission merchant received 180 pounds of butter and 45 cases of eggs, each containing 30 dozen. He sold the eggs at $.38 a dozen, and the butter at $.36 a pound. His commission was 6%. How much were the net proceeds?

INTEREST

If you hire a boat you pay rent for the use of it. If you live in a house belonging to another, or if you live on a farm owned by somebody else, you pay rent for the use of the house or farm. In like manner, if you borrow money to use for a time you must pay rent for the use of the money. This rent paid for money is called *interest.*

You often hear people speak of putting money in the savings bank and getting interest on it. You also know that people buy bonds which pay interest. It is plain that when you put money in the savings bank, the bank gets the use of your money. It is only proper that the bank should pay you interest for using it. When you buy a bond from the government, you simply make a loan to the government. Since the government is getting the use of your money for a time, it pays you interest for it.

The money for the use of which interest is paid is called the *principal.* The sum of the principal and the interest is called the *amount.* The interest itself is usually computed as a certain per cent of the principal for one year. This is called the *rate of interest.*

For example: John's father says that he borrowed \$100 at 6%. This means that he pays 6% of \$100, or \$6, as interest for 1 year. For using the money 6 months he pays \$3. For using it 4 months ($\frac{1}{3}$ year) he pays \$2. How much interest would John's father pay for using the money $1\frac{1}{2}$ years? What amount would he repay at the end of that time? What is the *principal* in this transaction? What is the *rate of interest?*

Exercise 37

1. What is the interest on \$200 for 1 year at 6%? At 5%?

2. What is the interest on \$90 for 2 years at 6%? At 7%?

3. What is the interest on \$500 for 1 year at $4\frac{1}{2}$%? At $3\frac{1}{2}$%?

4. What is the interest on \$400 for 6 months at 6%? For 3 months? For 1 year, 6 months?

5. What is the interest on \$1850 for 3 months at 6%? For 9 months?

Find the interest on:

	Principal	Time	Rate		Principal	Time	Rate
6.	$940	2 yr.	6%	**13.**	$1200	⅓ yr.	5%
7.	$360	1½ yr.	4%	**14.**	$185	4 mo.	6%
8.	$840	4 mo.	6%	**15.**	$90	6 mo.	5%
9.	$540	6 mo.	3½%	**16.**	$400	1 yr.	4¾%
10.	$780	6 mo.	4½%	**17.**	$850	1½ yr.	6%
11.	$584.60	1 yr.	5%	**18.**	$200	1 yr.	4¼%
12.	$250	4 mo.	6%	**19.**	$150	6 mo.	7%

20. Mr. Adams bought a $500 bond that pays 3½% interest. Find the yearly interest. How much interest is paid each six months?

21. The Third Liberty Loan bonds pay 4¼% interest. How much will Mary receive each year on two $100 bonds? How much will she receive on three $50 bonds?

22. Sam put $75 in a savings bank that pays 4% interest. How much interest will he receive in 1 year?

23. What is the interest per year on four $100 bonds of the Fourth Liberty Loan, the rate being 4¼%?

24. Was Henry right when he said that the interest on $450 for 1 year at 6% is $27? How much is the interest on that amount for 1 month at the same rate?

25. In order to pay his men, a contractor borrowed $3600 at 6%. How much interest was due in 3 months?

26. Cora deposited $250 in a bank paying 4% interest. At the end of six months she withdrew the principal and interest. How much did she withdraw?

27. After renting a farm, Henry Simmers borrowed $1600 with which to buy machinery and live stock. How much interest did he pay every six months, if the rate was 6% a year? After Mr. Simmers had paid $400 on the principal, by what amount was the yearly interest reduced?

To Find Interest for a Period of Less Than a Year

Banks do not usually lend money for as long as a year. They prefer to make a loan for thirty, sixty, or ninety days, then to renew the loan, if advisable. Since this is the case, it is well for you to know how to compute interest for an exact number of days. When finding the time that such a loan has to run, the bank computes the exact number of days. That is, if you borrow money on March 24 and repay it on May 9, the bank charges interest for 7 days in March, 30 days in April, and 9 days in May, a total of 46 days.

In computing interest for a period of less than a year, it is the general custom to count 360 days as a year. At that rate, the interest for 1 day is $\frac{1}{360}$ of the interest for 1 year. In these computations, a fraction of a cent equal to one-half or more is usually counted as an additional cent in the final result. A fraction of a cent amounting to less than one-half is usually discarded in the final result.

For example: Find the interest on $360 from June 4 to September 8 at 6%.

SOLUTION

$$\begin{array}{r l} \$360 & \\ .06 & \\ \hline \$21.60 & \text{interest for 1 year} \end{array}$$

The time from June 4 to September 8 is 96 days.

96 days $= \frac{96}{360}$ of 1 year, or $\frac{4}{15}$ of 1 year.

Since the interest for 1 year is $21.60, the interest for 96 days is $\frac{4}{15}$ of $21.60, or $5.76.

Exercise 38

Find the interest on:

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
1.	$200	60 da.	6%	**5.**	$750	19 da.	6%
2.	$360	30 da.	4%	**6.**	$340	60 da	5%
3.	$780	90 da	5%	**7.**	$175	84 da.	6%
4.	$500	132 da.	4%	**8.**	$3480	93 da	5%

9. $760 from June 3 to August 2 at 5%.

10. $90 from September 4 to October 17 at 6%.

11. $4850.60 from July 14 to September 2 at 5%.

12. $948 from March 3 to July 16 at 6%. At 5%.

13. $495 from April 4 to September 28 at 6%. At 5%.

14. $960 from March 18 to October 25 at $5\frac{1}{2}\%$. At 6%.

15. Mr. Jones borrowed $850 for 90 days at 6%. The loan was made June 14. When is the loan due? How much is the interest?

16. On April 14 a farmer borrowed $450 at 6%. He repaid the loan October 3. How much interest was due?

17. Sam's father asked him to compute the interest due on a loan of $650 at 6% made March 5 and repaid July 13. Sam said the interest is $14.75. Was Sam's answer correct?

To Find Interest for a Period of More Than a Year

Since interest is usually paid for periods of a year or less, in practice you will seldom need to compute it for a longer time. The method of computing interest for periods longer than a year varies with the community. Two methods commonly used are shown below. In solving the problems on page 98 use the method in common use by the business men of your community.

For example: Find the time from April 26, 1925 to August 20, 1927.

Solution I

The time from April 26, 1925 to April 26, 1927 is 2 years. From April 26, 1927 to August 20, 1927 is 116 days, according to the method explained on page 96. The entire time is 2 years, 116 days, and the interest is computed for $2\frac{116}{360}$ years.

Solution II

Year	Month	Day
1927	8	20
1925	4	26
2	3	24

August is the eighth month, April is the fourth. Since 26 days cannot be subtracted from 20 days, 1 month of the 8 months is changed to days, making a total of 50 days. Subtracting the date when the loan was made from the date when it was paid, you get 2 years, 3 months, and 24 days. Counting 30 days to the month, this equals 2 years, 114 days, or $2\frac{114}{360}$ years.

Exercise 39

Find the interest on:

1. \$350 from April 7, 1925 to October 8, 1927, at 6%.

2. \$1360 from July 14, 1925 to August 27, 1927, at 4%.

3. \$450 from May 3, 1926 to July 9, 1928, at 3½%.

4. \$340.60 from September 8, 1926, to December 17, 1927, at 5%.

5. \$98 from July 13, 1924 to August 19, 1928, at 5½%.

6. \$584.50 from May 3, 1926 to November 12, 1928, at 4½%.

7. On September 13, 1925, Walter borrowed \$560 at 6% to help pay his expenses through college. What amount will be due if the loan is repaid on June 14, 1929?

How to Use the Table for Finding Interest Periods

Bankers who have many interest problems to solve usually make use of a table like that shown in Fig. 9, to find the length of interest periods. The use of this table enables one quickly to determine the number of days from a certain day in a given month to the corresponding day of any other month.

From Any Day of	To the Same Day of the Next											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

Fig. 9

For example: How many days from April 3 to November 3? First find the month *April* in the list of names of the months at the left-hand side. Follow the line to the right to the number beneath the month *November.* This shows that from any given day in April to the same day of the following November is 214 days.

Bear in mind that if you are finding the length of any period that includes February of a leap year, you must add 1 extra day to the number indicated by the table.

For example: How many days from December 14 to March 14? The table shows the time to be 90 days. If February included in the period has 29 days, add 1 day to the total shown by the table, making 91 days.

This table may also be used to find the interval between any two dates whose difference is less than 365 days.

For example: How many days from March 7 to November 16? The table shows that from March 7 to November 7 is 245 days. From November 7 to November 16 is 9 days. The total time is 245 days+9 days=254 days.

Exercise 40

Use the table on page 98 to find the number of days between each of the following dates:

1. May 3 to January 3.

2. April 7 to December 7.

3. July 14 to November 14.

4. August 15 to December 15.

5. October 3 to January 3.

6. March 15 to November 15.

7. July 2 to June 23.

8. March 3 to October 3.

9. March 15 to October 26.

10. November 2 to January 2.

11. August 14 to December 29.

12. May 5 to November 26.

13. January 9, 1928 to November 15, 1928.

14. How many days from the opening day of your school in September to the closing day the following year?

15. How many days from your birthday to each of the following dates: Independence Day? Armistice Day? Christmas Day? Washington's Birthday?

How to Find Exact Interest or Accurate Interest

In computing interest, the United States government and most state governments use the exact number of days between dates. They allow 365 days to the year, and 366 days to each leap year. Interest computed by this method is called *exact interest* or *accurate interest.*

In practice, those who compute interest by this method use the interest table based on a year of 365 days. Banks usually compute interest based on the 360-day year. Exact interest is more commonly used when very large amounts are involved. When the principal is small, the difference between exact interest, and interest computed by using the 360-day year is not large.

For example: Compute the interest on \$600 at 6% for 90 days by each of the methods, then compare the results. Compute the interest on \$10,500,000 at 6% for 90 days by each of the methods. Compare the results.

In the interest problems you have been solving, you have allowed 360 days to the year. As you know, this is not exact because you fail to use 5 days, which are $\frac{5}{365}$, or $\frac{1}{73}$ of a year. By the method of exact interest, those 5 days are always reckoned. Therefore, exact interest is $\frac{1}{73}$ less than that computed on the basis of 360 days to the year.

The method of finding exact interest may be stated as follows:

Find the interest by the ordinary method, then diminish the amount by $\frac{1}{73}$ of itself.

Exercise 41

Find the exact interest on the following:

1. \$4,000,000 for 60 days, at 4%.
2. \$1,500,000 from April 1 to July 14, at $4\frac{1}{4}$%.
3. \$428,000 from September 23 to December 15, at $3\frac{1}{2}$%.
4. \$2,500,000 from July 15 to October 1, at $4\frac{1}{4}$%.

NOTE TO TEACHER: This exercise may be omitted if desired.

Special Methods of Computing Interest

The Six Per Cent Method

The interest on $1 for 1 year (12 months) at 6% = $.06
The interest on $1 for 1 month (30 days) at 6% = $.005
The interest on $1 for 6 days ($\frac{1}{5}$ month) at 6% = $.001
The interest on $1 for 1 day at 6% = $.000$\frac{1}{6}$

From the above it is plain that the interest on $1 for each year is $.06, for each month is $.005, for each day is $.000$\frac{1}{6}$. For any number of years, months, and days the interest will be multiples of the amounts given. Since these amounts are the interest on $1 for the indicated time, the interest on any principal will equal the interest on $1, multiplied by the number of dollars in the principal.

For example: Find the interest on $385 for 3 months and 24 days at 6%.
The interest on $1 for 3 months at 6% = $.015.
The interest on $1 for 24 days at 6% = $.004.
The interest on $1 for 3 months and 24 days at 6% = $.019.
The interest on $385 for 3 months and 24 days at 6% = 385 × $.019 = $7.32.

To use the six per cent method for finding the interest at any rate other than 6%, take $\frac{1}{6}$ of the interest at 6% and multiply by the rate given.

Exercise 42

Use the six per cent method to find the interest on the following at 6%:

1. $360 for 7 months.
2. $4800 for 3 months, 12 days.
3. $540 for 1 month, 18 days.
4. $945 for 11 months, 15 days.
5. $760 for 6 months, 3 days.
6. $8050 for 27 days.
7. $9400 for 1 year, 3 months.
8. $750 for 90 days.
9. $480 for 2 months, 18 days at 5%.
10. $150 for 5 months, 8 days at 5%.
11. $7895 for 6 months, 6 days at 3$\frac{1}{2}$%.
12. $9050 for 11 months, 24 days at 7%.

How to Use the Formula or Equation to Find Interest

You have learned to use the following rule when finding interest:

The interest is equal to the principal multiplied by the rate per year, multiplied by the time expressed in terms of a year.

The following formula, in which I = the interest, p = the principal, r = the rate, and t = the time in years or part of a year, is often used for finding interest.

$$I = p \times r \times t$$

For example: Find the interest on \$360 for 3 months at 5%. According to the formula:

$$I = \$360 \times \tfrac{5}{100} \times \tfrac{3}{12} = \$4.50$$

Find the interest on \$1200 for 1 year and 4 months at 7%.

$$I = \$1200 \times \tfrac{7}{100} \times \tfrac{4}{3} = \$112$$

It is evident that cancellation may be used to advantage in both the above examples.

Cancellation may often be used to advantage in solving problems like those in *Exercise 43.*

Exercise 43

Find the interest, using the method explained above:

	Principal	Time	Rate		Principal	Time	Rate
1.	\$450	9 mo.	5%	**10.**	\$90	7 mo.	5%
2.	\$300	$2\frac{1}{2}$ yr.	6%	**11.**	\$810	$3\frac{1}{2}$ yr.	7%
3.	\$740	3 mo.	4%	**12.**	\$650	40 da.	5%
4.	\$960	5 mo.	8%	**13.**	\$9600	90 da.	$5\frac{1}{2}$%
5.	\$2400	1 yr. 3 mo.	5%	**14.**	\$3600	1 yr. 10 mo.	7%
6.	\$1800	1 yr. 4 mo.	7%	**15.**	\$480	2 yr. 5 mo.	5%
7.	\$1500	1 yr. 9 mo.	$5\frac{1}{2}$%	**16.**	\$729	1 yr. 3 mo.	7%
8.	\$300	2 yr. 3 mo.	6%	**17.**	\$5400	2 yr. 6 mo.	$3\frac{1}{2}$%
9.	\$430	1 yr. 6 mo.	$3\frac{1}{2}$%	**18.**	\$96	1 yr. 9 mo.	7%

REVIEW: INTEREST

Use the formula to compute the interest in the following problems. Use cancellation whenever possible.

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
1.	$4800	36 days	6%	**4.**	$985	52 days	6%
2.	$94.60	62 days	5%	**5.**	$1700	30 days	8%
3.	$12,490	24 days	7%	**6.**	$24,350	90 days	4¾%

Use whatever method you prefer to find the interest in each of the following problems:

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
7.	$7560	60 days	5%	**13.**	$742.60	84 days	5½%
8.	$4750	90 days	6%	**14.**	$94.50	30 days	7%
9.	$3640	32 days	8%	**15.**	$25.80	76 days	7%
10.	$7480	28 days	5%	**16.**	$483.50	60 days	5½%
11.	$8700	90 days	4¾%	**17.**	$74.30	35 days	7%
12.	$9478	62 days	6%	**18.**	$485.60	30 days	6½%

19. Find the interest on $3600 for 90 days at 5%.

20. Find the interest on $850 for 60 days at 7%.

21. A loan of $960 made on June 7 is paid October 23. If interest at 6% is charged, how much is due?

22. Mr. Clark paid $8500 for a piece of property. At what price per month must he rent it to make 8% on the total investment, if he allows $400 a year for taxes and other expenses?

23. A merchant carries a stock that averages $60,000 in value, and has an average of $12,000 due on customers' accounts. If money is worth 6%, what is the monthly cost to the merchant for carrying his stock and the accounts?

24. Mr. Cook bought an automobile for $3200. If money is worth 6½%, what amount of interest does he fail to receive each month because he bought the automobile?

25. A loan for $3420 was made May 4, and was paid September 9. What was the interest at 6%?

26. What is the exact interest on $74,860,000 from July 1 to January 1, at $4\frac{3}{4}$%? What is the interest on the basis of 360 days to the year?

27. By home study a young man increased his weekly wages $6. What sum of money would he have to have on deposit for a year at 6% to earn in interest the same amount as this income from increased wages?

28. A boy living on a farm borrowed $45 from his father with which to buy a calf. The boy agreed to pay 6% interest. After 90 days the boy sold the calf and repaid the loan. How much did he owe his father for interest?

29. Howard lent $150 for 60 days at 6%, and Jack loaned $170 for 60 days at 5%. Who received the larger amount of interest? How much more did he receive?

30. Use the formula to find the interest on $450 at 6% for 9 months.

31. What is the interest on $2400 at 6% from March 23 to June 28?

32. If you receive $4\frac{1}{4}$% interest on $500, the interest being paid in equal amounts each half-year, how much do you receive every six months?

33. A man has an income of $150 a month. Can you find what principal invested at 5% will produce this income for 1 year? (How much interest will $1 produce in 1 year at 5%?)

34. A man borrowed $50 for 1 month, and was required to pay $1.50 interest for the loan. What was the rate of interest per year?

35. Mr. Harrison invested $10,000 at $5\frac{1}{2}$% and gave his son the interest to help pay college expenses. The son was required to earn the remainder of his expenses. If the son's expenses at college were $942 a year, how much did the boy have to earn?

GENERAL REVIEW

From time to time try the drill exercises on pages 7-18. Have you improved your speed and accuracy in computation since your first trials?

1. George's father works in a factory where the working day was decreased from 9½ to 8 hours. What was the per cent of decrease?

2. What per cent has your age increased in the past five years?

3. Is it possible for anything to increase more than 100% in value? If so, give examples.

4. Is it possible for anything to increase more than 100% in weight? If so, give examples.

5. Find what interest the banks in your community pay on savings deposits. Compute how much interest you would receive from $250 on deposit for 1 year.

6. When a dealer gains 25% on the selling price, what per cent does he gain on the cost?

7. When a dealer gains 25% on the cost, what per cent does he gain on the selling price?

8. Arrange the following numbers in the order of their value, placing the largest first. Find their sum. .0942 .00875 .5 .1098 .0375 .0099

9. Goods that cost $1.20 a yard were marked to sell at $1.80 a yard. A discount of 25% was given. What per cent of the cost was gained?

10. Give a short method for multiplying by 10. By 100. By 1000. Give examples of each method.

11. Give a short method for multiplying by 33⅓. By 37½, By 50. Give examples of each method.

12. Draw a square and a rectangle and tell how to find the area of each.

13. Which is better for the purchaser, discounts of 20% and 10%, or a single discount of 29%?

14. Frank lowered his record on the 100-yard dash from $13\frac{1}{2}$ seconds to 13 seconds. By what per cent did he lower his record?

15. What is the selling price of dresses marked $18.60 and advertised to sell at "one-third off"?

16. Fred planted corn on May 12. His father said it should be ready for use in 75 days. On what date might Fred expect to use the corn?

17. A classroom containing 38 pupils is 28 ft. long and 22 ft. wide. How many square feet of floor space has each person, including the teacher?

18. I bought 3 doz. eggs at $.38 a dozen, a broom at $.75, and 2 lb. of butter at $.46 a lb. How much change should I receive from a 5-dollar bill?

19. By studying at night Arthur increased his earning capacity from $.40 an hour to $.50 an hour. What was the per cent of increase? If Arthur works 8 hours a day and 6 days a week, what is the increase in his wages per week?

20. It is estimated that over a paved road a horse can pull a load $3\frac{1}{3}$ times as heavy as it can pull over a dirt road. If a load of 1650 lb. can be pulled over a dirt road, how heavy a load can be pulled over a paved road? What is the per cent of increase in the load?

21. It is said that the good roads in a certain county increased the value of farm land 10 per cent. A farm in the county is now worth $88 per acre. By what amount did good roads increase its value?

22. A dealer bought potatoes at $1.20 per hundred pounds, and sold them at $.30 per peck. What per cent of the cost did he gain?

23. It is estimated that fire losses in the United States during a recent year were $332,654,950. How many students, to the nearest whole number, could be given a four-year college course with this amount, allowing $650 per year for each student?

24. A furniture dealer bought goods at discounts of 10% and 20%, and sold at the list price. What per cent of the cost did he gain?

25. How do you find the selling price when you know the cost and the per cent of the cost that is gained?

26. How do you find the net price when you know the list price and the discounts?

27. What measurements would you take to find the area of a garden in the form of a rectangle?

28. The average weekly wages of the 420 men in a certain factory were $22.50. If the wages are increased 8%, what will the weekly payroll for these men amount to?

29. Carl and his father made a trip of 84.6 miles in $3\frac{1}{2}$ hours. What was the average rate per hour to the nearest tenth of a mile?

30. Mary says that discounts of 20% and 20% must be the same as a single discount of 40%, since 20% + 20% = 40%. If Mary is wrong, how do you show her error?

31. What common fraction is equivalent to each of the following? $33\frac{1}{3}\%$ 10% $66\frac{2}{3}\%$ $12\frac{1}{2}\%$ 20% $37\frac{1}{2}\%$ 50%

32. What per cent is equivalent to each of the following? $\frac{2}{5}$ $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{3}{5}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{7}{8}$

33. When the wind is blowing with a pressure of 6.5 lb. per square foot, what is the total wind pressure on a wall 42 ft. long and 18 ft. high? What is the pressure on a wall 180 ft. long and 60 ft. high?

34. A merchant bought goods at a discount of 20% from the list price, and sold them at the list price. What per cent of the cost did he gain? What per cent of the selling price?

35. After marking goods 30% above the cost price, a merchant gave a discount of 10%. What per cent of the cost did he gain?

36. One dealer offers an article at $3.60, less discounts of 20% and 10%. Another dealer offers an article of the same kind and quality at $4.00, less a discount of 35%. Which is the better for the purchaser? By how much?

37. What is the time in days between each of the following holidays this year? Christmas and Washington's Birthday. Independence Day and Armistice Day.

38. Read the following decimal fractions: $.0\frac{1}{3}$ 500.003 .503 $4.0\frac{2}{3}$ 400.004 .404 3.005 3.1416

39. What is the sum of the largest and the smallest integers of three digits each?

40. What is the largest pure decimal of three digits?

41. A history textbook states that the area of the United States in 1783 was 830,000 sq. mi., and that the area of the Louisiana Purchase was 875,000 sq. mi. What per cent of the area in 1783 was the area of the Louisiana Purchase?

42. The combined weight of George and Frank is $166\frac{1}{2}$ lb. George weighs $83\frac{3}{4}$ lb. How much does Frank weigh?

43. Susan told the class that she multiplied $\frac{7}{8}$ by another fraction and got a product of $\frac{35}{48}$. What was the other fraction?

44. A merchant paid a lawyer 3% of the amount collected for collecting 75% of a $480 account. How much did the merchant receive?

45. Mrs. S. W. Evans bought the following of R. C. Riley and Company: April 3, $3\frac{1}{2}$ yd. of ribbon at $.22 per yd., 4 yd. of lace at $.90 per yd. April 24, $1\frac{1}{2}$ doz. buttons at $.80 per dozen. Make out the bill. Receipt the bill.

46. A grocer bought apples at $3.40 per bbl. of $2\frac{1}{2}$ bu., and sold them at $.50 a pk. What per cent of the cost did he gain?

47. Make and receipt a bill listing the following items bought of L. D. Johnson & Company by R. C. Scott: 10 lb. of sugar at $\$.08\frac{1}{2}$ per lb., $1\frac{1}{2}$ bu. of potatoes at $.90 per bu., 2 lb. of coffee at $.35 per lb.

48. Each of a herd of 10 dairy cows averaged 21 lb. of milk daily. The milk contained 3.9% butter fat. If the butter fat sold at an average price of $.34 per lb., what were the weekly receipts from the herd?

49. A merchant bought goods at a discount of 40% from the list price, and sold them to gain 20% on the cost. What per cent of discount on the same list price could he give?

50. A piece of beef weighed $4\frac{1}{2}$ lb. before roasting and $3\frac{5}{8}$ lb. after it was roasted. What per cent of the weight was lost by roasting?

51. A dealer who sold goods at what would have been a profit of 20% on the cost, failed to collect for 20% of the sales. What per cent of the cost did he gain or lose?

52. An airplane flew 80 miles in 50 minutes. How many miles per minute did it fly? What part of a minute was required for it to fly 1 mile?

53. A dealer gained 20% of the cost of oranges when he sold them at $.54 a dozen. What per cent of the cost did he gain when he sold them at 4 for $.20?

54. Tell what you need to know, and how you would proceed to find each of the following:

1. The per cent of increase in population in your state in the last ten years.
2. The per cent of increase in your weight during the past year.
3. The area of a field in the form of a rectangle.
4. The amount of interest due on a savings deposit in a bank.
5. The average daily school attendance of your class for the last month.

55. Are any of these statements not true? Which ones? Why?

1. The amount of gain is always less than the cost.
2. A decimal fraction of three digits is always larger in value than a decimal fraction of two digits.
3. Multiplying both terms of a fraction by the same number does not change the value of the fraction.

56. What can you find, and how do you find it, if I tell you the following:

1. The amount of time a train travels, and its rate per hour.
2. The length and width of a rectangle.
3. The gain and the per cent of gain based on the cost.
4. The list price and the discount.
5. The side of a square.

57. Read the following and give the missing numbers.

	a	*b*	*c*
1.	1 qt. = ? pt.	1 ton = ? lb.	1 lb. = ? oz.
2.	1 bu. = ? pk.	1 sq. yd. = ? sq. ft.	1 doz. = ? things
3.	1 mi. = ? ft.	1 gross = ? things	1 acre = ? sq. rd.
4.	1 gal. = ? qt.	1 yd. = ? ft.	1 mi. = ? rd.
5.	1 yr. = ? da.	1 pk. = ? qt.	1 ton = ? cwt.

58. Read the following aloud.

1. During a recent year Canadian imports totaled $1,240,158,882; exports totaled $1,210,428,119.
2. During a recent year the value of the farm land in Texas was estimated at $3,263,296,630.
3. The world's production of sugar during a recent year was 36,827,954,240 pounds.

59. Find the time in years, months, and days since each of the following:

1. The beginning of the Great War.
2. The signing of the Declaration of Independence.
3. The admission of your state to the Union.

60. If the decimal point in any of the following examples is not correctly placed, read the answer as it should be.

	a	*b*
1.	34.025×1.48=50.35700	72.03×.308=221.8524
2.	2.07×3.05=63.135	6.005×.002=.01201
3.	.085×4.5 =.03825	.08×.008=.0064

61. The Revolutionary War began April 19, 1775, and ended January 10, 1783. How long did it last?

Most of the remainder of this book consists of the applications of numbers to the affairs of everyday life. Your attention will be centered chiefly on how such applications are made. In order to do this to greatest advantage, you must be very familiar with the processes used in making simple computations. In this work you must be speedy and accurate.

You should make frequent use of the drill exercises on pages 7-18. Frequently review the *Short Methods of Computation* on pages 19-25.

The following exercises are to be used as a review before you begin work on the remainder of the book, and for use as a drill from time to time throughout the year.

COMMON AND DECIMAL FRACTIONS: TESTS

At this stage of their work, many pupils can find the correct results for the examples in exercises I to VI in the time stated. This does not include the time used in copying the examples. How quickly can you do each exercise?

I. (*Time: 3 minutes.*)

	a	b
1.	$\frac{2}{3}+\frac{3}{4}$	$\frac{5}{12}+\frac{1}{4}$
2.	$\frac{4}{5}+\frac{5}{8}$	$\frac{3}{5}+\frac{1}{8}$
3.	$\frac{1}{2}+\frac{3}{5}$	$\frac{5}{12}+\frac{2}{3}$
4.	$\frac{5}{8}+\frac{2}{3}$	$\frac{3}{4}+\frac{7}{12}$

II. (*Time: 3 minutes.*)

	a	b
1.	$\frac{7}{8}-\frac{2}{5}$	$\frac{7}{15}-\frac{2}{5}$
2.	$\frac{3}{4}-\frac{3}{5}$	$\frac{9}{16}-\frac{3}{8}$
3.	$\frac{5}{6}-\frac{1}{5}$	$\frac{3}{5}-\frac{2}{15}$
4.	$\frac{8}{9}-\frac{2}{3}$	$\frac{3}{8}-\frac{1}{10}$

III. (*Time: 3 minutes.*)

	a	b	c	d
1.	$\frac{2}{3}\times\frac{3}{4}$	$\frac{7}{15}\times\frac{2}{5}$	$\frac{4}{5}\times\frac{14}{16}$	$\frac{5}{8}\times\frac{4}{5}$
2.	$\frac{3}{5}\times\frac{2}{3}$	$\frac{1}{2}\times\frac{4}{15}$	$\frac{8}{9}\times\frac{3}{5}$	$\frac{4}{5}\times\frac{15}{16}$
3.	$\frac{7}{8}\times\frac{3}{5}$	$\frac{6}{7}\times\frac{5}{3}$	$\frac{5}{6}\times\frac{14}{15}$	$\frac{3}{5}\times\frac{3}{4}$

IV. (*Time: 4 minutes.*)

	a	b	c	d
1.	$\frac{4}{5} \div \frac{8}{15}$	$\frac{5}{6} \div \frac{3}{5}$	$\frac{1}{2} \div \frac{1}{3}$	$\frac{5}{8} \div \frac{3}{4}$
2.	$\frac{1}{2} \div \frac{5}{6}$	$\frac{2}{3} \div \frac{5}{9}$	$\frac{7}{8} \div \frac{14}{15}$	$\frac{4}{5} \div \frac{7}{10}$
3.	$\frac{3}{5} \div \frac{9}{15}$	$\frac{5}{16} \div \frac{15}{32}$	$\frac{2}{3} \div \frac{8}{9}$	$\frac{3}{4} \div \frac{5}{6}$

V. (*Time: 3 minutes.*)

Add:

	a	b	c	d
1.	$4\frac{1}{2}$ $3\frac{2}{3}$	$7\frac{5}{6}$ $4\frac{3}{4}$	$19\frac{4}{5}$ $13\frac{1}{6}$	$24\frac{5}{6}$ $32\frac{7}{8}$
2.	$\frac{5}{12} \div \frac{7}{8}$	$\frac{3}{4} \div \frac{2}{3}$	$\frac{7}{8} \div \frac{21}{16}$	$\frac{4}{9} \div \frac{9}{4}$

VI. (*Time: 3 minutes.*)

Write the quotients on a sheet of paper:

	a	b	c	d
1.	.1042 ÷ .21	3.6 ÷ 12	2.2 ÷ .11	4.8 ÷ .08
2.	.009 ÷ .3	240 ÷ 1.2	.14 ÷ 14	.072 ÷ .9
3.	7.2 ÷ .09	40 ÷ .4	.075 ÷ .25	2.1 ÷ .07
4.	0.36 ÷ .09	8 ÷ .08	48 ÷ .8	.64 ÷ .8

VII

Solve the following, using short methods whenever possible:

	a	b	c	d
1.	$36 \times 33\frac{1}{3}$	$62\frac{1}{2}\%$ of 48	15×15	11×3048
2.	$16\frac{2}{3} \times \$72$	11×4865	19×16	65×65
3.	45×45	75×75	$62\frac{1}{2} \times 248$	99×7342
4.	$12\frac{1}{2}\%$ of \$42.40	125% of $\frac{5}{4}$	$16\frac{2}{3} \times \$42$	$166\frac{2}{3}\%$ of 48
5.	85×85	75% of 3600	$2\frac{1}{2} \times 84$	95×95
6.	11×4378	$37\frac{1}{2} \times 72$	99×746	$.375 \times 480$
7.	$12\frac{1}{2} \times 648$	25×37	$8\frac{1}{2} \times 8\frac{1}{2}$	25×346

LINES AND ANGLES

A line that has the same direction throughout its length is called a *straight line.*

A line is usually designated by placing a letter at each end, as shown in Fig. 10.

We read these as follows: The line AB The line CD. The line EF.

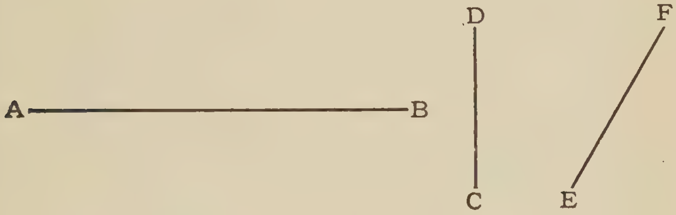


Fig. 10

Estimate the length of the line AB, then measure it to check your estimate.

Without the aid of ruler or other measure, draw lines of each of the following lengths, then check your work by measuring: 6 inches. 3 feet. 4 inches. $1\frac{1}{2}$ feet. $\frac{2}{3}$ of a yard. 18 inches.

Estimate the length of each line in Fig. 11, then check your estimates.

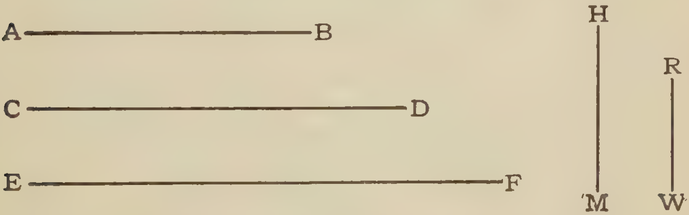


Fig. 11

Estimate the lengths of three lines which you can measure in your schoolroom. Check your estimates.

Estimate the following, then check each: The height of the top of the blackboard. The length of the teacher's desk. The height of some boy in the class. The length and width of the room. The height of the door.

Two or more lines that are the same distance apart, no matter how far they are extended, are called *parallel lines.*

The pairs of lines in Fig. 12 are parallel.

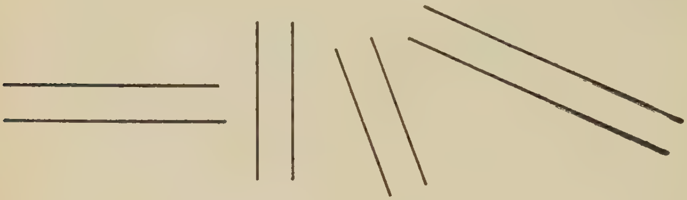


Fig. 12

Find parallel lines in your schoolroom.
Tell of parallel lines outside which you know of.
Draw four parallel lines.

When two lines meet they form an *angle.* In Fig. 13, the lines OA and OB form angle AOB. The lines DC and DE form angle CDE.

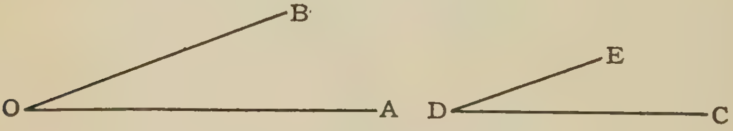


Fig. 13

The lines OA and OB are called the *sides* of the angle AOB. The point O is called the *vertex.*

Name the sides and vertex of the angle CDE.

Instead of writing *angle AOB*, we may write $\angle AOB$. *Angle CDE* may be written $\angle CDE$.

Trace $\angle CDE$ on a sheet of thin paper, and show that, if the vertex D is placed at O, and the side DC is made to fall along OA, then the side DE may be made to fall along OB.

Whenever this is true of two or more angles, they are said to be *equal.*

It is important to note that the size of an angle does *not* depend on the length of its sides.

If two straight lines meet so that the angles formed are equal, the lines are said to be *perpendicular* to each other. The equal angles thus formed are called *right angles.*

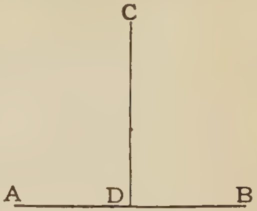


Fig. 14

In Fig. 14, the lines *AB* and *CD*, which meet at *D*, form the right angles *BDC* and *CDA*.

Name the four right angles in Fig. 15.

What right angles do you see in your schoolroom?

Hold two of your fingers to form one right angle. To form four right angles.

Fold a rectangular piece of paper in half, and crease. The crease will be perpendicular to the edges that it joins.

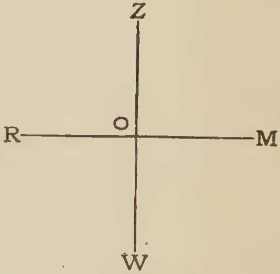


Fig. 15

An angle less than a right angle is called an *acute angle.*

An angle greater than a right angle, and less than the sum of two right angles, is called an *obtuse angle.*

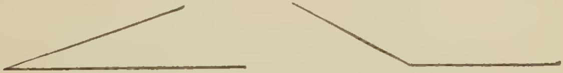


Fig. 16

Hold two of your fingers to form an acute angle.

Hold two of your fingers to form an obtuse angle.

Show the acute angles and the obtuse angles in Fig. 17.

Where are there streets or roads that form an acute angle?

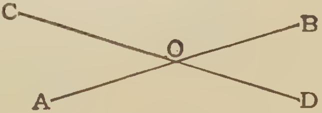


Fig. 17

Draw a triangle containing an obtuse angle.
Draw a triangle containing three acute angles.
Draw a triangle containing a right angle.

What kind of angle do the hands of a clock form at 1 o'clock? At 3 o'clock? At 5 o'clock? At 8 o'clock?

The Measurement of Angles

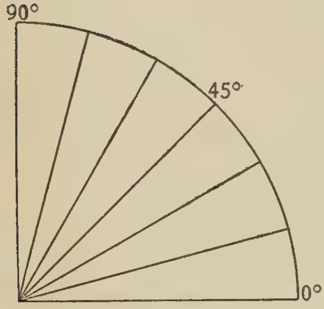


Fig. 18

Angles are measured by *degrees*.

As shown by Fig. 18, a right angle contains 90 degrees.

The sign for *degree* is °. 90 *degrees* is written 90°.

What part of a right angle is an angle of 45°? An angle of 30°? An angle of 15°?

Draw an angle of 45°. Of 90°.

Is any angle of less than 90° an acute angle?

Exercise 44

1. Write the following, using the sign for degree: 90 degrees, 10 degrees, 45 degrees, 15 degrees, 30 degrees.

2. How many angles of 45° are in one right angle? Prove by a drawing.

3. A road running due north and south crosses another running due east and west. What kind of angles do the roads form where they cross? Prove by a drawing.

4. If a third road from the northwest joins the two mentioned in the problem above at exactly the point where they cross, what kind of angle will it form with the east and west road? With the north and south road? Prove by a drawing.

5. How large an angle is formed by the hands of a clock at three o'clock? At one o'clock? At two o'clock? Prove each answer by a drawing.

PLANE FIGURES

You have learned how to find the area of a rectangle. At this time you should recall what you have learned about the rectangle and about the method of finding its area. You should also review the tables of linear measure and of square measure. You have observed that not all plane figures are rectangles. You are now to learn about other plane figures.

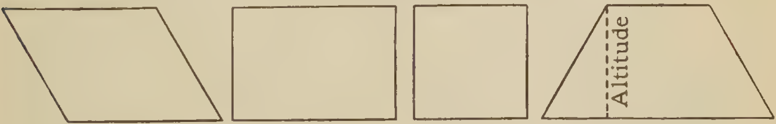
A flat surface bounded by four straight lines is called a *quadrilateral.*

A quadrilateral having its opposite sides parallel is called a *parallelogram.*

A parallelogram, all of whose angles are right angles, is called a *rectangle.*

A rectangle having all its sides equal is called a *square.*

A quadrilateral having only one pair of opposite sides parallel is called a *trapezoid.*



Parallelogram **Rectangle** **Square** **Trapezoid**

Fig. 19

The *altitude* of a quadrilateral is the perpendicular distance from the line that is regarded as the base to the opposite side.

In Fig. 19, the altitude of the trapezoid is indicated.

The *perimeter* of a quadrilateral is the sum of its sides.

To Find the Area of a Rectangle

How many square inches are contained in a rectangle 6 inches long and 3 inches wide?

Solution: The area of a rectangle 6 inches long and 3 inches wide = 3×6 sq. in. = 18 sq. in.

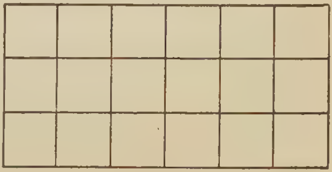


Fig. 20

How many square inches are contained in a rectangle of the same length, but three times as wide?

Find the area of a rectangle 4 feet long and 3 feet wide.

The area of a rectangle equals the product of its base and altitude.

When the product of two lines is spoken of, the product of the number of units in the lines is meant. Before applying the rule, the lines must be expressed in terms of the *same* unit of measure.

Exercise 45

1. If a rectangle 8 in. long and 5 in. wide is divided into square inches, how many square inches will there be along the base? How many such rows will there be? What is the area of the rectangle? Draw the figure.

2. Find the area of a rectangle 20 ft. long and 12 ft. wide.

3. A rectangular field is 40 rd. long and 25 rd. wide. What is its area in sq. rd.? In acres?

4. Find the cost of a rectangular walk 120 ft. long and 6 ft. wide, at $1.25 a sq. yd.

5. What part of an acre is there in a tennis court 78 ft. long and 36 ft. wide?

6. A football field is 100 yd. long and $53\frac{1}{3}$ yd. wide. Does the field contain more or less than an acre?

7. Find the area of the blackboard in your schoolroom.

8. If your school ground is rectangular, find the area.

9. If your school has a rectangular garden, find its area.

10. Lay off a square rod on your school grounds. Practice estimating the number of square rods in various areas. Check your estimates by measuring.

11. Estimate the area of the floor of your schoolroom. Check your estimate. How much floor space is there for each pupil?

12. Find the cost of the linoleum needed to cover a kitchen floor 15 ft. long and 12 ft. wide, at $1.30 per sq. yd.

13. Find the window area of your schoolroom. What fractional part of the floor area is the window area?

14. A rectangular field is 60 rd. long and 45 rd. wide. What is its value at $90 an acre?

15. Draw a rectangle 6 inches square. What is its area?

16. Draw several rectangles of different dimensions, each containing 6 square inches.

17. How many acres in a square field whose perimeter is $\frac{1}{2}$ mile?

18. A city street is to be paved at a cost of $1.40 per sq. yd. The street is 60 ft. wide. What will be the cost per mile?

19. How many acres in a township 6 mi. long and 6 mi. wide?

20. A roof is 44 ft. long and 14 ft. wide. How many squares of roofing does it contain? (A *square* of roofing = 100 sq. ft.)

21. Do all rectangles which have equal perimeters have equal areas? Prove your answer.

22. Do all rectangles which have equal areas have equal perimeters? Prove your answer.

23. Draw, if possible, a rectangle that is not a square.

24. Draw, if possible, a square that is not a rectangle.

To Find the Area of a Parallelogram

Draw a parallelogram with a base of 4 inches and an altitude of 3 inches. Cut off the triangle corresponding to the shaded portion in Fig. 21 and place it on the opposite side of the figure, as indicated by the dotted lines. What kind of figure has the parallelogram now become?

Compare the base of the parallelogram and that of the rectangle.

Compare the altitude of the parallelogram and that of the rectangle.

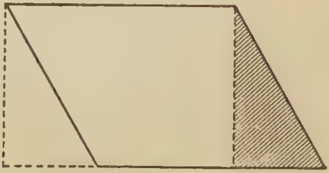


Fig. 21

It is evident that the area of a parallelogram is the same as that of a rectangle having the same base and altitude.

The area of a parallelogram is equal to the product of its base and altitude.

Exercise 46

1. Find the area of a parallelogram 24 in. long and 14 in. wide.

2. A field in the form of a parallelogram has a base of 36 rd. and a width of 22 rd. What is its value at $140 an acre?

3. Find the areas of parallelograms with the following dimensions:

	Base	Altitude		Base	Altitude
1.	42 in.	15 in.	*4.*	84 ft.	36 ft.
2.	34 ft.	12 ft.	*5.*	72 rd.	48 rd.
3.	27 rd.	16 rd.	*6.*	8.6 ft.	4.2 ft.

4. Draw, if possible, a parallelogram that is not a rectangle.

5. Draw, if possible, a rectangle that is not a parallelogram.

6. Draw a square, each side of which is 3 in., and a parallelogram with a base of 3 in. and an altitude of 3 in. Show that the figures have the same area.

7. Without using a ruler or yardstick, draw on the blackboard a parallelogram that contains 40 square inches. Check the accuracy of your drawing.

8. Two parallelograms have the same length of base, but the altitude of the first is three times as great as the altitude of the second. How do their areas compare?

9. One parallelogram has its base and its altitude three times as great as the base and altitude of another parallelogram. How do their areas compare?

10. If possible, draw a parallelogram all of whose sides are right angles.

11. If possible, draw a parallelogram all of whose sides are acute angles.

To Find the Area of a Trapezoid

As you have already learned, a plane surface like *ABCD* in Fig. 22 is called a *trapezoid*. To find the area of such a figure, draw the line *EF* through *O*, the middle point of the line *DC*, thus making the line *EF* parallel to the line *AB*. If the triangle *EOC* were then cut off and placed in the position *DOF*, what kind of figure would be formed?

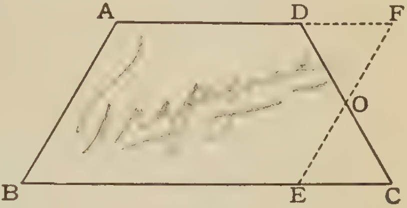


Fig. 22

Cut from paper several trapezoids of different sizes. Cut a triangle from each, as explained above, and place it as shown in Fig. 22. Show that the base of the parallelogram into which the trapezoid has been changed is equal to half the sum of the two parallel sides of the trapezoid, and that the altitude of the parallelogram is the same as the altitude of the trapezoid.

The area of a trapezoid is equal to half the sum of the two parallel sides times the altitude.

Exercise 47

1. The parallel sides of a trapezoid are 48 ft. and 64 ft., respectively. The altitude is 24 ft. What is the area? Make a diagram.

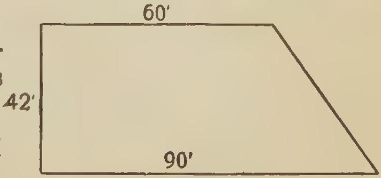


Fig. 23

2. A city lot has the dimensions shown in Fig. 23. What is the area?

3. The parallel sides of a field in the form of a trapezoid are 64 rd. and 82 rd. The distance between the parallel sides is 28 rd. What is the area in sq. rd.?

4. What trapezoids, if any, do you see in your classroom?

5. Find the areas of the trapezoids whose altitudes and parallel sides are given below:

	Altitude	One Side	Other Side
1.	14 in.	38 in.	49 in.
2.	26 ft.	65 ft.	94 ft.
3.	28 yd.	48 yd.	60 yd.
4.	380 yd.	960 yd.	1020 yd.

6. Draw on the blackboard a trapezoid that contains 250 square inches. Check your work.

7. Draw four trapezoids of different areas, then estimate the area of each. Check your estimates.

8. Cut from paper three trapezoids of different shapes, each of which has the same area.

9. Draw a square, a rectangle, a parallelogram, and a trapezoid having equal areas.

The Triangle

A figure bounded by three straight lines is called a *triangle.*

A *vertex* of a triangle is the meeting point of any two sides.

The *base* is the side on which the triangle rests.

The *altitude* is the perpendicular from the vertex to the base. See Fig. 24A. The altitude may in some cases be the perpendicular from the vertex to the base prolonged. See Fig. 24B.

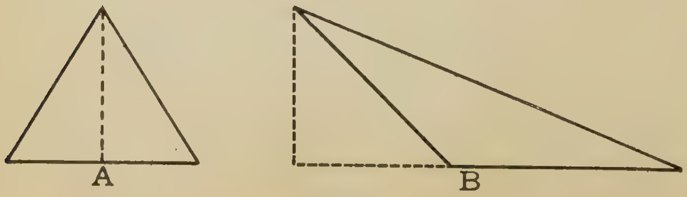
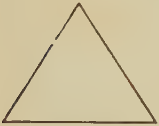
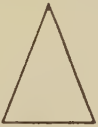
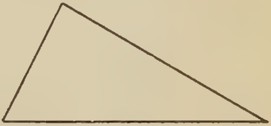


Fig. 24

A triangle having all three sides equal in length is *equilateral*.
A triangle with only two of its sides equal in length is *isosceles*.
A triangle with no equal sides is called a *scalene* triangle.


Equilateral


Isosceles

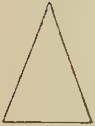

Scalene

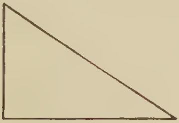
A triangle takes its name from the name of its largest angle. It may be acute, right, or obtuse.

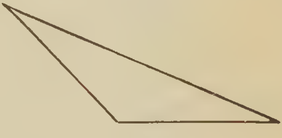
An *acute triangle* has all the angles acute.

A *right triangle* has one right angle.

An *obtuse triangle* has one obtuse angle.


Acute


Right


Obtuse

Draw three right triangles of different sizes.

Draw two equilateral triangles of different sizes.

Draw an isosceles right triangle.

Can you draw a triangle containing two right angles or two obtuse angles?

Do you know of any lot or field that is triangular in shape? If so, what kind of triangle is it?

Point out any triangles that you see in your schoolroom.

Report to the class any triangle that you see on your way to school.

Draw an equilateral, an isosceles, and a scalene triangle. Indicate the altitude of each by a dotted line.

Cut several kinds of triangles from paper. Take any of these triangles, and after cutting off two of the angles, place the two angles at C as shown in Fig. 25. What is the sum of the three angles of the triangle?

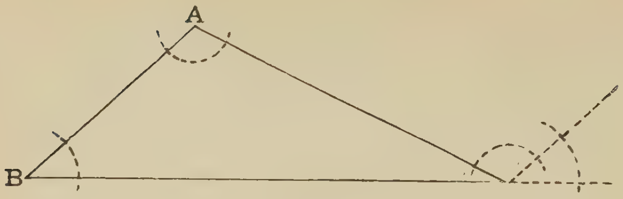


Fig. 25

Try this experiment with several paper triangles. Does the sum of the three angles of each triangle seem to be the same, no matter how large the triangle or what its shape may be?

To Find the Area of a Triangle

Draw a triangle similar to one of those shown in Fig. 26. From the vertex *A* draw a line parallel to the base, and from the point *C* draw a line parallel to the side *AB* to meet the line drawn from *A*. What kind of figure is *ABCD?* Compare the base and the altitude of the figure *ABCD* with the base and altitude of the triangle *ABC*.

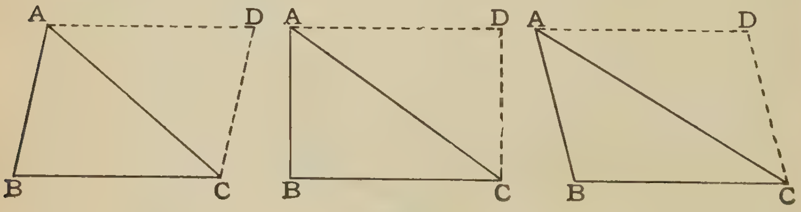


Fig. 26

Cut out the parallelogram *ABCD* that you have drawn, and cut it along the diagonal *AC*. What part of the area of the parallelogram is the area of the triangle *ABC?*

Since the area of a parallelogram is the product of its base and altitude, we may use the following rule for finding the area of a triangle:

The area of a triangle is one-half the product of its base and altitude.

Exercise 48

1. The base of a triangle is 12 in., and the altitude is 10 in. Find the area.

2. What is the area of a triangle having a base of 7 ft., and an altitude of 12 ft,?

3. A triangular field has a base of 34 rd., and an altitude of 20 rd. How many square rods does it contain?

4. A city park in the shape of a right triangle has a base of 264 rd., and an altitude of 110 rd. Find the area in acres.

5. Find the total area of the sides and ends of a barn, a cross section of which is shown in Fig. 27. The barn is 42 ft. long.

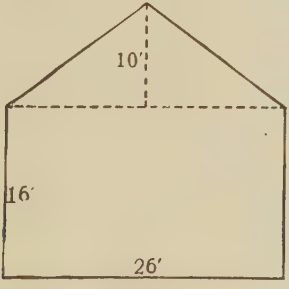


Fig. 27

6. Find the area of the field *ABCD* whose shape is shown by Fig. 28. $AC = 92$ rd., $DE = 55$ rd., $BF = 22$ rd.

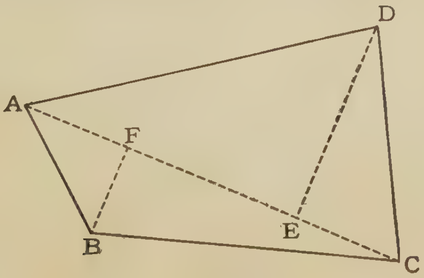


Fig. 28

7. Find the area of each triangle whose dimensions appear below.

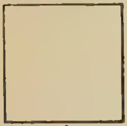
	BASE	ALTITUDE		BASE	ALTITUDE
1.	12 ft.	8 ft.	*5.*	92.5 rd.	45.6 rd.
2.	14.6 rd.	11.5 rd.	*6.*	8.6 in.	4.5 in.
3.	15 ft. 6 in.	7 ft. 9 in.	*7.*	14 ft. 4 in.	12 ft.
4.	380 ft.	240 ft. 6 in.	*8.*	26 ft. 9 in.	16.5 ft.

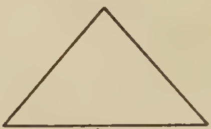
8. Cut out or draw six triangles of different areas. Estimate each area, then check your estimates.

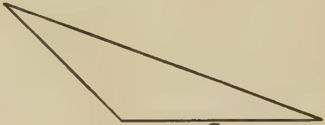
9. A girl made a school pennant in the shape of a triangle. The base of the pennant was 30 in., and the altitude was 64 in. What was the area?

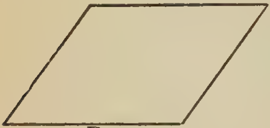
10. A street cuts from a city block a triangular area with a base of 142 ft., and an altitude of 95 ft. What will it cost to pave this area at $1.60 per sq. yd.? Make a diagram.

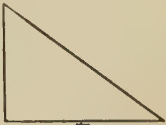
11. Name each of the following figures and tell how to find its area:

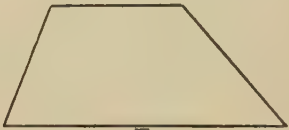

A


B


C


D


E


F

12. Do triangles with equal bases and equal altitudes necessarily have equal perimeters? Do they necessarily have equal areas? Illustrate by figures.

13. Is it possible for two triangles to have equal perimeters and yet have unequal areas? Illustrate by figures.

14. The end of a shed is in the form of a trapezoid. One of the parallel sides is 9 ft. long, the other is 7 ft. long. The shed is 18 ft. wide. What is the area of one end?

The Division of Land

In surveying the public lands of the United States, the surveyors first establish a line running north and south. This is called the *principal meridian.* They then survey a line running east and west, which crosses the principal meridian and forms right angles with it. This line is called the *principal base line.* At intervals of six miles, lines are established parallel to the principal meridian and to the principal base line. This system of lines divides the land into squares whose sides are six miles in length. These squares are called *townships* (*twp.*).

A row of townships that extends north and south is called a *range.* The townships in each range are numbered north and south from the principal base line. The ranges are numbered east and west from the principal meridian.

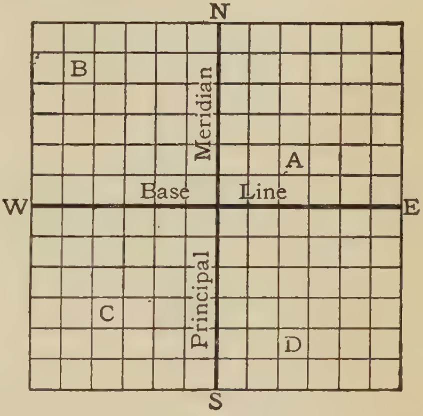


Fig. 29

To designate a township you must give the following information: (*1*) Its number and its direction from the principal base line; (*2*) the number and position of its range; (*3*) the name or number of the principal meridian.

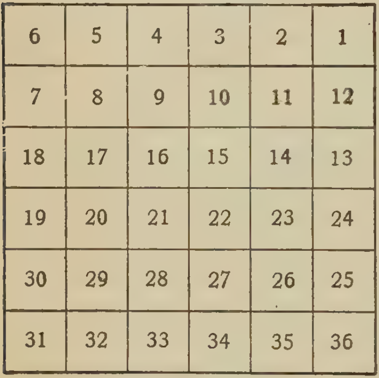
Township A, in Fig. 29, is designated as follows: 2 North, Range 3 east of the principal meridian. This designation is usually abbreviated as follows: *T. 2 N., R. 3 E.*

Give the designation of Twp. B in Fig. 29. Of Twp. C. Of Twp. D.

Use Fig. 29 to show the location of each of the following: T. 3 N., R. 4 E. T. 2 S., R. 2 W. T. 3 N., R. 2 W.

Draw a diagram similar to that of Fig. 29, and be prepared to indicate the location of any township designated.

A township is divided into *sections* one mile square, as shown in Fig. 30. The sections are numbered from right to left, back and forth, beginning at the northeast corner. Each section contains 640 acres, or one square mile. The sections are divided into *half-sections*, *quarter-sections*, and other fractional parts as shown in Fig. 31.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Fig. 30

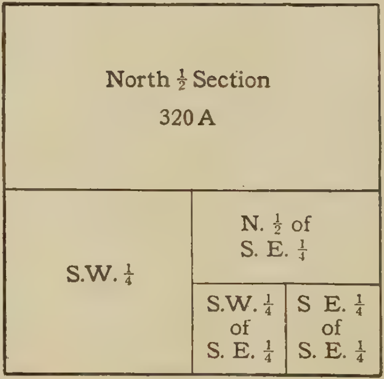

North $\frac{1}{2}$ Section
320 A
S.W. $\frac{1}{4}$
N. $\frac{1}{2}$ of S. E. $\frac{1}{4}$
S.W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$
S E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$

Fig. 31

Exercise 49

1. How many square miles are in a township? How many acres are in a township?

2. How many square miles are in a section? In a half-section? In a quarter-section? How many acres are in a half-section? In a quarter-section?

3. Read the following: (*a*) T. 3 N., R. 2 W. (*b*) T. 4 N., R. 2 S. (*c*) T. 3 S., R. 1 W. (*d*) T. 2 S., R. 4 E.

4. Make a drawing of a section, and locate each of the following tracts of land. Find the number of acres in each tract. (*a*) S. $\frac{1}{2}$ of N.E. $\frac{1}{4}$. (*b*) W. $\frac{1}{2}$ of S.W. $\frac{1}{4}$. (*c*) S.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$. (*d*) S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$. (*e*) N. $\frac{1}{2}$ of S.E. $\frac{1}{4}$. (*f*) W. $\frac{1}{2}$ of N.E. $\frac{1}{4}$.

5. How many rods of fence are required to inclose the W. ½ of N.E. ¼?

6. A farmer owns the north half of a section. What is the value of his farm at $110 an acre?

7. A man bought a quarter-section of land at $90 an acre. What was the total cost?

8. Draw a map of the section, then locate and find the value of the following piece of land at $120 per acre: N.W. ¼.

9. Mr. Hubbard owned a section of land. He sold the W. ½ of N.W. ¼ for $85 an acre. Draw a section and locate the part sold. How much did he receive? How much land had he left?

10. A farmer is offered $145 an acre for a quarter-section of land. What is the total amount offered?

11. A farmer who owned ¾ of a section of land, left the land to be equally divided among his five children. How many acres did each child receive?

12. Show by a drawing whether the following farms in the same section are adjoining: S.E. ¼ of the S.E. ¼ and the S.W. ¼.

13. Mr. Olsen owned a section of land. He sold the N.E. ¼ and also the S.E. ¼. After selling these two pieces of land, how many acres did he then own?

14. How many townships are in the county in which you live? What is the area of the county in square miles?

15. How many acres are in a farm that includes the S.W. ¼ of the N.W. ¼ and the N.W. ¼ of the S.W. ¼?

16. What is the perimeter of the following farm in rods: The S. ½ of the S.E. ¼?

17. The records show that Robert Carson owns the E. ½ of the S.W. ¼. Show the location on a drawing.

18. The grounds of a consolidated school are the S.W. ¼ of the S.W. ¼ of the N.E. ¼. How many acres are in the grounds?

19. Locate the following farms on a drawing: The W. ½ of the S.E. ¼ and the E. ½ of the S.W. ¼.

Carpeting, Papering, Painting, Plastering

Regardless of the trade you follow, or the business or profession in which you engage after leaving school, you will sometimes find it necessary to measure plane surfaces. This will be true even of those whose time is largely occupied with household affairs. When buying carpet or wall paper, when estimating the cost of painting or plastering, or when checking the merchant's or contractor's estimate for such work, a knowledge of this portion of arithmetic is necessary.

CARPETING: Carpet is usually sold by the running yard, regardless of its width. Usually, cheaper carpets are one yard wide, more expensive kinds are twenty-seven inches wide. Part of a width cannot be bought, because dealers refuse to split carpet. This must not be overlooked when estimating for carpet. In some cases allowance must be made for matching figured carpet.

WALL PAPER: Although wall papers vary in width, the better kinds being usually from thirty to forty inches wide, dealers and paper hangers usually estimate their charges on the basis of a standard width of eighteen inches. A *single roll* of wall paper contains eight yards; a *double roll* contains twice that amount. Part of a roll cannot usually be bought. In some cases allowance must be made for matching figured papers. Many paper hangers charge for their work on the basis of so much per 100 square feet; they estimate that three single rolls will cover that amount of surface. Some contractors make allowance for door and window openings when estimating; others do not. In solving the problems in *Exercise 50,* you should follow the custom of the contractors in your community.

PAINTING: The cost of painting is usually estimated by the square yard, no allowance being made for doors, windows, or other openings. Some contractors estimate their work by the *square,* which is 100 square feet.

PLASTERING: Plastering is usually measured by the square yard. There is no uniform practice among contractors regarding allowances for doors, windows, or other openings. In solving the problems given in *Exercise 50*, you should follow the custom of the contractors in your community.

The dimensions of a room are usually given in the following order: Length, breadth, height.

For example: The dimensions of a room 18 feet long, 14 feet wide, and 9 feet 6 inches high, are indicated as follows: 18 ft. × 14 ft. × 9 ft. 6 in. *or* 18′ × 14′ × 9′ 6″. What does 16′ × 14′ × 9′ 2″ indicate?

Exercise 50

1. Find the cost of plastering the walls and ceilings of three rooms, each 10 ft. in height, at the price per square yard now being charged in your community. Make no allowance for openings. The first room is 15′ × 12′, the second 14′ 6″ × 12′ 6″, the third 14′ × 12′.

2. Find the cost of papering the walls and ceilings of these three rooms at 50 cents a double roll, allowing 6 inches on each strip for matching.

3. How many gallons of paint are required for two coats on a signboard 42′ × 28′, if the first coat requires one gallon for 40 sq. yd., and the second coat one gallon for 48 sq. yd.?

4. How many square yards of plastering are required for the walls and ceiling of a schoolroom 32 ft. long, 24 ft. wide, and 12 ft. high, making an allowance of $33\frac{1}{3}\%$ for openings, blackboard, and baseboard?

5. The walls of a room are 8′ 6″ high above the baseboard, and the perimeter of the room is 64′. The paper for the room costs $1.10 a roll. What is the total cost of the paper, allowing 3 rolls to 100 square feet?

6. Find the cost of carpeting rooms having the following dimensions, with 27-inch carpet at $1.40 a yard, allowing 3 inches for matching each strip except the first.

1. 12 ft. × 18 ft.
2. 14 ft. 6 in. × 20 ft.
3. 16 ft. × 22 ft.
4. 24 ft. × 30 ft. 6 in.
5. 12 ft. 6 in. × 16 ft. 6 in.
6. 42 ft. × 60 ft.

7. Measure a room in your home, and find the cost of plastering and papering at current prices.

8. What will it cost to paint the floor of a room 24 ft. long and 18 ft. wide, at 45 cents per square yard?

9. A room is 18′×14′×9′. What will it cost to plaster the walls and ceiling, at 55 cents a square yard, making no allowance for openings?

10. If one gallon of paint is enough for two coats on 200 square feet of surface, how many gallons will be required to paint the walls and ceiling of your schoolroom?

11. As soon as convenient, visit a house or other building that is under construction, and inquire how the contractor estimates the following:

1. The amount of roofing required, and the cost?
2. The amount of plastering required, and the cost?
3. The amount of paint required, and the cost?

12. Using the methods of the contractor in your community, compute the cost of each of the following:

1. Plastering a room 24 ft. long, 16 ft. wide, and 9½ ft. high. The baseboard is 12 in. wide. There are two doors, each 7 ft. high and 3 ft. wide, and three windows, each 4 ft. high and 30 in. wide.
2. Plastering your schoolroom.
3. Painting the woodwork in your schoolroom.

13. How much will it cost to tint the walls and ceiling of a room 16 ft. long, 13 ft. wide, and 10 ft. high, at 16 cents a square yard?

14. Find the cost of painting a board fence 120 ft. long and 7 ft. high, at 38 cents a square yard.

15. If carpet $\frac{3}{4}$ of a yard wide is to be bought for a room 14 ft. long and 18 ft. wide, is it better to plan for laying the carpet lengthwise or crosswise of the room? Explain your answer.

16. What area can you cover with a single roll of wall paper 24 feet long and 18 inches wide?

THE BANK: WHAT IT IS AND HOW TO USE IT

Banks play an important part in the business life of everybody. Sooner or later you will have some sort of business with a bank, even though it is only so simple a matter as having a check cashed. It is well for you now to get some notion about banks, how they serve people, and how to transact business with them.

There are two chief kinds of banks: *Savings banks* and *commercial banks.* Most smaller banks, and many larger ones, provide both kinds of service. The savings bank accepts money for deposit, and pays interest on it. It therefore provides a safe place for keeping money, and at the same time makes saving profitable because of increased earnings.

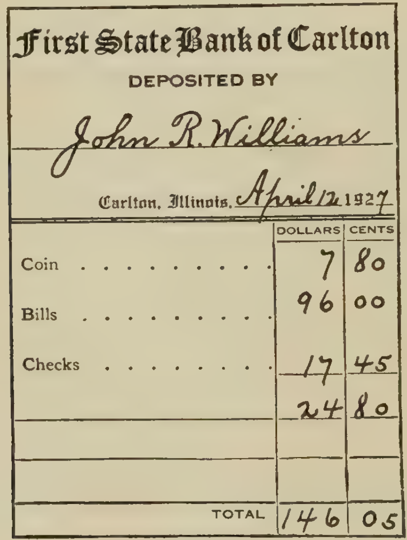
The commercial bank, as its name indicates, is organized to help people carry on their business. It performs many services, chief among them being the following: It receives cash and checks as deposits, it cashes checks made out by people who have money deposited in it or in other banks, it lends money to responsible people, it buys and sells notes. In our discussion of banking we shall first consider the transactions of the commercial bank, and later give some attention to the savings bank.

Opening a Bank Account: To open a bank account you must first be identified; that is, some one who is acquainted with you and also with the banker must introduce you. You then fill out an *identification card.* On this you write your name just as you expect to write it on the checks you write later. By comparing your signature on the identification card with any doubtful signature on a check, the clerk can usually tell whether you or somebody else signed a check.

You then receive a *pass book.* In this the bank clerk, called the *receiving teller*, records the date and the amount of money you deposit. The pass book should always be presented when a deposit is made.

Making a Deposit: All banks furnish blank *deposit slips* similar to the one shown in Fig. 32. To make a deposit you fill out a deposit slip, write your name, the date, and the amounts of coin, currency (paper money), and checks, together with the total. You then hand the deposit, the deposit slip, and the pass book to the receiving teller. He counts the deposit to see if the amounts agree with the sums stated on the deposit slip, records the total in the pass book, then returns the book to you.

Money deposited in a commercial bank is called a *checking account.* It can be drawn out by means of checks payable to the depositor or to any other person. A checking account does not usually bear interest.



First State Bank of Carlton

DEPOSITED BY

John R. Williams

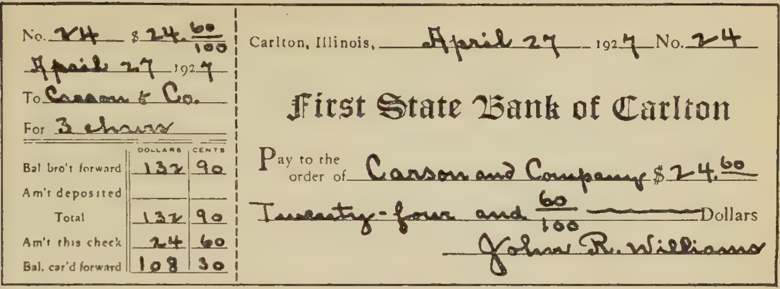
Carlton, Illinois, April 12 1927

	DOLLARS	CENTS
Coin	7	80
Bills	96	00
Checks	17	45
	24	80
TOTAL	146	05

Fig. 32

As shown by Fig. 32, who is depositing the money? How much did he deposit? How much coin? How much in bills? How much in checks?

Drawing a Check: The bank provides its depositors with blank *checks* similar to that shown in Fig. 33. Each check has a *stub* for use in making notations as to the amount of the check, the person to whom it is payable, the purpose for which the check is drawn, and the balance remaining in the bank after the check is paid.



No. 24 $24.60/100
April 27 1927
To Carson & Co.
For 3 chairs

	DOLLARS	CENTS
Bal bro't forward	132	90
Am't deposited		
Total	132	90
Am't this check	24	60
Bal. car'd forward	108	30

Carlton, Illinois, April 27 1927 No. 24

First State Bank of Carlton

Pay to the order of Carson and Company $24.60

Twenty-four and 60/100 Dollars

John R. Williams

Fig. 33

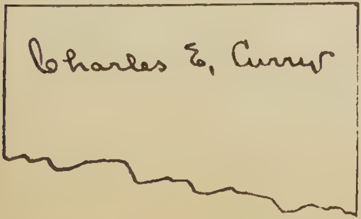
Fig. 33 shows that John R. Williams, the *drawer*, or *maker*, of the check, is paying Carson and Company, the *payee*, the sum of $24.60. Explain the use of the stub. Of what use are stubs to the owner of a pass book?

If a depositor wishes to draw money for himself from his own account, he should make the check payable to "Self" or "Cash."

CASHING A CHECK: To cash or deposit a check received from another person, you must sign your name on the back of the check. This is called *indorsing* the check. The signature is called an *indorsement*. The indorsement should always be written across the left end of the back of the check.

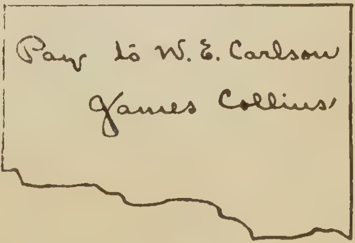
Fig. 34 shows the proper way to indorse a check. This kind of indorsement is called a *blank indorsement*. It makes the check payable to any one who holds it.

To make a check payable to a particular person, it must be specifically indorsed; that is, the payee must indorse it as shown in Fig. 35.



Charles E. Curry

Fig. 34



Pay to W. E. Carlson
James Collins

Fig. 35

When the bank cashes a check, the word "*Paid,*" and the date are stamped across the face, thus *canceling* it. The amount of the check is then deducted from the maker's balance on the bank's books. Canceled checks are returned to the maker, usually at the end of the month.

It is important that you remember the following about checks:

1. If possible, a check should be cashed or deposited on the day it is received.
2. A canceled check is a receipt of payment.
3. A check payable to bearer or cash may be cashed by any one who holds it.
4. A check bearing a blank indorsement may be cashed by any one who holds it.
5. Each person who indorses a check becomes in turn responsible for its payment.

Exercise 51

Use deposit slips and blank checks obtained from local banks, or made by pupils, to carry out the instructions of the following problems. Each pupil should use his own name if no other is suggested. Mark all checks "*Sample.*"

1. Make out a deposit slip showing the following deposit: 4 ten-dollar bills, 2 five-dollar bills, 1 one-dollar bill, 9 half dollars, 14 quarters, 3 dimes, 17 nickels, a check for $13.80, and one for $34.18.

2. Make out a deposit slip showing the following deposit: 5 ten-dollar bills, 7 five-dollar bills, 8 half dollars, 29 quarters, 14 dimes, 82 nickels, 38 pennies, and a check for $16.90.

3. An eighth-grade pupil deposits 2 five-dollar bills, 2 one-dollar bills, 9 quarters, 18 dimes, and 4 nickels. Make out the deposit slip. Use the name of some pupil whom you know.

4. If your deposits in a bank have been $38.90, $40.00, $15.30, $17.80, $9.60, and $3.85, and you have drawn checks for $19.25, $7.92, $13.25, and $18.20, what is your balance?

5. Write a check for $25, making yourself the payee and Henry M. Jones the maker. Indorse it.

6. At the beginning of the week a merchant's balance in the bank was $785.45. He made the following deposits during the week: Monday, $192.50, Tuesday, $154.35, Wednesday, $231.15, Thursday, $178.40, Friday, $216.90, Saturday, $298.15. He drew checks for the following amounts: $325.60, $114.38, $216.95, $82.40, $54.35, and $283.20. What was his balance?

7. On December 1 a boy's balance in the bank was $32.40. During the month he deposited the following amounts: $2.90, $3.85, $7.40, $1.50, and $5.40. He gave checks for $7.20, $3.95, $15.50, and $2.40. What was his balance at the end of the month?

8. Write a check for $12.50 to be paid to Richard Roe. Sign your name as maker. Who pays the money to Richard Roe? Who orders the money to be paid? Who gets the check when it is paid? Who finally gets the check that has been paid?

9. Suppose you have money deposited in a home bank, and wish to draw $12 for your own use. Write the check.

10. Why is the amount to be paid written in words and in figures?

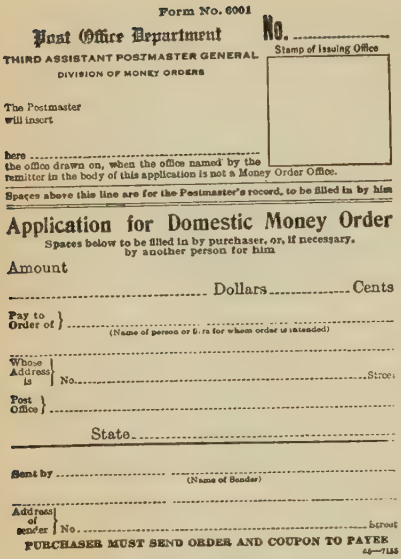
11. Why are no blank spaces left before or after the sum to be paid?

12. What should be written on the stub of the check mentioned in *Problem 7?*

Sending Money to a Distant Place

It is frequently necessary to send money to a distant place in order to pay a bill, or for some other purpose. This may be safely and conveniently done in any of the following ways:

REGISTERED LETTER: Money or valuable papers may be sent in a registered letter for a fee of fifteen cents in addition to the regular postage. Such letters are given particular care in the mail. It is never advisable to send coin by registered letter, as the coins may wear through the envelope and be lost. The postal authorities are not responsible in that case.



Form No. 6001

Post Office Department

THIRD ASSISTANT POSTMASTER GENERAL

DIVISION OF MONEY ORDERS

No. ....

Stamp of Issuing Office

The Postmaster will insert here .... the office drawn on, when the office named by the remitter in the body of this application is not a Money Order Office.

Spaces above this line are for the Postmaster's record, to be filled in by him

Application for Domestic Money Order

Spaces below to be filled in by purchaser, or, if necessary, by another person for him

Amount

.... Dollars .... Cents

Pay to Order of } ....
(Name of person or firm for whom order is intended)

....

Whose Address is } No. .... Street

Post Office } ....

State ....

Sent by ....
(Name of Sender)

....

Address of sender } No. .... Street

PURCHASER MUST SEND ORDER AND COUPON TO PAYEE

46—7155

Fig. 36

Postal Money Orders

These may be bought at the post office for any amount not to exceed $100. Any number, however, may be bought at one time. The order directs the postmaster in some stated place to pay the payee, or to some person indicated by him, a certain sum of money when the order is presented. A postal money order may be indorsed only once. The person who receives payment signs the order on its face.

To obtain a postal money order you must first fill out an application blank like that shown in Fig. 36. A small fee, that varies with the amount, is charged for each order. The cost of a postal money order is the sum of the amount and the fee.

Express Money Order: Express companies also sell money orders. These are payable at any office of the express company. An express money order may have any number of indorsements.

Table of Money Order Fees

Postal Money Order		*Express Money Order*	
Not over $2.50	5 cents	Not over $2.50	6 cents
Over $2.50 and not over $5	7 cents	Over $2.50 and not over $5	8 cents
Over $5 and not over $10	10 cents	Over $5 and not over $10	10 cents
Over $10 and not over $20	12 cents	Over $10 and not over $20	12 cents
Over $20 and not over $40	15 cents	Over $20 and not over $40	15 cents
Over $40 and not over $60	18 cents	Over $40 and not over $60	18 cents
Over $60 and not over $80	20 cents	Over $60 and not over $80	20 cents
Over $80 and not over $100	22 cents	Over $80 and not over $100	24 cents

TELEGRAPHING MONEY: Whenever immediate payment at a distance is necessary, money may be telegraphed. The amount to be sent, together with the fee for the service, is presented at a telegraph office. A telegram is then sent to the payee, instructing him to call at his local telegraph office and receive the money. It will be paid to him at that point, after he is identified. The person in charge of your local telegraph office will give you complete information about sending money by wire.

Exercise 52

1. How much will postal money orders cost for each of the following amounts: \$7.40, \$1.80, \$13.95, \$15.00, 90¢, \$25.85?

2. Which is the cheaper way to send \$32, by postal money order or by registered mail? How much cheaper?

3. Mr. Andrews, who lives in Texas, bought a camera for \$18.50 from a Chicago firm. Is it cheaper for him to remit by postal money order or by registered mail?

4. How much will an express money order cost for each of the following amounts: \$42.50, \$4.80, \$19.25, \$48.60? What would a postal money order for each amount cost?

5. If you wish to send \$8.40 by postal money order to a distant city, what must you do to obtain the order? How much will it cost?

6. Henry sent a postal money order for \$1.75 to Boston in payment for a book. How much did the order cost?

7. If you wished to pay a bill of \$54 in a distant city, would you use either a postal or an express money order? Why?

8. If you wished to pay a bill of \$29.60 in a distant city, and were to make payment either by registered letter or by postal money order, which method would you choose? Why? Would you choose the same method if the bill were \$79.50? Why?

9. How much will an express money order cost for each of the following amounts? 90¢ \$4.35 \$13.80 \$29.40

10. Mary's uncle in California sent her an express money order for $12 as a birthday gift. What was the cost of the order?

11. How much will a postal money order cost for each of the following amounts? 75¢ $2.45 $12.90 $32.40

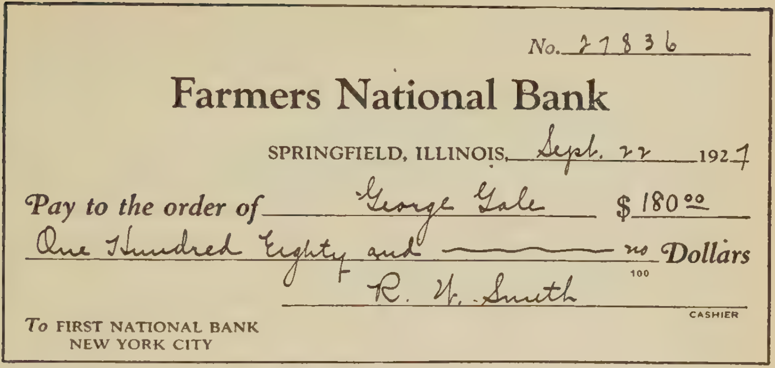
12. Is it a better plan to send money by registered letter or by postal money order? Why? Is it a better plan to use a registered letter or an express money order? Why?

13. For which of the following amounts will the cost of a postal money order and of an express money order be the same? $17.20 $5.60 $3.95 $1.70 $32.20

PAYING BY CHECK: It is possible, of course, to pay money at a distance by sending a check. Unless such checks are drawn against deposits in the banks of larger cities, usually New York City and Chicago, the payee frequently has to pay a fee of from five to fifteen cents for having them cashed. This fee is often charged to the maker of the check. The charge is called *exchange.*

PAYING BY BANK DRAFT: Most banks have deposits in the banks of some other cities, and draw checks against such deposits, just as you draw a check on a bank in which you have deposited money. An order drawn by one bank against its deposit in another bank is called a *bank draft.* A bank draft is preferable to a check for paying bills at a distance, (1) because there is greater certainty that the bank draft will be paid when presented for payment, (2) because a fee is seldom charged for collecting a bank draft.

When a bank sells a bank draft it gives the purchaser the benefit of its business relations with a bank in another city. For this service a charge is sometimes made. As in the case of the fee charged for collecting personal checks, this charge is also called *exchange.* The customary exchange fee is $\frac{1}{10}$ per cent of the face of the bank draft, the minimum charge being ten cents. Many banks do not charge their customers an exchange fee.



No. 27836

Farmers National Bank

SPRINGFIELD, ILLINOIS, Sept. 22 1927

Pay to the order of George Gale $ 180 00

One Hundred Eighty and —————— no/100 *Dollars*

R. H. Smith

CASHIER

To FIRST NATIONAL BANK
NEW YORK CITY

Fig. 37

Fig. 37 shows the usual form of bank draft. Who is the maker? Who is the payee? Where was the bank draft made? Where can it be cashed? Who is sending the money?

The Promissory Note

Banks keep on deposit only about thirty per cent of the money deposited. The remainder is lent to responsible people to help them carry on their business. Such loans are usually made for periods of 30, 60, or 90 days. Sometimes the borrower agrees to repay the loan "on demand," that is whenever the bank asks him to do so.

Before the bank delivers money on loan, the customer is required to sign a *promissory note.* This is a promise to repay the amount borrowed, on some future date, and to pay interest at a certain rate for the duration of the loan. Banks often require the borrower to get others besides himself to sign such a note. The additional signers are called *sureties.* They are responsible for paying the amount of the note in case the borrower himself fails to do so.

The person to whom the money is lent is called the *drawer* or *maker.* The one to whom the promise to pay is made is called the *payee.* The sum named in the note is the *face.* The date on which the note falls due is called the *date of maturity.*

If the place of payment is not stated, the note must be presented at the maker's place of business, or to him personally.

When a note merely bears the words *with interest*, but does not specify the rate of interest, interest is payable at a rate specified by law in the state in which the note is made.

If a note that does not draw interest is not paid when due, it will bear interest from the date when it is due until the date when it is paid.

If a note is payable so many months after date, you must count the calendar months from that date.

For example: A note dated October 16, due in two months, is due December 16.

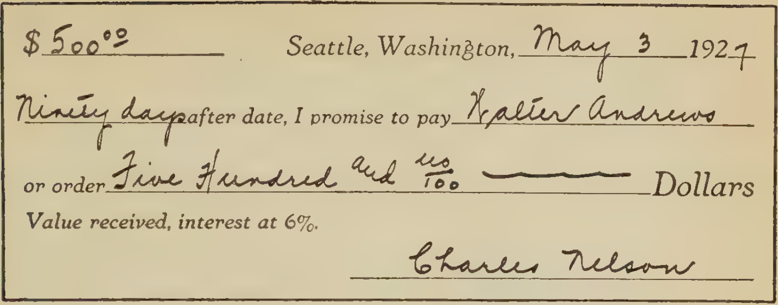
If a note falls due on Sunday or on a legal holiday, it is payable on the next preceding or on the next following business day, depending on the law of the state.

A note payable to bearer can be presented for payment by any one who holds it when it is due.

A note payable to a certain person, or to his order, can be transferred to another person or to his order. In such a case, the holder of the note must indorse it. Notes are indorsed in the same manner as checks. Notes that can be transferred to other persons are called *negotiable notes.* Negotiable notes can be bought and sold.

It is important that you remember the following about a promissory note:

1. It should state the place where and the time when it is given.
2. It should promise to pay to a certain person, to his order, or to bearer.
3. It should promise to pay a specified amount. The amount should be specified in both figures and words.
4. It should state the time the note has to run.
5. It should state the rate, if interest is to be paid.
6. It should state where payment is to be made.
7. It should state that value has been received.
8. It should be signed by the maker.



$500.00 Seattle, Washington, May 3 1927

Ninety days after date, I promise to pay Walter Andrews

or order Five Hundred and 00/100 —— Dollars

Value received, interest at 6%.

Charles Nelson

Fig. 38

Exercise 53

1. What is the face of the note in Fig. 38?

2. Who is the maker and who is the payee?

3. What is the date of maturity?

4. What amount must be paid on the date of maturity?

5. Is the note negotiable? Why?

6. If the note is sold to Charles M. Cook, by whom and how should it be indorsed?

7. Write a negotiable 60-day promissory note for $500, making Carl W. Olson the payee and yourself the maker. What is the date of maturity? If the note draws interest, what total amount must be paid on the day of maturity? If the note does not draw interest, what must be paid on the day of maturity?

8. Write a 90-day promissory note for $450.

9. A 60-day promissory note is dated May 14. When is it due?

10. Write a 90-day promissory note for $250, with interest at 6 per cent. Use the name of one of your classmates as payee. Write the word "*Sample*" across the note so that it cannot later be collected from you.

11. When is a promissory note due if it is dated March 7, and is payable in 3 months? In 6 months? In 90 days?

12. Write a promissory note payable to bearer.

13. Write a promissory note payable on demand.

14. Write the following notes. Tell who is maker, payee, and indorser. State the face, time, rate, and amount to be paid when each is due.

1. A note for $350 for 90 days, with interest at 6%.
2. A note for $605 for 60 days, with interest at 6%.
3. A note for $750 payable on demand, without interest.

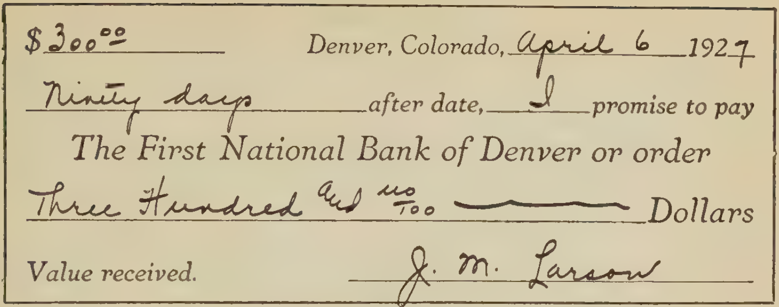
15. George Freeman borrowed $780 at 7% from the First National Bank on May 3, 1927. Mr. Freeman signed a note at the bank. He repaid the loan July 9, 1927. What was the total amount due on the note?

16. Fred Williams signed a demand note for $260 at 6% on June 5, 1928. The bank asked for payment on November 14, 1928. What was the total amount due?

Bank Discount

Most bank loans are made payable either on demand, or in 30, 60, or 90 days. Banks usually collect interest on loans at the time the loans are made. In such cases the bank is said to *discount* the note, and the amount deducted by the bank as interest is called the *bank discount*. The amount left after the discount has been deducted is called the *proceeds* of the note. Bank discount is computed on the *maturity value* of the note. If a note does not bear interest, its maturity value is the face. If a note bears interest, the maturity value is the sum of the face and the interest.

If J. M. Larson gives a bank the note shown in Fig. 39, and the bank is discounting notes at 6 per cent, the bank deducts ninety days interest, or $4.50, and gives Mr. Larson the proceeds, or $295.50. This note bears no promise to pay interest, because the interest is prepaid.



$300.00 Denver, Colorado, April 6 1927

Ninety days after date, I promise to pay

The First National Bank of Denver or order

Three Hundred and no/100 —— Dollars

Value received. J. M. Larson

Fig. 39

What is the maturity value and what is the bank discount on the note shown in Fig. 39? When does the note mature? How much, and to whom must Mr. Larson pay when the note matures? What is the face of the note? Who is the maker? Who is the payee?

Walter Andrews, holder of the note shown in Fig. 38, needs the money, so he discounts the note at 6 per cent on May 21. The note matures August 1. The date of discount, May 21, is 72 days before the note matures. When the note matures, the maker, Charles Nelson, must pay the face and the interest, a total of $507.50. The bank discount is, therefore, computed on $507.50 at 6 per cent for 72 days, the term of discount. The discount is $6.09, and the proceeds $501.41.

If Walter Andrews had not discounted the note until June 26, what would the proceeds have been? If the note had not been an interest-bearing note, on what amount would the bank discount for 72 days have been computed?

A note is not necessarily discounted at the rate of interest which it draws. Thus, a note that draws interest at 5% may be discounted at 6%, 7%, or even 8%.

To find *bank discount*, compute the interest on the maturity value of the note for the term of discount.

To find the *proceeds*, subtract the bank discount from the maturity value.

Exercise 54

1. Find the bank discount and the proceeds of a note for $800, without interest, due in 60 days, discounted at 6% on the day the note was drawn.

2. Find the bank discount and the proceeds of a note for $75, without interest, due in 90 days, discounted at 7% on the day the note was drawn.

3. Find the bank discount and the proceeds of a 90-day note for $1250, with interest at 7%, discounted 42 days before the date of maturity, at 7%.

4. Find the bank discount and the proceeds of a 6-months note for $250, with interest at 6%, discounted 18 days before the date of maturity, at 7%.

5. I hold a 90-day note for $360, dated April 19, bearing interest at 6%. I discount this note at a bank at 6% on May 23. What are the proceeds?

6. On March 18 Mr. Lewis sold a house and lot for $6800, and took from the buyer a promissory note for 60 days, with interest at 6%. Mr. Lewis discounted the note at 7% at a bank on April 1. What were the proceeds?

7. A farmer held a public sale on April 18. Among his receipts were nine 60-day notes totaling $3640, with interest at 6%. He discounted these notes at a bank, at 6%, on April 23. What were the net proceeds?

8. I gave my 90-day note for $2400 to a bank. What were the proceeds if the rate of discount was 6%?

9. Find the proceeds of the following notes:

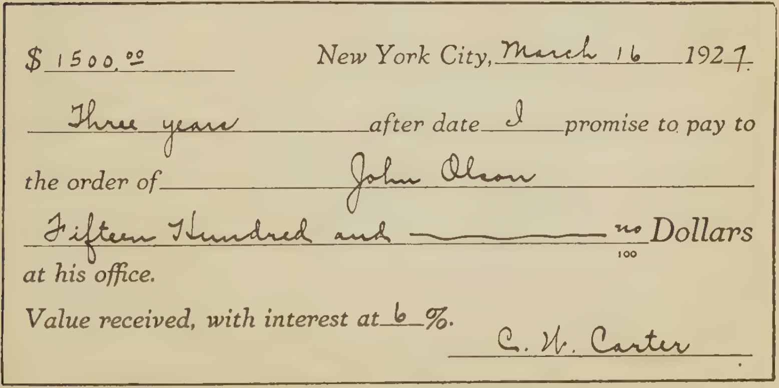
	Face	Date	Time	Interest	Date of Discount	Rate of Discount
1.	$800	Oct. 26	60 days	5½%	Nov. 18	6%
2.	$490	May 2	6 months	5%	Aug. 14	6%
3.	$780	Mar. 14	3 months	6%	May 29	6%
4.	$1470	June 8	90 days	6%	Aug. 20	6%

Partial Payments

One who gives his note for a long-time loan commonly pays the interest at regular intervals. At such times the borrower is often allowed, by special agreement, to make payments that will reduce the principal. The amounts so paid are called *partial payments*. The date and the amount of each payment are written on the back of the note.

If partial payments have been made on a note, the amount due on the day of settlement is usually computed by the United States Rule for Partial Payments. This rule is most often used when the time of the note exceeds a year. It has been made legal in most states. According to the United States Rule:

1. Payments must be applied to pay the interest due before any deduction can be made from the principal.
2. Payments that do not pay all of the interest due leave the principal unchanged until other payments are made which are sufficient to pay all interest then due. Interest is not charged on unpaid interest.
3. Any amount which is paid in excess of the total interest then due is applied to reduce the principal.



$ 1500.00 *New York City,* March 16 *192*7

Three years *after date* I *promise to pay to*

the order of John Olson

Fifteen Hundred and no/100 *Dollars*

at his office.

Value received, with interest at 6 *%.*

C. H. Carter

Fig. 40

Note to Teacher: This topic may be omitted if desired.

The note shown in Fig. 40 had the following payments recorded on the back: May 3, 1927, $200; June 14, 1928, $75; July 2, 1929, $650. What amount is necessary to pay this note at maturity?

SOLUTION

Interest on $1500 from March 16, 1927 to May 3, 1927 = $12.
$200, the first payment − $12 = $188, amount to be applied to reduce the principal.
$1500 − $188 = $1312, unpaid principal.
Interest on $1312 from May 3, 1927 to June 14, 1928 = $87.90.
The payment of $75 does not pay the interest due, so the principal is not changed.
Interest on $1312 from June 14, 1928 to July 2, 1929 = $82.66.
Total interest due July 2, 1929 is $87.90 + $82.66 or $170.56.
Total payments to date since last reduction of the principal are $75 + $650 = $725.
$725 − $170.56 = $554.44, amount to be applied to reduce the principal.
$1312 − $554.44 = $757.56.
Interest on $757.56 from July 2, 1929 to March 16, 1930 is $32.45.
$757.56 + $32.45 = $790.01, the amount due when the note matures.

Exercise 55

1. Find the amount due at maturity on a three-year note for $600, dated October 7, 1925, with interest at 6%, having the following payments recorded: December 9, 1925, $100; November 3, 1926, $25; October 18, 1927, $250.

2. Find the amount due at maturity, January 18, 1928, on a 6% note for $1450, dated July 18, 1926, having the following payments recorded: April 30, 1927, $35; December 6, 1927, $560.

3. Mr. Robinson gave his note for $5000 on April 12, 1926, with interest at 6%. The note matured in 3½ years. What was the amount due at maturity, if the following payments were made: November 1, 1926, $175; January 20, 1927, $250; April 30, 1927, $1800; December 1, 1927, $1000?

4. A note for $8450, with interest at 6%, was dated March 7, 1925. The following payments were recorded on the back of the note: January 15, 1926, $500; September 18, 1926, $450; January 8, 1927, $2500. The note was paid June 1, 1927. What was the amount due on that date?

5. Find the amount due on the date of settlement of a note for $2400, dated June 8, 1925, with interest at 6%. Payments were recorded as follows: December 12, 1925, $850; May 17, 1926, $200; October 24, 1926, $500; May 6, 1927, $1000. Final settlement was made December 20, 1927.

6. On May 7, 1925, C. W. Carlson gave a three-year note for $4250, with interest at 6%. The following payments were recorded: November 9, 1925, $400; February 18, 1926, $850; January 3, 1927, $750; November 6, 1927, $100. What amount was due when the note matured?

7. H. G. Davis gave his note for $12,780 on June 6, 1925, with interest at 6%. The note matured in four years. The following payments were recorded: June 28, 1926, $2450; November 12, 1926, $750; May 9, 1927, $1850; June 6, 1928, $5250. What amount was due when the note matured?

8. Find the amount due at maturity on a three-year note for $1860, dated March 9, 1926, with interest at 6%. Payments were recorded as follows: November 3, 1926, $100; January 7, 1927, $250; September 8, 1927, $650.

9. A two-year note for $6750, with interest at 6%, was dated July 7, 1925. The following payments were recorded: November 2, 1925, $250; March 3, 1926, $100; August 14, 1926, $2500. What amount was due when the note matured?

10. Write a demand note for $750 with interest at 6% payable semi-annually. George Harrison is the maker of the note, and Harry Moore is the payee. Date the note July 9, 1926. Record the following payments on the back of the note: February 18, 1927, $150; July 9, 1927, $90; March 8, 1928, $300. What amount was due on July 9, 1928?

11. Write a note for $1250 dated September 17, 1926, and payable three years from date, with interest at 6% payable semi-annually. Record the following payments on the back of the note: March 17, 1927, $260; September 17, 1927, $100; March 17, 1928, $350; September 17, 1928, $150. What amount was due when the note matured?

SAVING AND INVESTING MONEY

If you will bear in mind that "The penny speaks a language every man can understand," and if you will remember that more than half the people who reach the age of sixty are supported by relatives, friends, or charity, you may be impressed with the importance of saving, and so gain a special interest in this part of arithmetic.

If you are ever to be financially successful, indeed, if you are ever to be very successful at all, you must form the habit of saving a part of what you earn, you must save regularly, and you must invest your savings wisely. Every wise person urges you to save. Safe and profitable places are provided for your savings, no matter how small they may be.

Careful systematic saving should be followed by prudent investment. The first thing you should consider about an investment is its safety. Any banker is glad to give you sound advice about the safety of any investment you may consider. Safe investments do not usually pay more than the current rate of interest, and they can generally be sold for cash within a short time, in case one needs cash. Investments that promise an unusually high rate of interest are seldom safe.

The following pages tell of some methods provided to help you save and invest, and they furnish problems about savings and investments. Your local banker will doubtless be glad to give you information about these matters. He can also tell you about interesting savings plans not mentioned here.

The Postal Savings System

The postal savings system, established by the United States Government and carried on in conjunction with each post office, is a profitable savings plan available to everybody, no matter how little may be saved from time to time. Although this system pays no interest on amounts less than a dollar, it does provide a way for you to save as little as ten cents at a time. The postmaster will tell you all you need to know about this matter, and he will help you arrange to start a postal savings account.

Postal Savings Stamps: These are provided to encourage the saving of small amounts. They cost ten cents each. They do not draw interest, but when ten such stamps have been purchased and placed on a *Postal Savings Card,* the card may be exchanged for a *One Dollar Interest-bearing Postal Savings Certificate.*

Exercise 56

1. If I open a postal savings account on November 14, when does interest on my deposit begin? When is the first payment of interest due? Get this information at the post office.

2. If I open a postal savings account with $6 on October 8, what certificates will I receive, and when will interest on my certificates begin?

3. My uncle gave me postal savings certificates of $10 and $50, which he bought December 24. When did the interest on these certificates begin?

4. If I open a postal savings account on January 18 with a deposit of $28, what certificates will I receive?

5. Howard buys a 10-cent postal savings stamp each week day. What will be the total of his investments at the end of 52 weeks?

6. George deposited $10 in a postal savings account on June 26, and $25 on July 18. When will each amount begin to draw interest?

7. Explain how I may save 60¢ in a postal savings account.

8. If you know of any boy or girl who has a postal savings account, ask him to tell you how much interest the account has earned. Tell the class about it.

9. The pupils of the seventh grade organized a Thrift Army. They bought postal savings stamps. So soon as a pupil had enough stamps to exchange for an interest-bearing certificate, he was made an officer. Those who had no interest-bearing certificates were privates. What was the least amount an officer could have?

10. James, Fred, and Mary were the only officers when the Thrift Army was organized. James deposited $10, Fred, $7, and

Mary, $5. How many interest-bearing certificates did each have? How many savings stamps did the certificates represent?

11. There were 41 pupils in the Thrift Army. If each bought a postal saving stamp every Monday, how much money was turned in to the teacher on that day?

12. Peter buys 1 postal saving stamp each week. At that rate, how long will it be before he becomes an officer? How long will it be before Mabel becomes an officer, if she buys 2 postal savings stamps each week?

13. At the end of the school year of 38 weeks, it was found that every pupil had bought an average of 1 postal saving stamp weekly throughout the school year. How much money had been saved by the pupils of the room?

14. By the end of the year, every pupil had become an officer of the Thrift Army. How much had each pupil saved?

The Savings Bank

As we have learned, most banks have a savings department which makes it possible and profitable for people to save even small amounts. The bank's method of handling savings accounts differs from that used for checking accounts, which we have studied. You should find out the practices of the savings banks in your community, because the regulations regarding deposits, withdrawals, interest periods, and the computation of interest vary.

Most savings banks will let you start an account with one dollar. Interest at the rate of from two to five per cent is paid on deposits. Some savings banks compute interest on the smallest balance on hand during the interest period, which is generally one of three months or six months. In some banks, each deposit not made on the first of the month begins to draw interest on the first day of the month following the deposit. The depositor is allowed to withdraw money from a savings account. The pass book should be presented when the withdrawal is made. Checks are not used for savings accounts. Savings departments are now maintained in a great many schools.

DATE	DEPOSITS		INTEREST		WITHDRAWALS		BALANCE	
1926								
July 1	$340	50					$340	50
July 16	50						390	50
Aug 3					$60		330	50
Nov 8	160	50					491	
Dec. 23					50		441	
1927								
Jan. 1			$6	61			447	61
Mar. 22	85						532	61
July 1			8	94			541	55

Fig. 41

Fig. 41 shows a page from a savings account pass book. This bank pays interest on savings deposits semi-annually. Interest is paid on the smallest balance of each interest period.

How many deposits were made during the first interest period? What was the date and the amount of each? How many withdrawals were made during this period? What was the date and the amount of each? What was the smallest balance on this account during the first interest period? On what amount will the bank compute interest for this period? At 4 per cent, what interest did this depositor receive for the first interest period? What was done with the interest? What was the balance on this pass book at the opening of the second interest period? Is it usually profitable to disturb a savings account by making withdrawals? Give your reasons. If the interest is not withdrawn by the depositor, will it draw interest for the next interest period?

Compound Interest on Savings Accounts

The most desirable way to handle a savings account, because it is the most profitable, is to place the interest, when paid, in the savings deposit. If this is done, the interest draws interest, thus increasing the amount on deposit.

For example: If you have $50 in a savings bank that pays 4% a year semi-annually, then at the end of six months you receive interest at 2% on $50. This amounts to $1. By adding this $1 to the principal of $50, the new principal becomes $51. On this amount you receive interest at 2%, payable at the end of the next six months. This interest amounts to $1.02.

The table shown in Fig. 42 shows the amounts due at the end of different periods, if you make monthly deposits of from $1 to $10, interest being computed at 2½% every six months, and in every case added to the principal. A study of this table should impress you with the importance of saving even small amounts.

Monthly Deposits of	1 Year	2 Years	3 Years	4 Years	5 Years	6 Years	7 Years	8 Years	9 Years	10 Years
$1.00	$ 12.31	$ 25.25	$ 38.84	$ 53.12	$ 68.12	$ 83.88	$100.44	$117.83	$136.11	$155.31
2.00	24.62	50.50	77.68	106.24	136.24	167.76	200.88	235.66	272.22	310.62
3.00	36.93	75.75	116.52	159.36	204.36	251.64	301.32	353.49	408.33	465.93
4.00	49.24	101.00	155.36	212.48	272.48	335.52	401.76	471.32	544.44	621.24
5.00	61.55	126.25	194.20	265.60	340.60	419.40	502.20	589.15	680.55	776.55
6.00	73.86	151.50	233.04	318.72	408.72	503.28	602.64	706.98	816.66	931.86
7.00	86.17	176.75	271.88	371.84	476.84	587.16	703.08	824.81	952.77	1087.17
8.00	98.48	202.00	310.72	424.96	544.96	671.04	803.52	942.64	1088.88	1242.48
9.00	110.79	227.25	349.56	478.08	613.08	754.92	903.96	1060.47	1224.99	1397.79
10.00	123.10	252.50	388.40	531.20	681.20	838.80	1004.40	1178.30	1361.10	1553.10

Fig. 42

Interest computed in this way is called *compound interest.* In computing compound interest, the interest is added to the principal at the end of each interest period, to form a new principal for the next interest period. The difference between the final amount, called the *compound amount,* and the original principal, is called the *compound interest.* Interest may be compounded annually, semi-annually, or quarterly.

The computation of compound interest for long periods of time is very laborious, if made according to this method. To save time, and to avoid possible mistakes in making calculations, compound interest is usually computed by means of tables like that shown in Fig. 43.

Amounts of $1.00 compounded annually at various rates.

Periods	1%	1½%	2%	2½%	3%	3½%	4%	4½%	5%	6%
1	0.010000	1.005000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016
4	1.040604	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279	1.316932	1.3[illegible]862	1.407100	1.503630
8	1.082857	1.126493	1.171659	1.218403	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479
10	1.104622	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848

Fig. 43

Compound Interest Table.

For most practical purposes the figures shown in Fig. 43 are used only to *three* decimal places.

For example: The compound amount on $1 for 6 years at 4% correct to three decimal places is $1.265.

What is the compound interest on $360 for six years at 4%, if the interest is compounded annually?

Solution (*Using the Compound Interest Table*)

First find the column marked "4%." Run down this column to the line opposite the figure 6 in the column at the left-hand side, marked "Periods." The figures found there, $1.265319, show the amount of $1 at compound interest for 6 years at 4%. Multiply the principal, $360, by this amount to find the amount of $360 at interest which is compounded annually for six years at the rate stated.

$455.51 (compound principal) − $360 (original principal) = $95.51.

If interest is compounded semi-annually, use half the given rate and twice the number of years and proceed as above.

Exercise 57

When solving the problems of this exercise you should use the method commonly used by the savings banks of your community.

1. A pupil deposited $3 in a savings bank each month, and made no withdrawals. The bank paid 4% interest, compounded semi-annually. How much did the pupil have in the bank at the end of 3 years?

2. A girl who had a balance of $164.90 in a savings bank on July 1 made the following deposits and withdrawals during the remainder of the year. *Deposits:* August 1, $15, September 18, $12.40, December 1, $10. *Withdrawals:* October 23, $25, December 21, $14. What was the balance on January 1, with interest at 3½% a year, computed semi-annually?

3. A boy had a balance of $48.60 in a savings bank on January 1. He made the following deposits and no withdrawals: February 6, $15, March 18, $34, May 3, $12.50. What was his balance on July 1, interest being computed semi-annually at the rate of 3% a year?

4. What will $300 amount to at the end of 3 years, if it draws interest at the rate of 4%, the interest being compounded semi-annually? What will the compound interest amount to? What will the simple interest amount to at the same rate and for the same time?

5. On July 1 Henry deposited $150 in a savings bank which paid 4% interest, compounded semi-annually. If he made no withdrawals, what amount had he to his credit 4 years later?

6. Two brothers were left $1200 each by an uncle. One deposited his money in a savings bank which paid 4% interest compounded semi-annually. The other lent his to a friend at 4½% simple interest. Which had received the larger return for his money at the end of 5 years?

7. If you put $100 in a savings bank paying 3% interest, compounded semi-annually, and make no withdrawals, what amount will be to your credit in 5 years?

8. Which will pay more, and how much, \$300 at 5% interest compounded semi-annually for 5 years, or \$325 at $4\frac{1}{2}$% simple interest for 5 years?

9. What will \$380 amount to if it draws interest at 3% compounded semi-annually for 4 years? (Use the table.)

10. What will \$500 amount to if it draws interest at 5% compounded semi-annually for 5 years? (Use the table.)

11. What will \$4500 amount to if it draws interest at 5% compounded semi-annually for 3 years? (Use the table.)

Bonds as an Investment

You have learned that when you borrow money you must usually give your note, which is a promise to repay the sum borrowed at a certain time, and an agreement to pay interest for its use during that time. Large business concerns (corporations) often need to borrow money, too. So does the government of the nation, and those of the states, counties, cities, and school districts. Often these organizations want to borrow so much money at one time that no one person can supply enough. Hence they must arrange to borrow from anyone who is willing to loan to a responsible organization. To each of those people the borrower in this case gives a *bond*. A bond is simply a special form of note, an agreement to repay borrowed money at some future time, and to pay interest on the money at a certain rate and at stated intervals.

The person who lends you money on your note often requires other signatures besides your own in order to assure payment. Such other signers are called *sureties*. The corporation that issues bonds also has to furnish some guarantee of repayment, and of payment of the interest as it falls due. Instead of getting people to act as sureties, the corporation promises to turn over its property to a representative of the bondholders, in case it does not repay the borrowed money when the bonds become due. Property pledged as a guarantee of payment is called *security*.

Like notes, bonds too are *negotiable*, that is, they may be bought and sold. In fact, the money that the corporation wants is obtained by selling its bonds through bankers and other persons engaged in the business and known as *brokers*. Those who buy and sell bonds charge a fee for their services. This fee, which is usually $\frac{1}{8}$ of 1 per cent of the face value of the bond, is called *brokerage*.

Bonds are issued in various denominations ranging from $50 to $1000 or more. Interest is paid quarterly, semi-annually, or annually, according to the agreement in the bond itself. The interest is always computed on the face of the bond. Brokers usually refer to bonds as 4's, 5's, 6's and so on, according to the rate of interest the bond bears. Thus, *City of Washington 5's* refers to bonds issued by the City of Washington, bearing interest at 5 per cent.

Most bonds have small certificates or *coupons*, there being one for each interest payment. When an interest payment is due, the coupon bearing the proper date is clipped from the bond and presented at the office of the corporation that issued the bonds, where it is redeemed for cash. Banks will also cash such coupons.

Registered bonds do not have coupons. The name of the owner of each registered bond is written on the bond itself, and also on the books of the issuing corporation. This kind of bond cannot be transferred to another person without the indorsement of the original owner. When the interest on a registered bond falls due, the issuing corporation mails a check to the owner.

Throughout the country bonds are bought and sold in large numbers, just as other things. When times are good many people have money to invest. There are then a greater number of persons who want to buy bonds, and as a consequence, the prices of good bonds go up, just as the price of any article increases whenever a number of people want to buy it. On the other hand, when times are not so prosperous, and many people are out of work, holders of bonds who need cash are often willing to sell the bonds for less than they paid for them. At such times, as at others when many people want to sell the same kind of thing, the price falls and one can buy some bonds for less than they cost when issued.

Bonds are usually issued *at par*, that is, at the face value of the bonds themselves. Bought at that rate, a $100 bond bearing interest at 5% will net the holder $5 a year in interest. If a bond sells *below par*, that is at less than its face value, it nets the holder more than the agreed interest return, because the interest is always paid on the face of the bond. That is, if a $100 bond bearing interest at 5% could be bought for $80, the holder receives an income of $5 on an investment of $80, which amounts to $6\frac{1}{4}\%$. If the same bond sold *above par*, say for $110, the income would be only $4\frac{6}{11}\%$. The market value of both stocks and bonds is usually either above or below par.

The date of maturity of a bond should also be considered when computing the rate of income on the investment, but in solving the problems in an elementary book like this it is unnecessary to consider that factor.

Unless a bond is bought on the day when one of the interest payments falls due, the buyer must pay, in addition to the price at which he buys the bond, such interest as has accrued on the bond from the date of the last interest payment up to the date of purchase. This is called *accrued interest.*

Since the savings bank usually pays not more than 4 per cent on deposits, it is a good plan to invest your savings in a good bond as soon as you can purchase one. The banker in your town is always willing to sell you first class bonds. He will tell you some things about bonds that you have not learned from this book, and he will give the following reasons and others why bonds are a good investment.

It is important that you remember the following about bonds:

1. They can be bought in small denominations.
2. The income from bonds is reasonably certain.
3. The interest is usually paid promptly.
4. Good bonds can usually be sold without much delay.
5. Bonds may be used as security when borrowing money.
6. Most bonds pay a higher rate of interest than do savings accounts.
7. The income from many bonds is not taxed.

Exercise 58

1. In order to build and equip a school building, \$260,000 worth of 20-year bonds paying interest at $5\frac{1}{2}\%$ were sold at par. Find the total amount of interest to be paid on these bonds. What annual interest will the owner of 4 such bonds receive, each bond having a face value of \$100?

2. A city issued eight hundred \$100 bonds paying 5% interest for 25 years. Mr. Adams bought 7 of these bonds at $102\frac{1}{2}$, including brokerage. What was the total cost? What was his yearly income from the bonds?

3. What will eight \$100 bonds of the Fourth Liberty Loan cost at $101\frac{1}{2}$, including brokerage, if no accrued interest is due? These bonds pay $4\frac{1}{4}\%$ interest. What are the yearly dividends on the eight bonds?

4. I bought twelve \$100 bonds of the Third Liberty Loan when these bonds were quoted at $91\frac{7}{8}$, brokerage $\frac{1}{8}$ of 1 per cent. What was the total cost? These bonds pay $4\frac{1}{4}\%$ interest. What is my annual income from them?

5. If possible, find in a newspaper the present market price of Fourth Liberty Loan bonds. How many \$50 bonds of this loan could I buy for \$400, not counting brokerage, and how much money would be left over?

6. What rate of income are Third Liberty Loan bonds actually yielding on the investment at the present quoted price?

7. Mr. Olson bought a \$50 bond at $52\frac{1}{2}$, including brokerage. The bond pays $5\frac{1}{2}\%$. What rate does the bond yield on the investment?

8. The United States government sold First Liberty bonds amounting to \$3,035,226,850. These bonds pay $3\frac{1}{2}\%$ interest per year. What is the amount of the annual interest payment?

9. Mr. Knox bought a 5 per cent bond for \$90. The par value of the bond was \$100. What is the rate of income on the investment?

The Stock of a Corporation

Some large concerns, such as railroads, steamship lines, and great manufacturing establishments, require so much money for carrying on business that no one person or small group of persons can furnish the needed funds. Such concerns find it necessary to obtain money from those who are willing to furnish funds for carrying on the business in return for a chance to share in the profits, if any are made. A business concern so organized that a great number of people may contribute money and share in its profits, without taking an active part in the management of the business, is called a *corporation*. The corporation must be organized according to the law of some state. The permit to do business which the corporation receives from the state is called a *charter*. The charter clearly states all the regulations that are to govern the important affairs of the corporation.

The *capital stock* of a corporation is the money paid in by all the members in order to start and carry on business. The capital stock is divided into *shares*, each of which has the same value, usually $100. The face value of a share is called the *par value*.

Each person who buys shares becomes a *stockholder*. Each stockholder receives a *certificate of stock*, signed by the officers of the company. This certificate indicates the number of shares that he holds. For each share, the stockholder is usually allowed one *vote* at the election of the *directors*, who manage the affairs of the company, declare dividends, and determine the business policy of the corporation.

After a corporation has carried on its business for a certain length of time, all debts and expenses having been paid or arranged for, all or a part of the profits, if there are any, may be divided among the stockholders. Such a division of profits is called a *dividend*. A dividend is usually expressed as a certain per cent of the par value of the stock. Any undivided profits are called *surplus*.

Many companies issue two kinds of stock: *Common* and *preferred*. *Preferred stock* entitles the owner to a certain agreed per cent of dividend, if there are enough profits to pay such a dividend, but it does *not* assure a dividend. *Common stock* entitles the owner to share in the profits of the corporation only after all other claims have been taken care of. Some stocks pay no dividends at all when the company earns no profits.

Stocks are bought and sold every day, just as bonds are. The sales are made in the *stock exchanges* located in the larger cities. There the brokers buy and sell to each other, and bid for stocks much as people bid at an auction sale. Brokers buy and sell stocks for their customers, who are called *clients*. The broker charges a fee for his services. The client who sells stocks through a broker, and the one who buys from a broker, must each pay a brokerage fee.

People buy stocks because of their desire to share in the profits of a corporation; they may sell them because they are unwilling to do without profits at times when the corporation is not doing a profitable business. When a corporation is making large profits, and declaring large and frequent dividends, its stock usually sells *above par*. When the dividends are infrequent or small, many people sell the shares of stock they hold. As in the case of bonds and other things, when many people want to sell the same thing at the same time, the price of the article—stock, bond, or anything else—goes down.

The daily newspapers usually give prices of the better known stocks. When a stock is priced or *quoted* at 95¾, this means that a share may be bought for $95.75. A stock quoted at 111⅛ means that a share may be bought for $111.12½. Fig. 44 shows the usual form in which the prices of stocks, called the "quotations," appear on the financial page of a newspaper.

	Open	High	Low	Close	Previous Close
American Woolen	95¾	96½	95½	96	95½
Baldwin Locomotive	126¼	127½	126¼	127¼	126¼
Baltimore & Ohio	48¾	48¾	48	48	49
Chesapeake & Ohio	71½	73	71½	72	71
General Electric	192	194	191	194	193
United States Steel	108½	109¾	108	109½	108

Fig. 44

In the table which appears at the foot of page 162 "American Woolen" indicates the stock of the American Woolen Mills Company, a concern that produces woolen cloth. "Baldwin Locomotive" indicates the stock of the Baldwin Locomotive Company, a concern engaged in making locomotives, railroad cars and equipment, and other steel and electrical products. Brokers and others keenly interested in stocks are familiar with the facts concerning companies whose stocks are listed on the stock exchange. "Open" indicates the opening sale price of the stock on the day the quotation was made. "High" indicates the highest sale price, "low" the lowest sale price, and "close" indicates the price of the last sale for that day. "Previous close" indicates the last price offered on the business day preceding the date of quotation.

As shown by the quotations given at the foot of page 162, what was the first price paid for a share of American Woolen Mills stock? What was the lowest price paid during the day's trading? What was the highest price paid? If you had bought 10 shares of this stock at the low price, what would it have cost you? If you had sold it at the high price on the same day, what would you have received for it? How much profit would you have made? Would this have been a net profit? Why?

Exercise 59

In solving the problems of this exercise, a share of stock should be regarded as having a par value of $100, unless some other value is given. Broker's fee should be reckoned according to the following rates: Shares selling under $10, fee $7\frac{1}{2}$ cents; shares selling between $10 and $125, fee 15 cents; shares selling at $125 and above, fee 20 cents. Minimum fee for any sale, $1.

1. J. M. Arnold and C. W. Peterson organize The Superior Hammer Company, with a capital stock of $100,000. Mr. Arnold invests $25,000, and Mr. Peterson invests $30,000. How many shares does each receive? How many shares remain to be sold? To how many votes are Mr. Arnold and Mr. Peterson together entitled? What is the importance of this in determining the way in which the affairs of the company shall be run?

2. If The Superior Hammer Company declares a dividend of 8%, how much will the holder of 12 shares receive?

3. L. C. Weber bought 20 shares of stock in The Superior Hammer Company at par. What is the face value of his shares? To how many votes is Mr. Weber entitled at the stockholders' meetings?

4. At the end of the year a dividend of 6% was declared on the stock. How much did Mr. Weber receive?

5. If 40% of the stock of The Superior Hammer Company is preferred stock, and the remainder is common, what is the par value of all the common stock?

6. How many shares of preferred stock were issued by The Superior Hammer Company? (See *Problem 5.*)

7. During one year, the directors declared a dividend of 9% on the common stock, after paying a dividend of 6% on the preferred stock. L. W. Harrison owns 12 shares of common stock and 15 shares of preferred stock. What total dividend did he receive?

8. If $100 is the par value of each of the stocks listed on page 162, which ones have a market value above par, and which ones have a market value below par?

9. I bought 100 shares of General Electric at the opening price, and sold at the closing price. How much did I gain? Allow brokerage for both buying and selling.

10. I bought 20 shares of American Woolen at the highest price quoted, and sold two days later for $99\frac{1}{4}$. How much did I gain? Allow brokerage for both buying and selling.

11. I bought 50 shares of United States Steel at $108\frac{1}{2}$, and sold one day later at 106. How much did I lose? Allow brokerage for both buying and selling.

12. What was the difference in the highest and lowest prices for each of the stocks quoted?

13. Including brokerage, find the cost of each of the following: 45 shares of Northern Pacific at $76\frac{3}{8}$, 70 shares of Western Union at $124\frac{1}{2}$, 15 shares of Canadian Pacific at $151\frac{1}{2}$, 50 shares of International Harvester at $112\frac{3}{8}$.

14. How much should a broker remit to a customer after selling 80 shares of Illinois Central stock at 110½?

15. I bought 60 shares of American Radiator stock when it was quoted at 125½, and sold it when quoted at 94¾. Allowing brokerage for both buying and selling, how much did I lose?

16. A company with a capital stock of $250,000 declares a 6% dividend. What is the total amount of the dividend? How much does the holder of 20 shares receive?

17. What is the income from 250 shares of stock paying a 5½% dividend?

18. How much is a 7% dividend on 35 shares of stock?

19. A company having a capital stock of $400,000 declared a dividend of $32,000. How much did the owner of 25 shares receive?

20. A manufacturing company with a capital stock of $300,000 earned $27,000 above all expenses. It was decided to put $6,000 in a surplus fund, and to pay the rest in dividends. What was the rate of dividend?

21. I bought stock at 60, including brokerage. The stock paid a 5% dividend. What rate did my investment yield?

22. Stock which cost me $150 a share, including brokerage, pays a dividend of 12%. What rate does the investment yield?

23. A friend told me that a stock which cost $120, including brokerage, and which pays a dividend of 8%, will yield 6% on the amount invested. If the friend was mistaken, what was the amount of his error?

24. I bought 40 shares of stock at 125, including brokerage. A dividend of 8% was declared. What income did I receive from the stock? What rate did my investment yield, including brokerage?

25. An electric company is paying quarterly dividends of 2½%. What annual income does the holder of 30 shares of this stock receive, if the par value is $50 a share?

REVIEW: STOCKS AND BONDS

1. A company having $80,000 capital stock pays a 6% dividend. Find the total dividend, and the amount paid to the owner of 30 shares of stock.

2. If I buy 50 shares of stock at 119$\frac{7}{8}$, and sell them the following day at 121$\frac{1}{2}$, what is my gain? (Do not include brokerage.)

3. A man owns 20 of the 2000 shares of the capital stock of a bank. What per cent of the stock does he own?

4. Why are stocks sometimes above par and sometimes below? Give an example.

5. A broker sold 250 shares of stock at 115$\frac{1}{2}$. After deducting brokerage, how much will the owner of the stock receive?

6. When 6% stock is selling at 120, including brokerage, what rate does the investment yield? (6% stock is stock that pays 6% a year on the par value.)

7. I bought 10% stock at 80, including brokerage. What rate did the investment yield?

8. I am satisfied to have my money earn 5%. How much more or less than this rate will I get from 8% stock at 120, including brokerage?

9. How much can a man afford to pay per share for 8% stock, if he is satisfied with 6% on his investment?

10. What is the annual income from $4260 invested in 5% bonds bought at $106.50, including brokerage?

11. A company with a capital stock of $100,000 sold $40,000 worth of 5% bonds. The net earnings for the year were $12,600. What surplus was left after paying the interest on the bonds, and a 7% dividend on the stock?

12. The bonds of the First Liberty Loan pay 4$\frac{1}{4}$%. What interest is due semi-annually on $5000 worth of these bonds?

13. A man has an annual income of $480 from 4% bonds. How many such bonds does he own, if the par value is $100?

14. A town issues $80,000 worth of $5\frac{1}{2}\%$ bonds in order to build a high school. How much interest is paid to the bondholders annually?

15. Find what bonds have been issued by your city, county, or school district recently. What rates of interest do they bear? What was the total amount of each issue? When do the bonds mature? For what purposes were the bonds issued?

16. A woman wishes to invest a sufficient amount in 5% bonds, par value $50, to yield $800 annually. The price of the bonds is 48, including brokerage. How much must she invest?

17. The Central Milling Company, with a capital stock of $400,000, declares a dividend of 6%. What is the total amount of the dividend? Mr. Peterson owns 140 shares, par value $50. What dividend will he receive?

18. A company with a capital stock of $150,000 distributes $12,000 in dividends. How much will be paid in dividends to the holder of 35 shares with a par value of $50?

19. The par value of the preferred stock of a certain company is $100, and the market value is $115.25. What is the par value of 50 shares? What is the market value of 50 shares? What is the difference between these values?

Real Estate Investments

If you have money to invest, and care to do so, it is often possible to lend it to persons who own real estate, and who are in need of cash. Such persons are willing to give you a claim on their property as a pledge that they will repay the loan, and pay the interest when due. The real estate becomes the security for the loan. Such a pledge is called a *mortgage*.

If the taxes and other charges against the real estate have been paid, and if the borrower asks for not more than 40% or 50% of the estimated value of the property, such loans are usually safe and profitable. Real estate mortgage loans usually pay from 5% to 7% interest.

Exercise 60

1. Mr. Landis owns a house and lot valued at $6000. He borrowed $2500 at 6%, giving a mortgage on the property as security. Was this probably a safe investment for the lender? Why? What yearly interest did Mr. Landis pay on the loan?

2. A man owns a $4000 mortgage bearing 5½% interest, a $5800 mortgage bearing 6% interest, a $2500 mortgage bearing 5½% interest, and a $7200 mortgage bearing 6% interest. How much is his average monthly income from these investments?

3. A farmer borrowed $12,000, and paid 6% interest each year. He repaid the loan at the end of 5 years. What was the total amount of interest that he paid?

4. What must be the monthly rental on a property valued at $7200 to yield a gross return of 10%?

5. A house costing $4800 was built on a lot that cost $1400. The taxes, insurance, repairs, and depreciation are estimated at $192 per year. What monthly rental must be charged to net the owner 6% on the investment?

6. A property that rents for $28 a month costs $132 a year for upkeep. How much can I afford to pay for this property if I expect to realize 6% on my investment?

7. If a man buys a house for $5400, he will need to borrow $1500 at 6%, and he will have to sell a $3900 mortgage that pays 5½%. The taxes and insurance on the house are $92 a year. The repairs and depreciation are estimated at $90 a year. The man now rents a house at $45 a month. How much will he gain or lose a year by buying the house?

8. A woman bought a house for $4800, giving a mortgage for $2500 at 6%, and paying the remainder in cash. She paid $120 a year for taxes, insurance, and repairs, and rented the property for $36 a month. After two years she sold the property for $5400 cash, and paid the mortgage. How much more or less would her money have earned if she had invested it at 6%?

9. A man bought a farm for $9250. He spent $750 for improvements, and four months later sold the farm for $11,660. If he received no yield from the farm, and paid no taxes or insurance, what was his per cent of gain on the total investment, allowing 6% for the use of his money for the four months?

Common Sense in Investments

No matter how little you may have to invest, you will be offered chances to place your money in the keeping of others. Not everyone who solicits you to invest is interested in helping you to invest wisely. All sorts of schemes are proposed to prospective investors, and many of them are planned to get your money and to give little or nothing in return. The experience of years does not protect some people against loss through worthless and poor investments. The inexperienced should be particularly alert and careful. Each year millions of dollars are lost by investors, and as you may suppose, most of this money is lost by those who are so honest they do not suspect others of trickery, and who can least afford to lose.

Prudence is not only the better part of valor, it is also the better part of wisdom. This is especially true of making investments of any kind. You should always carefully and deliberately investigate a prospective investment, and even though you are shrewd, you can profitably take the advice of others whose training and experience qualify them to know, and whose honesty assures you that they are telling you the truth. Take plenty of time to get information, to weigh and consider every possibility. The really good and safe investments are not all going to be snapped up today. Some of the best will be available tomorrow. So do not buy in haste and repent at leisure. Make your dollars work as hard for you as you work for your dollars.

The following problems will test your judgment in some of these matters. If you cannot determine what to do, ask some one who knows to help you decide. Your judgment will be shown even by your choice of the person whose advice you ask.

Exercise 61

1. The First National Bank of a certain city advertises that it will pay 4% interest on savings deposits. The bank was established in 1870. Do you think it is probably safe to deposit your savings in the bank? Why?

2. The street car company in a manufacturing city whose population doubled from 1900 to 1920 offers its stock at 105. For the past fifteen years the company has paid annual dividends of 4% to 7%. Is this stock apparently a good investment? Give your reasons.

3. A salesman whom you do not know is selling the stock of the Rock Ridge Mining Company. He offers to sell you any amount of the stock at $50 cash per share, each share having a face value of $100. The company is said to own the land where the mine is to be sunk, and it is said that as soon as machinery is bought work will be started. Is this probably a safe investment? Give your reasons.

4. If you had $500 to invest, and were to choose one of the following, which would you choose and why?

1. A six per cent $500 bond issued by a progressive city.
2. A personal loan of $500, at 8%, on a promissory note given by a person whom you very recently met.
3. Stock in a newly organized company which advertises that it will pay at least 10% on the investment.

5. If an agent tried to sell stock to you, and told you that the stock would yield 12% on the investment, whose advice would you seek before you seriously considered the investment? Why?

6. If your mother had $7500 left to her, would it be better for her to invest it in six per cent bonds recommended by a reliable banker, or in stock which an agent says will pay 15%? Why?

7. What is the very first thing to consider when making an investment? Name some other things that must be taken into consideration.

THE PROTECTION OF SAVINGS

Fire Insurance

Thousands of people having small incomes save carefully and invest as wisely as they can in order finally to get together enough money to build a home. When at last their savings are brought together and spent for a dwelling, or invested in some other piece of property, such people undertake a great risk. Money lying in the bank or invested in good bonds is reasonably safe; but money invested in a building may be lost in a few minutes by fire. For this reason, the owner of a building usually insures it with some company that agrees to pay most of its value, in case it is burned, in return for a sum of money paid as a fee for the service. Such a company is called a *fire insurance company*.

You may wonder how any company could afford to make such an agreement. You must remember, however, that many thousands of buildings are insured, and on each of these a fee is charged. On the other hand, comparatively few buildings are destroyed by fire in any one year. Usually the amount collected as fees enables the insurance company to lay up a *surplus* from which losses are paid. The company also pays the expenses of doing business, and in most cases earns a profit besides.

The insurance company's written agreement to pay a certain amount in case of loss is called a *policy*. The person who insures a building, and so takes out a policy, is called a *policyholder*. The fee paid to the company for the insurance is called a *premium*. The *term* of the insurance is the amount of time over which the agreement extends. The *face* of the policy is the amount of insurance named in the policy.

The cost of fire insurance varies according to the structure of the building, its location, and the purpose for which it is used. When you recall that the company receives a premium because it is willing to assume the risk, you will understand that the premium must increase as the risk increases.

The premium on a frame dwelling house located in a city having good fire protection will usually be less than the premium on the same sort of house located on a farm where no fire protection is possible. If the dwelling house in the city were converted into a garage, the premium would increase. Do you know why? What is the advantage of a so-called fireproof building? Is the premium likely to be higher on a dwelling that stands near the center of a town, surrounded by business buildings, or on the same sort of dwelling located at the edge of town, beyond the reach of the city water system? Explain your answer.

Many fire insurance policies are written for 75% or 80% of the value of the property. Policies are written for terms of from 1 to 5 years, the cost per $100 of insurance decreasing with the length of the term.

Can you give reasons why a fire insurance company should conduct its business in this way?

The cost of fire insurance is usually stated as so much for each $100 of insurance.

For example: If the rate on a certain building is 30 cents per $100 of insurance, the owner who insures that building for $4000 must pay a premium of 40×$.30 or $12 per year.

Exercise 62

1. A house worth $5500 is insured for 80% of its value, at $.28 per $100. What is the annual premium?

2. A factory worth $120,000 is insured for $\frac{3}{4}$ of its value, at $.40 per $100. What is the annual premium?

3. A city building is insured for $70,000, at $.85 per $100 for 1 year. What is the annual premium?

4. A garage was insured for $7500, at $1.40 per $100. What was the premium?

5. A house was insured for $6000, for 5 years, at $1.12 per $100. How much was the premium?

6. A building worth $75,000 is insured for 80% of its value; the machinery and stock worth $60,000 are insured for 75% of their value. If fire completely destroys the building and contents, what is the net loss to the owners?

7. A farmer insured his house worth \$5000, for $\frac{4}{5}$ of its value, at \$.36 per \$100; the contents of the house, for \$2000, at \$.38 per \$100; his barn, for \$2200, at \$.50 per \$100; the contents of the barn, for \$6000, at \$.54 per \$100. What were the premiums?

8. A house valued at \$12,000, and insured for \$9000, was completely destroyed by fire. The premium was $\frac{1}{4}\%$. What was the total loss to the owner, including premium, if one premium had been paid?

9. A school building valued at \$180,000 is insured at the rate of \$12.75 per \$1000. How much is the annual premium?

10. A merchant insured his stock, valued at \$35,000, for 80% of its value, at a premium of $\frac{1}{2}\%$. How much was the premium?

11. A boat is insured for \$40,000, and its cargo for \$18,500. The rate is $\frac{3}{4}\%$. How much is the premium?

12. Find the premium on each of the following policies:

1. Face \$800, rate $1\frac{1}{2}\%$
2. Face \$4800, rate $1\frac{3}{4}\%$
3. Face \$12,000, rate $1\frac{1}{4}\%$
4. Face \$1800, rate $1\frac{3}{4}\%$
5. Face \$2400, rate $1\frac{1}{2}\%$
6. Face \$54,000, rate $1\frac{7}{8}\%$

13. At the next meeting, report to the class the rate of insurance on your home or on some other building. Name a fire insurance company that has agents in your community.

14. Tell of any recent fire loss partially covered by insurance.

Casualty and Liability Insurance

The destruction of a building by fire is by no means the only thing likely to cause loss to a person who has invested his money in that sort of property. A building may be destroyed or seriously damaged by a windstorm, by lightning, by a flood, or by some other means. Some insurance companies sell policies protecting the property owner against such losses. This kind of insurance is called *casualty* insurance.

Automobile insurance is one of many kinds of casualty insurance. The owner of an automobile may insure his car against loss by theft, fire, or damage due to his own carelessness or that of others. He may also carry *liability* insurance. This is the agreement of an insurance company to assume financial responsibility in case the owner of a car injures or kills anyone, or damages another car or other object, while driving his own car.

There are a great many applications of casualty and liability insurance. You should talk to some one who is familiar with this business, learn about some of the conditions that are covered by such insurance, then make problems like the following.

Exercise 63

1. A plate glass window that cost \$135 is insured for its full value for \$11.20 a year. What per cent of the value of the glass is the annual cost of the insurance? If the glass is destroyed within the period covered by the policy, what amount will the owner receive?

2. A farmer who had 25 acres of wheat insured the crop for \$20 an acre. The rate of the premium was 2% of the face of the policy. What was the total cost of the insurance? What amount of insurance will be paid if the crop is completely destroyed? What amount will be paid if $\frac{2}{3}$ of the crop is destroyed?

3. A farm building, valued at \$8500, was insured at the rate of \$.75 per \$100 for 90% of its value for three years against damage by tornado. Seven months after the policy was taken out, the building was completely destroyed by a tornado. What amount of insurance must the company pay for the loss?

4. A city dwelling house, valued at \$9000, is insured for three years against damage by tornado. The insurance covers 90% of the value, and the rate is \$.40 per \$100. If a tornado completely destroys this building, what amount will the company have to pay the owner?

5. A lawyer took out an accident insurance policy for \$7500, at an annual cost of \$25 per year. After carrying the policy for $5\frac{1}{2}$ years he was accidentally killed. How much more did the insurance company pay than it had received in premiums?

6. A premium of $15.20 was paid for insuring a boiler for $5000, for a year. The boiler exploded, causing a total loss of $6400 to the boiler and the building in which it was located. What amount of insurance should be paid to the owners? The premium on a 3-year policy for $5000 is $38.00. What per cent of the total premium would be saved by taking out a 3-year policy instead of three 1-year policies?

7. Mr. Mitchell took out burglary insurance on his residence, at an annual cost of $22 per $1000. What per cent would he have saved by taking out a 3-year policy for $59.40?

8. Get information from some insurance agent regarding the insuring of elevators, flywheels, automobiles, and other things. Make some insurance problems from this information.

Life Insurance

Life insurance policies issued by various companies differ somewhat in details, but in general they may be classified as *ordinary life*, *limited payment life*, or *endowment*. The person who takes out a life insurance policy is called the *insured;* the written agreement or contract with the company is called the *policy;* the person to whom the policy is to be paid when due is called the *beneficiary;* the amount paid each year for the insurance is called the *premium*.

The great number of people who hold life insurance policies may be taken as some proof of the value of life insurance. During a recent year there were 71,760,000 life insurance policies in force in the United States; these policies represented more than forty-four billion dollars of insurance.

Ordinary Life Policy: The holder of an ordinary life policy pays a premium each year from the time he takes out the policy. At his death, the policy is payable to the person in whose favor it was taken out.

Limited Payment Policy: The holder of a limited payment policy pays an annual premium for a specified number of years, usually 10, 15, or 20. If he dies before he has paid the required number of premiums, the policy is payable. If he outlives the period during which he makes payments, he pays no more after paying the last premium specified. In that case also, the face of the policy and any accumulated earnings are paid when he dies.

ENDOWMENT POLICY: The holder of an endowment policy pays the premium for a specified number of years, usually 10, 15, or 20. At the conclusion of the premium-paying period, the face of the policy and the accumulated earnings are turned over in cash to the policyholder. In this case the policyholder gets the use of the money. Should the insured die before payments on the policy have been completed, the face of the policy and the accumulated earnings up to the date of death are paid. This type of policy is much used by those who buy life insurance as an investment, or use it as a means of saving.

The *rate of insurance* is the amount charged for each $1000 of insurance. It varies with the type of policy and the age of the insured at the time of taking out the policy. The annual premium does not change after a policy is taken out. The younger the insured, the smaller the premium. Can you explain why this is the case?

The following table, issued by one of the large insurance companies, shows the rate of insurance for each $1000 on certain kinds of policies for persons of various ages.

AGE	ORDINARY LIFE	20-PAYMENT LIFE	ENDOWMENT	
			20 YEAR	25 YEAR
21	$18.76	$28.73	$48.45	$38.03
25	20.55	30.63	48.86	38.50
30	23.31	33.44	49.57	39.34
35	26.88	36.85	50.64	40.64
40	31.56	41.10	52.33	42.76
45	37.82	46.57	55.15	46.25
50	46.36	53.86	59.87	52.02

Fig. 45

Exercise 64

When necessary use the table in Fig. 45 to solve these problems.

Find the annual premium on a $1000 policy and on a $5000 policy when the age of the insured and the kind of policy are as stated:

	Age	Kind of Policy	Kind of Policy
1.	21	Ordinary Life	20-Payment Life
2.	30	25-Year Endowment	Ordinary Life
3.	40	20-Payment Life	Ordinary Life
4.	25	20-Year Endowment	20-Payment Life
5.	50	Ordinary Life	20-Payment Life

6. For a given age, which kind of policy listed in Fig. 45 costs the most, and which the least? Why?

7. Why does a 20-year limited payment life policy cost less for a given age than a 20-year endowment policy for the same age?

8. At the age of 25, a man took out an ordinary life policy for $5000. What is the annual premium?

9. On his thirtieth birthday, a man took out a 20-payment life policy for $4000, and five years later he took out an ordinary life policy for $5000. He died after making 16 payments on his first policy. How much did the insurance paid by the company exceed the premiums which the insured paid? (Do not consider the interest.)

10. At the age of 25, a man took out an ordinary life policy for $5000. He died after making 36 annual payments. How much more or less would he have paid in premiums, if he had taken the policy at the age of 30?

11. Mr. Moore took out a 20-year endowment policy for $3000 on his twenty-first birthday. What must be his average savings per month to pay the annual premium?

12. Name two life insurance companies that have agents in your community.

COLLECTING AND SPENDING PUBLIC MONEY

You have learned the importance of saving part of what you earn so that you may have means to live well, to provide against sickness and accident, to provide for old age, and to profit by opportunities when they appear. You have doubtless been impressed by the need for money, if you expect to get along in the world, and you know that each person must have some means if he is to be successful.

Each of us needs many other things, however, for which one alone cannot provide. We need roads, bridges, and paved streets, courthouses, libraries, hospitals, and asylums; unfortunately we must have prisons and houses of correction. Schools, colleges, and universities are a necessity, and we must have teachers and teaching equipment. Police and other officers are needed to protect our property against those who do not respect the rights of others; firemen and equipment must be provided to fight destructive fires. Courts must be maintained, judges and other officials paid. These are only a few of the expenses that must be met, if people are to live together in happiness and safety.

But who is to furnish the money for these many things? No single person is willing to do so, some are unable to do so because nearly all they earn is required to enable them to live. Each should and must pay his share, however, and it is only right that each should pay to the extent that he enjoys the advantages we have mentioned.

Since all these things are common needs, provided for the use of all alike, they are furnished by the government, which represents the people. The government of city, county, state, and nation raises the money needed for public use. Such money is called *taxes;* these are *levied* or *imposed.*

For purposes of taxation, property is divided into two classes: *Real estate*, or fixed property, includes land and buildings; *personal property* consists of movable possessions such as household goods, domestic animals, farm products, manufactured articles, mortgages and notes, stocks and bonds, money, jewelry, and clothing. A tax on *income* is a tax levied on all or part of what one earns each year. In some states a *poll tax*, or head tax, is levied on each person who has reached the voting age.

For purposes of taxation, the value of property is usually determined by officers called *assessors*. The value assigned to property is called the *assessed value* or *valuation*. The *rate of taxation* is the amount that must be paid on each $100 of assessed value.

In some communities property is assessed at its full value. In others it is assessed at $\frac{2}{3}$, $\frac{1}{2}$, or even $\frac{1}{4}$ of its value. Would the practice followed in any community affect the amount of any person's taxes? In what way?

After all assessments have been made, the taxing department of the government is notified of the assessed value of property within a certain area. Another department of the government, having already estimated the amount of money necessary for carrying on the business of the government during the coming year, determines the rate of taxation by dividing the estimated expenses by the assessed value.

For example: A small city has an assessed value of $6,163,200. The estimated expenses for the coming year will be $77,040. The tax rate is whatever per cent the amount to be raised, $77,040, is of the assessed value, $6,163,200, or $1\frac{1}{4}\%$.

The tax rate is usually expressed as so many mills on each $1 of assessed value, or so many cents on each $100, or so many dollars on each $1000. Sometimes the tax rate is expressed in per cent, as in the *Example* given above.

A tax rate of $1\frac{1}{4}\%$ may be expressed as $12\frac{1}{2}$ mills per $1, or $1.25 per $100, or $12.50 per $1000.

For example: If the rate is $12\frac{1}{2}$ mills per $1, the tax on property having an assessed value of $3000 will be $3000 \times \$.0125 = \$37.50$. This is the same as $1\frac{1}{4}\%$ of $3000. Expressed as $1.25 per $100, or $12.50 per $1000, the result is the same.

Whenever a township, city, or county makes some improvement that increases the value of certain property, the owners may be required to pay a special tax in addition to the general tax. This special tax is called a *special assessment.*

For example: If a city paves a street or lays a sewer, most of the cost is assessed upon the owners of property that is directly benefited. The tax paid by each person is proportionate to the amount of benefit he receives from the improvement. In such a case the city usually pays for paving street intersections; the money used for this purpose comes from the general tax.

Exercise 65

1. People frequently exclaim, "Oh, if it weren't for the taxes!" Do you think people are usually justified in complaining about their taxes? Explain your answer.

2. What benefits do the residents in a school district get from the part of the taxes spent for school purposes?

3. Name some of the ways in which various city, county, and state officers serve the people.

4. A man's property is assessed at $7400, and the tax rate is 8 mills. What are his taxes on the property?

5. The assessed valuation of a certain city is $92,000,000, and the amount to be raised by taxes is $1,702,000. What is the tax rate?

6. What are the taxes on each of the following?

1. $4800 at a rate of 11½ mills on $1.
2. $24,550 at a rate of 9½ mills on $1.
3. $5860 at a rate of 15½ mills on $1.
4. $12,700 at a rate of $1.85 on $100.
5. $3500 at a rate of $2.10 on $100.
6. $15,400 at a rate of $1.95 on $100.

7. Appoint one in your class to find out from the county auditor, or by consulting a tax receipt, the rate of taxation in your community for school purposes, for county purposes, and for state purposes. Use your local rates to determine the amount of a man's taxes in each of the three classes, if his property is assessed at $5500.

8. The tax rate is 16.5 mills. What is the tax on property assessed at $6500?

9. A man's property located just outside the city limits was assessed at $18,500. The tax rate was 17.5 mills. The property was taken into the city where the tax rate was 24.5 mills. What was the increase in taxes? Tell some of the benefits the man will receive for his extra taxes.

10. If you live in a city or town, state the present tax rate in mills. State the rate as so much per $100.

11. What was the last total assessed value in your city or county?

12. Name several purposes for which the money raised by taxation in your city and county last year was spent.

13. When are the taxes collected in your community? Who collects them?

14. The school taxes in a certain school district are 8 mills on $1. What school tax is paid by a man whose property is assessed at $8500?

15. The value of the real estate in a certain city is $14,500,000, and that of the personal property $7,100,000. All of the property is assessed at 50 per cent of its real value, and the tax rate is $.45 per $100. What is the amount of tax levied?

16. A man owns a farm of 160 acres, assessed at $90 an acre, and personal property amounting to $1800. His county tax rate is 5 mills, school tax 8 mills, road tax 4 mills. What is the total amount of his tax for these purposes?

17. The tax rate in a certain city is 21.5 mills on each $1. What taxes are to be paid by the citizens whose property is assessed as follows?

1.	Henry Osborn	$5,850
2.	John Adams	$14,350
3.	Carl Olson	$3,800
4.	Fred Smith	$26,560
5.	Sarah Brown	$12,400
6.	Sam Parker	$25,600
7.	Mary Lewis	$14,900
8.	Walter Lynch	$58,790

Taxes Levied by the United States Government

The United States Government has to meet certain expenses unlike those of the states, counties, cities, and towns. Among these are the upkeep of the army and navy, the maintenance of the post offices and United States courts, the support of the Indians on reservations, the payment of pensions to disabled soldiers and sailors and to the widows and orphans of those who died in war or as a result of service in the army and navy. The salaries of government officials must be paid, rivers and harbors must be improved and kept in repair, and buildings of many kinds must be erected and maintained. The public debt, mostly in the form of bonds issued at various times, must be paid as the bonds become due, and the interest payments must be kept up. These expenses, and many others, require an outlay of billions of dollars, all of which must come from the people.

To meet these expenses the United States obtains its money chiefly from *import duties*, *internal revenue*, and *income tax*.

IMPORT DUTIES: Congress has the right to levy a tax or *tariff* on goods imported from other countries. Such a tariff is called an *import duty*, or more commonly, a *customs duty*. When the amount of the duty is based on the cost of the goods at the place of purchase, it is called an *ad valorem duty*. When the amount of the duty is so much per unit of the goods imported, that is, so much per pound, per ton, per bushel, and so on, it is called a *specific duty*. Some articles, such as perfume, are now subject to both duties. Articles on which no duty is charged are said to be on the *free list*.

Import duties are collected by customs officials located at many places throughout the country, but chiefly at seaport cities. A place where customs duties are collected is called a *port of entry*.

INTERNAL REVENUE: In general, this is a tax levied on certain products manufactured within the United States itself, collected by means of stamps which must be bought from the government's internal revenue officers, and fastened on the articles to be taxed, before these are sold. Among the articles commonly subject to an internal revenue tax are tobacco in all its manufactured forms, such as cigars and cigarettes, playing cards, and oleomargarine. In order to meet some of the greatly increased expenses due to the Great War, special taxes were imposed on certain drugs, toilet preparations, notes and other legal papers, soft drinks, and tickets to places of amusement.

Internal revenue taxes and customs duties are sometimes called *indirect taxes*, because they are not levied directly on the people as a whole, although the people finally pay them.

For example: If an importer of cigars has to pay a customs duty of $3 per 1000 cigars, he adds that to the cost of the cigars when he bills them to the retailer. The retailer, in turn, increases the selling price to his customer, who, after all, is the one that pays the duty.

INCOME TAX: Part of the money needed by the United States government is now raised by a tax on income. Every citizen of the United States, whether he resides within this country or abroad, and every person living in the United States must pay the government 2 per cent of his income above a certain amount fixed by law. Incomes derived from certain sources are exempt from this tax. Those whose taxable incomes exceed a certain larger amount fixed by law are taxed more heavily. There is also an extra tax, known as a *surtax*, which increases in rate as the amount of the income increases. The income tax is based on the theory that those having large incomes should pay a larger part of the expenses of government than those having small incomes.

Because of frequent changes made in the law relating to income taxes, only a brief treatment of the subject is practical in an elementary book. Income tax payers usually consult some banker or some attorney to learn the latest rules for computing the tax. If you wish to learn more about the income tax the banker in your town will no doubt be glad to give you information.

A man has a taxable income of $4000 a year. How much income tax does he pay?

Solution: He pays 2% on $4000, or $80.

NOTE TO PUPIL: The income tax rate is subject to frequent change.

Exercise 66

1. When the duty on olive oil is 20 cents a gallon, what is the duty on 1500 gallons? Is this an ad valorem or a specific duty?

2. When the duty on toys is 35% of their value, what is the duty on toys valued at $4800? What kind of duty is this?

3. When the duty on ready made clothing is 35% of the cost, what is the duty on a suit that costs $32 in England?

4. When the duty on a certain kind of jewelry is 60%, what is the duty on a shipment of this jewelry costing $24,850?

5. How much is the duty on 4800 bushels of barley at 15 cents a bushel? What kind of duty is this?

6. At 20%, what is the duty on jewelry valued at $28,000?

7. At $1.00 per 1000, what is the duty on 65,000 tulip bulbs?

8. A man has a taxable income of $4800. At 2%, how much is his income tax?

9. A man has a taxable income of $3800 a year, and his brother has a taxable income of $2200. How much income tax does each pay?

10. Mr. Smith pays an income tax of 2% on $14,000, and an income surtax of 15% on $8000. How much is his total income tax?

11. When the duty on fur coats is 40%, what is the amount of the duty on a fur coat valued at $750?

12. How much duty must be paid on an oriental rug valued at $450 when the duty is 45%?

13. The duty on a certain kind of chinaware is 55%. How much duty must be paid on a shipment of this china valued at $4760?

14. It is sometimes said that the collection of indirect taxes is expensive. If you can, suggest why this statement may be true.

15. Illustrate how the indirect tax is passed on to the consumer and finally paid by him.

THE FORMULA IN ARITHMETIC

From the beginning of your work in arithmetic you have used a kind of shorthand writing. You use the sign + instead of writing the word "plus"; you use the sign − for "minus," ÷ for "divided by," and = for "equals." You also abbreviate the word "pound" by writing *lb.*, the word "dollars" by writing *$*. As you know, the purpose of all these signs and abbreviations is to save time. Their use enables you to express an idea equally well in less space and with fewer words.

It is possible to express many rules of arithmetic by using signs and abbreviations. The rule for finding the area of a rectangle, "*The area of a rectangle equals the base multiplied by the height*," may be stated as $A = b \times h$. Such a statement is called a *formula*. In this formula, A represents the number of units in the area, b represents the number of units in the base, and h represents the number of units in the height. Formulas are much used in industrial work, in books of science and mechanics, in trade journals and in magazine articles.

In writing formulas, many signs that you have been accustomed to use are omitted. The multiplication sign, ×, in such an expression as $b \times h$, is usually omitted, and the expression is written bh. Thus, the formula $A = b \times h$ is usually written $A = bh$. In like manner, $5 \times a \times b$ may be written $5ab$. a^2 means $a \times a$, and is read "*a square*." a^3 means $a \times a \times a$, and is read "*a cube*." $\frac{a}{3}$ means "*a divided by 3*." Just as 4 doz. + 3 doz. + 2 doz. = 9 doz., so $4d + 3d + 2d = 9d$. 8 doz. − 6 doz. = 2 doz., or $8d - 6d = 2d$. In either of the last two cases, d may have any value whatever.

In using formulas, we must give certain letters definite values.

For example: What is the value of A in the formula $A = bh$, if $b = 5$ and $h = 4$?

To solve this we must first remember that bh means $b \times h$. Then, substituting the values as indicated, the formula $A = bh$ becomes $A = 5 \times 4$, or $A = 20$.

To find the value of A in the formula $A = \frac{bh}{2}$, when $b = 6$ and $h = 10$, we must remember that $\frac{bh}{2}$ means bh divided by 2. Then, by substituting the indicated numbers for the letters b and h we have: $A = 6 \times 10 \div 2$ or 30.

Exercise 67

1. Find the sums:

a	b	c	d	e
$4d$	$7d$	$4n$	$5x$	$8m$
$3d$	$8d$	$2n$	$7x$	$2m$
$5d$	$5d$	$7n$	$3x$	$6m$

2. Find the differences:

$12d$	$4b$	$100n$	$15h$	$5x$
$7d$	$3b$	$60n$	$11h$	$3x$

3. Find the value of each of the following when $a=7$, $b=6$:

	a	b	c	d
1.	$2a+b$	$6ab$	$\frac{a}{b}$	$2a+4b$
2.	$5b+7$	$2b+3a$	$25ab$	$7ab$

4. Find the value of the following when $a=3$, $b=5$, $c=8$:

1.	$a+b+c$	abc	$\frac{abc}{2}$	$\frac{bc+2}{a}$
2.	$2a+4b-c$	$5a-b+2c$	$3bc-a$	$4ab+c$

5. A rectangular playground is l feet long and w feet wide. Its area in square feet is lw. What is the area when $l=150$ ft., and $w=60$ ft.? When $l=130$ ft., and $w=50$ ft.?

6. The base of a rectangle is b ft., and the height is h ft. What is the area? What is the area when $b=6$ ft., and $h=3$ ft.?

7. If an apple costs d cents, what will n apples cost at the same rate?

8. The loss equals the cost minus the selling price. Representing loss by l, cost by c, and selling price by s, this becomes $l=c-s$. Make three problems by assigning values to l, c, and s.

9. The formula for the area of any rectangle is $A=lw$ when l = the length and w = the width. State this formula in words.

10. The area of a triangle is $A=\frac{bh}{2}$, when b = the base and h = the height. State this formula in words.

11. Write a formula that may be used for finding the entire surface of a cube when the edge is known. State the formula as a rule, letting e represent the edge.

12. Write a formula for finding the perimeter of a square when a side is known. Write a formula for finding the area of a square. State each formula in words.

13. If y represents the number of yards of cloth bought, and c the number of cents which each yard cost, write a formula for finding the total cost. State the formula in words.

14. Write a formula for finding the number of inches in a given number of feet. The number of feet in a given number of yards.

15. The number of times that a bicycle wheel of known circumference in feet will turn in going a mile is given by the formula $N=\frac{5280}{C}$. State the formula in words.

16. George learns that the number of tons of hard coal in a bin l ft. long, w ft. wide, and d ft. deep is $\frac{1}{35}l\times w\times d$. Write this as a rule, and find the number of tons in a bin 14 ft. long and 6 ft. wide, filled to a depth of 5 ft.

17. The formula for finding the interest i on p dollars for t years at r per cent is $i=prt$. Find the value of i when $p=600$, $t=2$, and $r=6$.

18. The formula for the area of a trapezoid is $A=\frac{1}{2}h\ (b+c)$, in which A = the area, h = the altitude, and b and c = the parallel sides. Find the area of a trapezoid whose altitude is 8 inches, whose lower base is 14 inches, and whose upper base is 12 inches.

19. The formula $S=\frac{1}{2}gt^2$ represents the distance that a body will fall in t seconds. In this formula g has a value of 32.16. S represents the space through which the body will fall. Find S when t is 3 seconds.

PLANE FIGURES (*Continued*)

To Find the Area of a Polygon

The name *polygon* is applied to any figure lying on a smooth surface and bounded by straight lines.

A triangle is a polygon with three sides. A quadrilateral is a polygon with four sides. A *pentagon* is a polygon with five sides. A *hexagon* is a polygon with six sides. Draw these figures.

A polygon whose sides and angles are all equal is called a *regular polygon.*

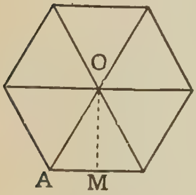


Fig. 46

What special name is applied to a regular polygon with three sides? With four sides? (See pages 117 and 122.)

In the regular polygon shown in Fig. 46, *O* is called the *center*, and *OA* is a *radius.* A perpendicular drawn from the center to any side is called an *apothem.* *OM* is an apothem.

Architects and designers frequently use the regular hexagon in their work. It may be constructed as follows:

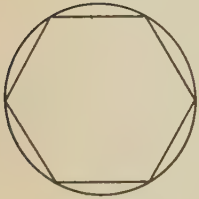


Fig. 47

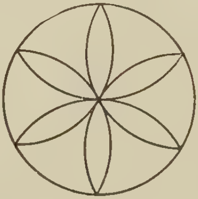


Fig. 48

Draw a circle. (See page 191.) Use the radius as a unit to mark off the circle as shown in Fig. 47. This divides the circle into six equal arcs. Connect the points by straight lines (*chords* of the circle) to form a regular hexagon.

Draw several circles of different sizes, and construct a regular hexagon in each. Find out how Fig. 48 was constructed, then construct a figure like it.

Construct a regular hexagon having each side 2 inches long. Construct another having each side $1\frac{1}{2}$ inches long.

Suggest a way to construct an equilateral triangle, after marking off a circle for the construction of a regular hexagon.

Locate on a circle the vertices of a regular hexagon, then make a six-pointed star like the one shown in Fig. 49.

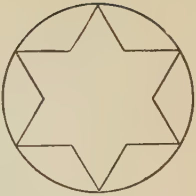


Fig. 49

A study of the patterns used in linoleums, rugs, and wall paper will reveal many interesting designs, some of them similar to the ones that you have studied here. After marking off a circle into six equal parts, construct three original designs using either straight or curved lines.

A study of Fig. 46 will show that a regular polygon is made up of equal triangles, and that the area of the polygon is the sum of the areas of the triangles. Explain why the following statement is true.

The area of a regular polygon is equal to one-half the product of the perimeter and the apothem.

Exercise 68

1. Find the area of a regular hexagon, one of whose sides is 10 in., and whose apothem is 8.66 in.

2. The apothem of a regular hexagon with a side of 20 in. is 17.32 in. What is the area?

3. A regular polygon with 8 sides (an *octagon*) has an apothem of 12.07 ft. Each side measures 10 ft. What is the area?

4. What is the area of a regular polygon of 5 sides (a *pentagon*), each side measuring 5 ft., the apothem being 2.75 ft.?

5. Lay out a flower bed in the form of a regular hexagon, each side of which measures 8 ft. Estimate the area. Take measurements and find the area.

Practical Measurements: Angles and Plane Figures

1. Point out 12 right angles in your schoolroom.

2. What kind of angle do the hands of a clock form at 3 o'clock? At 5 o'clock? At 2 o'clock?

3. Estimate the number of square feet in the top of your own or the teacher's desk. Check your estimate.

4. Try to draw on the blackboard, without the aid of a ruler, a square containing 36 square inches. Check your work.

5. Use a ruler to draw a right triangle containing 6 square inches. Draw one containing 10 square inches.

6. Draw three triangles of different shapes having equal bases and equal altitudes. Compare their areas.

7. Draw two triangles on the same side of a common base, one of which has twice the area of the other. Do the same, if possible, with two squares.

8. A square and a triangle are drawn on the same side of the same base. The altitude of the triangle is twice that of the square. Compare their areas.

9. Tell what you need to know, and how you then proceed, to find the area of each of the following: Parallelogram. Trapezoid. Triangle. Square. Rectangle. Regular hexagon.

10. Draw an equilateral triangle. A scalene triangle. An isosceles triangle. A regular hexagon.

11. What is the floor area of your schoolroom? How many square feet of space, on the average, does this allow each person?

12. Are the angles of a regular hexagon acute or obtuse?

The Circle

As shown by Fig. 50, a *circle* is a closed curve lying on a plane or smooth surface, and such that all its points are equally distant from a point within called the *center*. In Fig. 50 the letter *O* indicates the center.

The length of the curved line is called the *circumference*.

A line which passes through the center with both its ends in the circle is called a *diameter*. See Fig. 50.

A line from the center to the circle is called a *radius*. (Plural: *radii*.) See Fig. 50.

NOTE TO PUPIL: Formerly the surface enclosed by the curved line was called the circle, and the curved line was called the circumference. This usage is still rather common.

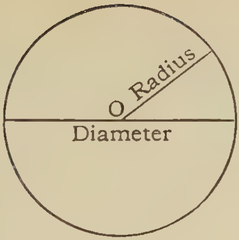


Fig. 50

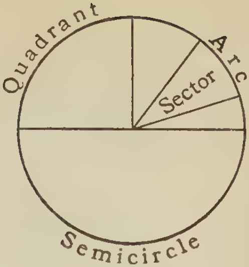


Fig. 51

Any portion of a circle is called an *arc*. See Fig. 51.

A figure bounded by an arc and two radii is called a *sector*. A sector is indicated in Fig. 51.

An arc equal to one half of a circle is called a *semicircle*. An arc equal to one fourth of a circle is called a *quadrant*. See Fig. 51.

Draw several circles of different sizes and show in each the center, a radius, a diameter, an arc, a quadrant, and a semicircle.

It is best to use a pair of compasses for drawing circles. If you do not have a pair of compasses, you can use a strip of stiff cardboard about $1\frac{1}{2}'' \times 6''$. Punch holes in the cardboard, as shown in Fig. 52.

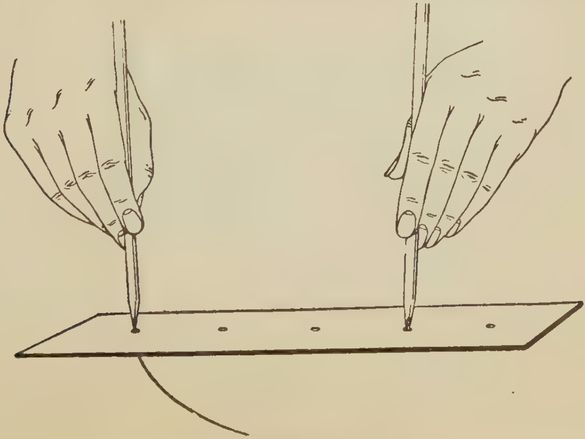


Fig. 52

To draw a circle, lay the cardboard on a paper. Place a pencil through one hole, and hold it firmly at the point that is to be the center of the circle. Place another pencil through one of the other holes, and rotate the cardboard about the first pencil. The different holes enable you to draw circles of different radii.

A string tied to a piece of crayon may be used for drawing circles on the blackboard.

The Measurement of Circles

Draw a circle, and within it draw two diameters that cross each other at right angles, as shown in Fig. 53. From this drawing it is plain that there will be four right angles, no more and no less.

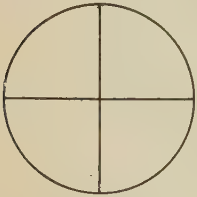


Fig. 53

You have already learned that angles are measured by degrees, and that a right angle is an angle of 90°. Since this is true, every circle must measure 360°, because a circle consists of the arcs of four right angles.

How many degrees are in a semicircle? In a quadrant? What part of a circle is a degree? How many degrees in the arc of a right angle drawn at the center of a circle with a diameter of 2 feet? Of 4 feet? Of 6 feet?

For measuring the arcs of circles, and the sizes of angles, an instrument called a *protractor*, illustrated in Fig. 54, is commonly used. This consists of a semicircle equally divided into 180°. They are usually numbered from each side.

To measure an angle, place the straight edge (*AB*) of the protractor along one side of the angle with the point *A* at the vertex of the angle. The point on the protractor crossed by the other line of the angle indicates the size of the angle being measured. As shown here, *BAC* is an angle of 30°.

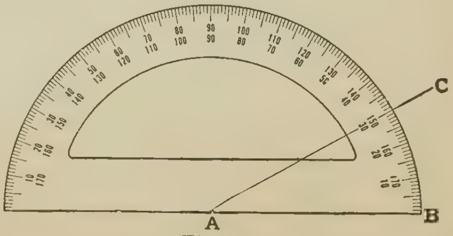


Fig. 54

To construct an angle of any size, draw one line, lay the straight edge of the protractor along that line with the point A of the protractor at the vertex of the angle to be formed. Find the required number of degrees on the arc of the protractor, place a point there, then draw a line from that point to the vertex of the angle to be formed.

How will you measure the size of an obtuse angle? Does an obtuse angle contain more or less than 90°?

Exercise 69

1. Use the protractor to construct angles of the following sizes:
30° 45° 90° 145° 20° 10°

2. Draw a right triangle, then measure the size of the acute angles. What is their sum?

3. Draw an equilateral triangle, then measure the size of each angle. What is the sum of the three angles? How do the angles compare in size?

4. Draw an isosceles triangle. What is the sum of the three angles? How do the angles opposite the equal sides compare in size?

5. The eight points of the compass are shown in Fig. 55. Name these points. How many degrees in the angle made by lines pointing southeast and northeast? How many degrees between the west and the southwest points? Between the southeast and the northwest points?

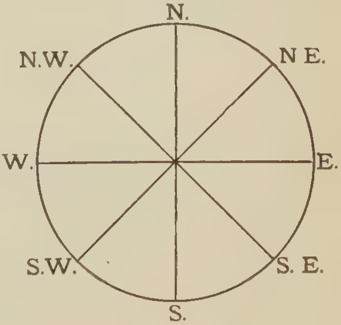


Fig. 55

6. Show by the use of the protractor that the size of an angle does not depend upon the length of its sides.

Note to Pupil: On page 285 is a printed protractor and full directions for making it available for use.

8. Does the size of an angle depend on the intersection of two straight lines? Give examples.

9. Draw a sector whose arc is 30°. Draw a sector whose arc is 70°.

10. Find the sum of the angles in each of several squares. In each of several rectangles. In each of several parallelograms. In each of several trapezoids.

11. What is your conclusion as to the sum of the angles in a quadrilateral?

To Find the Circumference of a Circle

The approximate circumference of a circle may be found as illustrated in Fig. 56. On a circle mark a point *P*. Place this point over a point *A* of a straight line, then roll the circle along the line until the point *P* again falls on the line. Mark this last point *B*. Measure the distance *AB*.

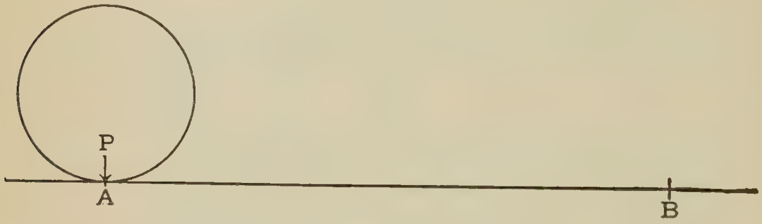


Fig. 56

From cardboard cut circles with various diameters and find the circumference of each according to the method illustrated in Fig. 56.

The following experiment gives an interesting result. Find the circumference and the length of the diameter of the objects mentioned in Fig. 57. Record the measurements in a table like that shown. In each case, divide the circumference by the length

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Note to Pupil: For purposes of measurement in surveying and astronomy the degree is divided into 60 equal parts, each of which is called a *minute* (′). The minute is divided into 60 equal parts, each of which is called a *second* (″).

of the diameter and record the quotient. Find the quotient correct to three decimal places. Find the average of all the quotients.

Object	Circumference	Length of Diameter	Quotient
Saucer			
Silver Dollar			
Plate			
Pail			
Phonograph Record			
Flower Pot			

Fig. 57

You will observe that the quotients are almost the same, although the circles vary in size. If the measurements were absolutely accurate, the quotients would be exactly the same.

It can be proved by geometry that the quotient obtained by dividing the circumference by the length of the diameter is 3.1416. Since this quotient is always the same, a special sign is used to represent it in formulas. The sign used is the Greek letter π (pronounced *pi*). In measurements where great accuracy is not required, the value of π may be taken as $\frac{22}{7}$.

The formulas below state some facts about the circle.

The circumference divided by the length of the diameter $= \pi$.

If C represents the circumference, D the diameter, and R the radius, we have $C \div D = \pi$, or $\frac{C}{D} = \pi$, therefore $C = \pi D$.

Since the diameter is twice the radius, we may say that $C \div 2R = \pi$, or $\frac{C}{2R} = \pi$, therefore $C = 2\pi R$.

If you know the diameter of a circle, you can find the circumference by multiplying the diameter by π.

If you know the radius, how can you find the circumference?
If you know the circumference, how can you find the diameter?
If you know the circumference, how can you find the radius?

Exercise 70

In solving the following problems use $\pi = 3\frac{1}{7}$ whenever this value can be used to advantage.

1. Find the circumference of each of the following circles:

1. Radius, 16 ft.
2. Diameter, 21 yd.
3. Radius, 140 ft.
4. Diameter, 6 in.

2. Find the diameter of a circle with a circumference of 44 ft.

3. Find the diameter of each of the following circles:

1. Circumference, 28 in.
2. Circumference, 56 ft.
3. Circumference, 35 yd.
4. Circumference, 42 in.

4. A circular table has a diameter of 42 in. What is its circumference?

5. Find the circumference of a circular table mat with a radius of 5 in.

6. An automobile tire has a diameter of 34 in. What is the distance covered each time the wheel makes one turn?

7. How do the circumferences of two circles compare, if the first circle has a radius of 3 in., and the second a radius of 6 in.?

8. Compare the circumferences of two circles, one of which has a radius of 4 in., and the other a radius of 8 in.

9. The diameter of a locomotive wheel is 80 in. How many revolutions will this wheel make in going a mile?

10. The diameter of a flywheel on an engine is 6 ft. If this wheel makes 1000 revolutions per minute, does a point on its rim travel more or less than 3 miles a minute?

11. If the earth were perfectly round, and had a diameter of 8000 miles, what would be the length of the equator?

12. Mary has a piece of silk that measures 38 in. Will this reach around the edge of a circular lamp shade with a diameter of 12 in.?

13. What is the diameter of a circular pond whose circumference is 770 ft.?

14. The diameter of a bicycle wheel is 28 in. How many revolutions will the wheel make in going 5 miles?

15. One automobile has 34-inch wheels and another has 36-inch wheels. In going 1 mile, how many more times do the smaller wheels turn than the larger ones?

16. How much lace will be needed as the edge for a circular doily with a diameter of 20 in.?

17. A roller is 32 in. in diameter. How many rotations will it make in traveling a distance of 120 rods?

18. A merry-go-round is 35 ft. in diameter. How far does a point on the circumference travel while the merry-go-round makes 30 revolutions?

19. What is the circumference of a circus ring with a diameter of 28 ft.?

20. To how many decimal places is the value of π correct when $\frac{22}{7}$ is used?

To Find the Area of a Circle

Cut a circle from a piece of cardboard. Draw diagonals dividing the circle into equal sectors as shown in Fig. 58. You will observe that each of the sectors resembles a triangle. The arc of the sector corresponds to the base of the triangle, and the radius of the circle corresponds to the altitude of the triangle. The sum of the bases of the sectors is the circumference of the circle.

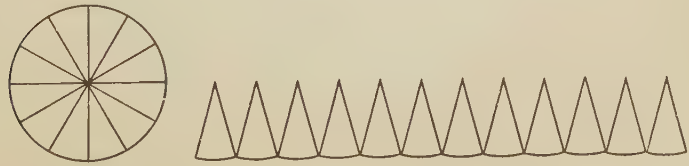


Fig. 58

If these small areas were triangles, the total area would be $\frac{1}{2}$ the altitude (which is the radius) multiplied by the sum of the bases (which is the circumference). We can prove by geometry that this is the area.

The area of a circle is equal to the product of $\frac{1}{2}$ the radius and the circumference.

The following formulas, in which A represents the area of the circle, R represents the number of units in the radius, and C represents the number of units in the circumference, are useful in solving problems that have to do with the areas of circles.

$$A = \frac{RC}{2}$$

However, since $C = 2\pi R$, we may put this value in place of C in the above formula, and so get the following:

$$A = \frac{\cancel{2}\pi RR}{\cancel{2}} = \pi R^2$$

To use the formula $A = \frac{RC}{2}$, you must know both the radius and the circumference; to use the formula $A = \pi R^2$ you need to know only the radius.

Find the area of a circle having a radius of 6 inches.

Solution

The area of any circle $= \pi R^2$.
The area required $= \frac{22}{7} \times 36 = 113\frac{1}{7}$.
This circle, therefore, has an area of $113\frac{1}{7}$ square inches.

Find the area of a circle having a circumference of 88 inches.

Solution

Since $C = 2\pi R$, $R = \frac{C}{2\pi}$.
$R = 88 \div 2 \times \frac{22}{7} = 14$.
The area, $\pi R^2 = \frac{22}{7} \times 196 = 616$ square inches.

Exercise 71

Use $\pi = \frac{22}{7}$, unless otherwise stated.

1. Find the areas of circles with the following radii:

1. 7 ft.
2. 56 in.
3. 28 in.
4. 10 in.
5. 1.4 in.
6. $3\frac{1}{2}$ ft.
7. 42 in.
8. $4\frac{1}{2}$ mi.

2. Find the areas of circles with the following circumferences:

1. 44 in.
2. 352 ft.
3. 132 yd.
4. 176 rd.

3. George tied a stone to a string $3\frac{1}{2}$ ft. long. When he swings the string around, how long is the path of the stone?

4. The inside diameter of a water pipe is 4 inches. What is the area of the hole?

5. What is the circumference, and what is the area, of the circle that may be traced on the ground by using a string 14 ft. long? (Use $\pi = 3.1416$.)

6. A circular silo has a base 16 ft. across. What is the area of the base?

7. Compare the areas of two circles, the radius of one being twice that of the other.

8. Draw a 4-inch square, then draw in it the largest possible circle. Find the area of the circle.

9. What is the area of the largest circle that can be cut from a 14-inch square? (Use $\pi = \frac{22}{7}$.)

10. A circular pond with a diameter of 60 ft. is surrounded by a walk 4 ft. wide. Find the cost of the walk at $1.30 per sq. yd.

11. Cora drew the largest possible circle in a square having a side of 10 inches. She discovered a way to find the area of one of the four equal parts left when the circle was cut out. Find the area of one of these parts.

12. Sam said that the circumference of the circle mentioned in *Problem 11* is greater than the perimeter of the square. Oscar said it is smaller. Show who is right.

13. What is the total area that may be protected by a revolving gun which has an effective range of 3 miles?

14. A searchlight has a range of $2\frac{1}{2}$ miles. What is the area of the circle over which its light may be thrown?

15. If a circle were drawn with a point in your school yard as a center, and a radius of 1 mile, how many square miles would be included in it? If the radius were 2 miles, how many times larger would the included area be?

16. How many yards of lace will be required to trim half a dozen round doilies, each of which is 8 in. in diameter? Allow $\frac{1}{9}$ yard to each doily for gathering.

17. A rotating sprinkler sprinkles a circular area with a radius of 24 ft. What is the area?

18. The diameter of the circular piston in an automobile engine is $3\frac{3}{4}$ in. What is the area of the surface of the piston?

19. George was asked to draw a circle having twice the area of a circle with a 1.5 ft. radius. He drew a circle with a radius of 3 ft. Did he do as he was asked? If not, what error did he make?

20. What is the wind pressure against a circular sign board with a diameter of 12 ft., when the wind is blowing squarely against it with a pressure of 30 lb. per sq. ft.?

21. At 30¢ per sq. ft., what will it cost to refinish the top of a circular table having a diameter of 42 in.?

22. The circumference of a tree at a certain height is 11 ft. What will be the area of the top of the stump, if the tree is sawed through at this place?

23. The diameter of the head of a drum is $2\frac{1}{2}$ ft. How many square inches of skin are in the two circular heads?

24. By how many square inches does the area of a 12-inch phonograph record exceed that of a 9-inch record?

25. Estimate the area of four circular objects at your home, then find the area of each.

26. What must you know, and what must you do, to find the area of a circle?

SQUARES AND SQUARE ROOTS

If a square has a side of 4 inches, it has an area of 16 square inches. Therefore, 16 is called the *square* of 4, and 4 is called the *square root* of 16. If a square has an area of 36 square inches, the number of inches in each side is the square root of 36 or 6.

From the above examples the following is plain:

1. **The number of units in the side of a square is the *square root* of the number of units in its area.**
2. **The *square root* of any number is the number which multiplied by itself produces the given number.**

The square of 4 is written 4^2. This is read "*four square.*"

Read the following and give the value of each: 2^2, 7^2, 9^2, 10^2, 3^2, 8^2, 4^2.

The sign $\sqrt{\ }$ indicates that we are to take the square root of the number written beneath it. $\sqrt{16}$ is read, "*the square root of* 16."

Read the following and give the value of each: $\sqrt{64}$, $\sqrt{9}$, $\sqrt{4}$, $\sqrt{49}$, $\sqrt{100}$, $\sqrt{81}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{144}$, $\sqrt{16}$.

A number which is the exact square of a whole number, a mixed number, or a fraction is called a *perfect square.*

For example: 16 is the square of 4. 1.96 is the square of 1.4. .09 is the square of .3. $\frac{9}{25}$ is the square of $\frac{3}{5}$.

Many numbers are not perfect squares. Thus, 3 is not a perfect square. However, the number 1.732 squared is 2.999824, which is almost 3. We may say, therefore, that the square root of 3 is 1.732 *nearly.*

In some cases, like 4, 9, 16, 25, you are able to state the square root at a glance. However, if you are asked to find the square root of a larger number, such as 2025, you cannot do so at sight. In this case you may find the square root by factoring.

For example:

$$\begin{aligned} 2025 &= 5\times5\times9\times9 \\ &= 5\times9\times5\times9 \\ &= 45\times45 \end{aligned} \qquad \begin{array}{r} 5\overline{)2025} \\ 5\overline{)405} \\ 9\overline{)81} \\ 9 \end{array}$$

Therefore the square root of 2025 is 45.

It is possible to estimate the square root of many numbers by comparing squares whose square roots are known.

For example: To estimate the square root of 20, we recall that 16 is the square of 4, and 25 is the square of 5. Therefore, the square root of 20 is more than 4 and less than 5.

Exercise 72

1. Find the squares of each of the following numbers.

	a	*b*	*c*	*d*	*e*	*f*
1.	9	14	30	1.6	.05	12.05
2.	.04	.7	6.05	1.8	.18	.0018

2. Name at sight the square root of each of the following:

1.	36	64	81	25	144	$\frac{16}{25}$
2.	.09	.25	.01	$\frac{49}{121}$	$\frac{16}{49}$	$\frac{1}{36}$

3. Find the square roots of the following by factoring:

1.	3025	324	1296	5184	1764	1024
2.	576	225	256	7225	2025	729

4. Estimate to the nearest whole number the square root of each of the following:

1.	82	39	200	318	180	124
2.	60	97	120	230	360	500

5. Find the sides of the square whose area is:

1. 169 sq. in.
2. 225 sq. in.
3. 2304 sq. ft.
4. 3025 sq. ft.
5. 10,000 sq. ft.
6. 5184 sq. yd.
7. 9025 sq. in.
8. 225 sq. yd.
9. 144 sq. ft.

6. A square has an area of 7225 sq. ft. What is the length of the side?

7. Write the squares of the whole numbers from 1 to 10.

8. Write the squares of all whole numbers from 11 to 25.

9. Can a number that ends in 7 be a perfect square?

10. What is the square of the smallest number containing two digits? Of the largest number containing two digits?

11. Can the square of a number containing two digits contain five digits? Show why.

12. Make a table of the squares of all numbers from 10 to 20. Use it to estimate to the nearest whole number the square root of each of the following: 168 327 260 298 317 386

13. A square garden has an area of 4225 sq. ft. What is the perimeter of the garden?

14. In one corner of a square field containing 1024 sq. rd. a farmer fenced off a square garden containing 169 sq. rd. Draw a plan of the field and of the garden within it, and find the perimeter of each.

15. A farmer buys a square plot of woodland that contains 10 acres. He wishes to fence this woodlot with wire fencing that costs $.64 a rod. How many rods of fencing will he need, and how much will it cost?

How to Find the Exact Square Root

You have learned that you can find the square root of some numbers by observation. The square root of others may be found by factoring, and that of others may be estimated. It is often necessary to find the exact square root of a number that cannot be easily factored. The following method may be used.

Find the exact square root of 1764.

$$\begin{array}{rr}
 & 4\;\;2 \\
 & \overline{17}\;\overline{64} \\
 & 16 \\
82\big| & \overline{1}\;\overline{64} \\
\big| & 1\;64
\end{array}$$

Begin at units, and point off the number 1764 into groups of two figures each, as shown here. These groups are called *periods*. The largest perfect square which is contained in 17, the first period at the left, is 16. The square root of 16 is 4. This is the first figure of the square root. Subtract the square of 4, that is 16, from 17. To the remainder, 1, annex the next period, 64, thus making 164. Double 4, the part of the square root already found, thus making 8, which is to be used for a *trial divisor*. Divide 164, exclusive of the right-hand figure, by 8 to find the next figure of the root. Annex the quotient, 2, to the trial divisor, 8, thus making a complete divisor, 82. Multiply 82 by 2, the last figure of the root found, and subtract the product, 164, from 164. As nothing remains, the square root of 1764 is 42.

$$\begin{array}{r|rrr}
 & 3 & 5 & 6\\
 & \overline{12} & \overline{67} & \overline{36}\\
 & 9 & & \\ \hline
65 & 3 & 67 & \\
 & 3 & 25 & \\ \hline
706 & & 42 & 36\\
 & & 42 & 36\\ \hline
\end{array}$$

In the example to the left, the method explained on page 203 has been used to find the square root of 126,736. Study the solution carefully, and work through it step by step.

Observe that in finding square root, the decimal point in the square root is placed above the decimal point of the number whose root is to be found, as shown in the example to the right.

$$\begin{array}{r|rr}
 & 8. & 7\\
 & \overline{75}. & \overline{69}\\
 & 64 & \\ \hline
167 & 11 & 69\\
 & 11 & 69\\ \hline
\end{array}$$

$$\begin{array}{r|rrr}
 & 1. & 7 & 3\\
 & \overline{3}. & \overline{00} & \overline{00}\\
 & 1 & & \\ \hline
27 & 2 & 00 & \\
 & 1 & 89 & \\ \hline
343 & & 11 & 00\\
 & & 10 & 29\\ \hline
 & & & 71
\end{array}$$

The example to the left illustrates a method for finding the square root of a number, correct to any number of decimal places. In such examples, periods of two zeros each may be annexed to the number until the root has been found to the required number of decimal places.

When finding the square root of a mixed decimal, begin at the decimal point and point off in groups of two figures to the left and also to the right.

For example: $\overline{27}.\overline{43}\,\overline{65}$ and $\overline{3}.\overline{74}\,\overline{8}$

Exercise 73

1. Find the square root of each of the following:

	a	*b*	*c*	*d*
1.	7569	1521	3481	3721
2.	1849	6241	9801	104976

2. Find the side of a square having an area of 6241 square feet

3. Find the square root of:

54.76 2.0449 .017424 9.4864

4. Find, to the nearest hundredth, the square root of 2. Of 5. Of 10. Of 40. Of 7.

Test the truth of the following statements, and illustrate each by three examples.

5. The squares of all numbers between 10 and 100 lie between 100 and 10,000.

6. The square root of a proper fraction is always less than 1, but it is always larger in value than the fraction itself.

7. The square of a mixed number is a mixed number.

8. Perfect squares that end in 6 have their square roots ending in 4 or 6.

The Use of Squares and Square Roots

In your study of angles and plane figures, you learned about the right triangle. As shown by Fig. 59, the line *BC* is the *base*, *AB* is the *altitude*. The line *AC* in this figure is called the *hypotenuse*. Your knowledge of square root will enable you to find the length of any one of these sides when the lengths of the other two sides are known.

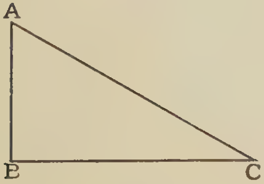


Fig. 59

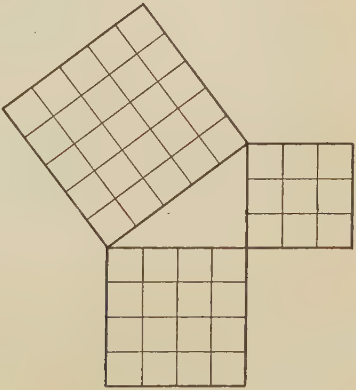


Fig. 60

Construct a right triangle with a base of 4 inches and an altitude of 3 inches. Measure the length of the hypotenuse. Construct squares on the three sides of the triangle, as shown in Fig. 60. Compare the area of the square constructed on the hypotenuse with the sum of the areas of the other two squares.

Try the same thing with a right triangle whose sides are 6 inches, 8 inches, and 10 inches.

Whatever the size of the right triangle used, the result will always be the same. This can be proved by geometry.

In a right triangle, the square on the hypotenuse equals the sum of the squares on the other two sides.

If h represents the number of units in the hypotenuse, a the number of units in the altitude, and b the number of units in the base of a right triangle, the following formulas may be developed:

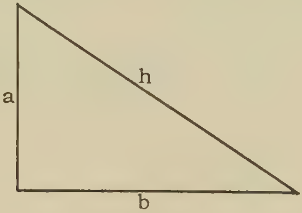


Fig. 61

I. $h^2=a^2+b^2$ Therefore $h=\sqrt{a^2+b^2}$
II. $a^2=h^2-b^2$ Therefore $a=\sqrt{h^2-b^2}$
III. $b^2=h^2-a^2$ Therefore $b=\sqrt{h^2-a^2}$

Exercise 74

1. State each of the above formulas in words.

2. If $h=25$, and $b=20$, find a.

3. If $a=5$, and $b=12$, find h.

4. If $a=21$, and $b=28$, find h.

5. How long is the diagonal of a room which is 32 ft. long and 24 ft. wide?

6. Measure the length and width of your classroom, then find the diagonal by using the formula given on page 206. Check your result by measurement.

7. What distance is saved by going diagonally across a rectangular lot 60 ft. by 150 ft., instead of going around the lot? Find the result to the nearest foot.

8. A wire is fastened to a telegraph pole 25 ft. from the ground, and to a stake at the ground 16 ft. from the foot of the pole. Find the length of the wire to the nearest 0.1 ft.

9. A rectangular sheet of paper is 10 in. long and 8 in. wide. Find to the nearest 0.1 in. the length of the longest straight line that can be drawn on this sheet.

10. A wire that is attached 8 ft. below the top of a flag pole 40 ft. high, is fastened at the ground to a stake 24 ft. from the base of the pole. Making no allowance for slack, what is the length of the wire to the nearest 0.1 ft.?

11. A baseball diamond is 90 ft. square. What is the shortest distance from first base to third base?

12. The bottom of a 30-foot ladder is 12 ft. from a wall against which the top of the ladder rests. How high does the ladder reach on the wall?

13. Two automobilists meet at the crossroad. How far apart are they after $1\frac{1}{2}$ hr., if one drove east at the rate of 20 mi. an hour, and the other drove north at the rate of 24 mi. an hour?

How the Workman Uses Square Root

Workmen employed in the building trades, and others as well, have many uses for a knowledge of square root. You should try to discover some of the cases where you could use square root to advantage.

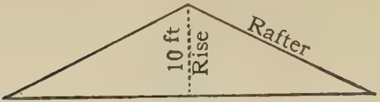


Fig. 62

THE PITCH OF A ROOF: The *pitch* of a roof means the slant of a roof. It is necessary to know the pitch in order to determine how long to cut rafters. The *rise* of a rafter is the distance above the base of the gable as shown in Fig. 62. The pitch of a roof is found by dividing the rise of the rafters by the width of the base of the gable. If the rafters rise 10 feet and the base of the gable is 40 feet, the pitch of the roof is $\frac{10}{40}$ or $\frac{1}{4}$. If the rafters rise 12 feet and the base of the gable is 32 feet, the pitch of the roof is $\frac{12}{32}=\frac{3}{8}$.

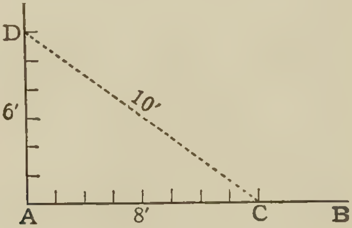


Fig. 63

"SQUARING" A CORNER: Workmen frequently "square" corners by means of a right-angled triangle, as shown in Fig. 63. To construct a line at right angles to *AB* at *A*, first measure a distance of 8 feet on *AB*, from *A* towards *B*. This locates the point *C*. Fasten a cord or tape measure at *A*, and draw it into such a position that the end of it, *D*, will be 6 feet from *A*, and 10 feet from *C*. Then *AD* is perpendicular to, or at right angles with, *AC*.

This method may be used to advantage in laying out a tennis court. Builders sometimes use this method when laying a foundation.

Exercise 75

1. What is the pitch of a roof, if the width of the gable is 45 ft., and the rise of the rafters is $11\frac{1}{4}$ ft.? What is the length of the rafters if they extend 1 ft. beyond the eaves?

2. With the help of a classmate, "square" several corners by using the method described on page 208.

3. How long must the rafters be cut for a roof 30 ft. wide, if the pitch of the roof is $\frac{1}{2}$, and the rafter is to extend 1 ft. beyond the eaves?

4. Carpenters often find the pitch of a roof by using a rule and a carpenter's square. Ask some carpenter to show you how this is done.

5. A workman wants to brace a pole whose top is 24 ft. above the ground. He decides to attach a wire 4 ft. from the top of the pole and fasten it to a stake 30 ft. from the bottom of the pole. How long must he cut the wire if he allows 2.5 ft. at each end for fastening?

6. Henry's father told him to cut a board to brace a fence gate. The gate is 8 ft. long and 4 ft. high. The brace is to extend diagonally across the gate. How long must the board be?

7. Some boy scouts find that they are 4 miles east and 3 miles south of their camp. When returning to camp, what distance will they save by taking a diagonal route instead of traveling by roads that run at right angles?

8. George put up a 35-foot aerial for his wireless outfit. He braced the aerial by a wire attached to a stake on the ground 20 feet from the base. He attached the other end of the wire 3 feet from the top of the aerial. How much wire did George use, if he had to allow 4 feet for connections?

9. Is it correct for a carpenter to "square a corner" by using a right triangle whose sides are 6 inches, 8 inches, and 10 inches?

10. Use a right triangle to "square a corner" on the schoolroom floor or playground.

THE MEASUREMENT OF SOLIDS

To Find the Volume of a Rectangular Solid

Earlier in your work you studied certain solids and learned how to find their capacity. You are now to review the method of finding the capacity, or *volume*, of certain solids, and you are to learn how to find the volume and the lateral surface of several kinds of solids that you have not previously studied.

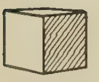


Fig. 64

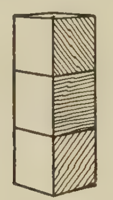


Fig. 65

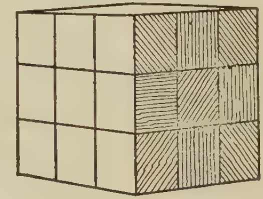


Fig. 66

A solid that has six rectangular faces is called a *rectangular solid.* Fig. 64 shows a rectangular solid 1 unit long, 1 unit wide, and 1 unit high.

The volume, or the number of units, in Fig. 65 is $3\times1\times1$, or 3 cubic units. The number of units in a solid 4 units long, 3 units wide, and 5 units high is $4\times3\times5$, or 60 cubic units.

The volume of a rectangular solid equals the product of its three dimensions.

If the rectangular faces of a solid are all squares, the solid is called a *cube.*

For example: Fig. 64 is a cube 1 unit long, 1 unit wide, and 1 unit high. Its volume is 1 cubic unit. Fig. 66 is a cube 3 units long, 3 units wide, and 3 units high. Its volume is $3\times3\times3$, or 27 cubic units. The number of cubic units in this cube may be expressed as the product of $3\times3\times3$, or as 3^3.

The volume of a cube equals the cube of its edge.

Illustrate this rule by examples of your own. Is the statement always true?

Exercise 76

1. Find the volumes of rectangular solids having the following dimensions:

	a	*b*	*c*
1.	2 in., 3 in., 4 in.	2.5 ft., 3 ft., 4 ft.	4 in., 2 in., 6 in.
2.	9 ft., 4 ft., 5 ft.	8 ft., 12 ft., 16 ft.	8 ft., 7 ft., 10 ft.
3.	3 yd., 4 yd., 7 yd.	3.8 yd., 4.5 yd., 3.5 yd.	9 ft., 12 ft., 6 ft.
4.	6 ft., 4 ft., 2.5 ft.	9.5 in., 6.2 in., 3.5 in.	8 ft., 4 ft., 6 ft.

2. How many cubic inches are in a cube having an edge 3 in. long? 5 in. long? 8 in. long?

3. How many cubic inches are in a cube having an edge 12 in. long? How many cubic inches are in a cubic foot?

4. How many cubic feet are in a cube having an edge 3 ft. long? How many cubic feet are in a cubic yard?

5. At 60¢ a load, what will it cost to excavate for a cellar 24 ft. by 27 ft. by 7 ft.? (Consider a cubic yard as a *load.*)

6. The inside measurements of a farm wagon-box are 10 ft. long, 3 ft. wide, and 26 in. deep. What is its volume?

7. Find the volume of a chalk box or of some other rectangular box in your schoolroom.

8. Find the volume of some box or bin at your home.

9. A *cord* of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. How many cubic feet does a cord contain?

10. How many cords of wood in a pile 48 ft. by 8 ft. by 4 ft.?

NOTE TO PUPIL: A cube may be constructed from cardboard by cutting out a diagram like the one shown in Fig. 67, bending the cardboard on the dotted lines, and pasting the flaps.

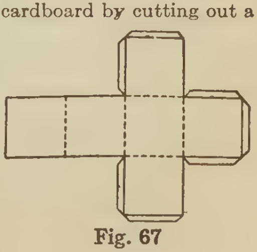


Fig. 67

11. When estimating the hay in a mow 32 ft. long and 16 ft. wide, a farmer allowed 512 cubic feet to the ton. How many tons of hay were in the mow if the depth of the hay was 8 ft.?

12. For a distance of half a mile, a contractor removed an average depth of 15 in. of earth from a street 60 ft. wide. How many cubic yards of earth were removed?

13. The dirt from a cellar 24 ft. by 28 ft. by 7 ft. is used to fill in a level lot 60 ft. by 150 ft. To what depth will the lot be filled if the dirt is evenly spread?

14. A gallon contains 231 cubic inches. How many gallons are in a cubic foot? (Find answer correct to 0.1 gallon.)

15. A freight car with inside dimensions of 36 ft. by 8 ft. 4 in. is filled with grain to a depth of 5 ft. Allowing $1\frac{1}{4}$ cubic feet to the bushel, what is the weight of the grain at 60 pounds to the bushel?

16. A level garden is 60 ft. long and 32 ft. wide. How many gallons of water fell on it during a month when the rainfall was 4 in.? What was the weight of the water? (Allow 62.5 lb. to the cu. ft.)

17. How many cubic feet of air space in your schoolroom? What is the average number of cubic feet per person?

18. A unit of measure called the *perch* is sometimes used for measuring stone and stone walls. A perch is 16.5 ft. long, 1.5 ft. wide, and 1 ft. high. How many cubic feet does a perch contain?

19. A wall is 66 ft. long, 3 ft. wide, and 3.5 ft. high. Find the cost of building the wall at $6.40 a perch.

20. Ice 15 inches thick is cut from a rectangular area 250 ft. by 460 ft. If a cubic foot of this ice weighs $56\frac{1}{4}$ lb., how many tons of ice are cut from this area?

21. A corncrib 24 ft. long and 8 ft. wide is filled with corn on the cob to a depth of 12 ft. How many bushels of corn are in the crib, allowing $1\frac{1}{4}$ cubic feet to the bushel?

22. The storage bin of a grain elevator is 64 ft. long, 32 ft. wide, and 48 ft. high. What is its capacity in bushels, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

23. The inside measurements of a gasoline tank on an automobile are as follows: 26 in. wide, 14 in. deep, 28 in. long. Allowing 231 cu. in. to the gallon, what is the capacity in gallons to the nearest tenth?

24. How many cubic feet of concrete are needed for a wall 60 ft. long, 6 ft. high, and 8 in. thick?

25. How many gallons of water are required to fill a swimming pool 42 ft. long and 20 ft. wide to an average depth of 5 ft.?

26. Make a list of the facts that you can discover in regard to the number of edges, corners, and faces of a cube.

27. George heard a speaker say that when an inch of rain falls on an acre of ground the weight of the water is more than 110 tons. George checked the statement. You find how much that amount of water weighs, counting 1 cu. ft. of water as 62.5 lb., and an acre as 43,560 sq. ft.

28. Four boys were hired to shovel a 5-inch fall of snow from the sidewalks of a small park. If the walks were 5 ft. in width and 1200 ft. in length, and each boy shoveled the same amount, how many cubic feet of snow did each shovel?

29. What is the cost of digging a sewer ditch 1800 ft. long, 3 ft. wide, and 6 ft. deep, at $.32 a cu. yd.?

Practical Measurements: Lumber Measure

Lumber measure, used by lumbermen, carpenters, contractors, and others engaged in the purchase or sale of lumber, is in reality only a special application of the principles you have learned for determining the volume of a rectangular solid. The *board foot* is the unit of lumber measure. It equals the volume of a board 1 foot square and 1 inch thick. Even though boards are less than an inch thick they are usually regarded as being a full inch in thickness, when computing board feet.

In estimating the amount of flooring required for a room, the length and width are measured. If the boards are four inches in width, one-fourth extra is usually allowed for matching; if the width is three inches, one-third extra is usually allowed.

The following rule is commonly used for finding the number of board feet in a piece of lumber.

Multiply the length in feet by the number of inches in the thickness by the number of inches in the width, then divide that product by 12 to find the number of board feet.

For example: To find the number of board feet in a board 9 ft. long, 8 in. wide, and 2 in. thick, $2 \times 8 \times 9 = 144$. $144 \div 12 = 12$, the number of board feet. The amount of lumber in any number of boards of the size indicated is found by multiplying the number of board feet in one board by the number of boards.

Lumber is priced at so much per thousand board feet. The sign M is used for thousand.

For example: $48 per M means $48 per 1000 board feet.

Exercise 77

1. Find the number of board feet in each of the following:

1. 16 planks, each 14 ft. long, 8 in. wide, 2 in. thick.
2. 24 pieces, each 16 ft. long, 12 in. wide, 8 in. thick.
3. 96 boards, each 16 ft. long, 8 in. wide, $\frac{7}{8}$ in. thick.
4. 18 joists, each 14 ft. long, 6 in. wide, 2 in. thick.

2. What is the cost of 24 pieces of lumber, each 16 ft. long, 6 in. wide, and 3 in. thick, at $56 per M?

3. What is the cost of 36 joists, each 14 ft. long, 8 in. wide, and 2 in. thick, at $48 per M?

4. Find the number of square feet of 4-inch flooring required for a room 28 ft. by 24 ft.

5. At $84 per M, find the cost of the boards for a floor in a room 24 ft. long and 20 ft. wide, if flooring 3 in. wide and 1 in. thick is used. Allow $\frac{1}{3}$ extra for matching.

6. At $125 per M, what will be the cost of 1-inch flooring for a hall 60 ft. by 42 ft., if flooring 3 in. wide is used? Allow $\frac{1}{3}$ extra for matching.

7. Find the cost of the following:

Pieces	Description	Price per M
280	2″×6″ soft pine, 8 ft. long	$45
84	10-in. boards, 12 ft. long	$54
8	3″×6″ oak, 10 ft. long	$80
15	3″×10″ planks, pine, 16 ft. long	$75

8. How much lumber will be required for a board fence around a lot 60 ft. by 150 ft., if the fence is to consist of two strips of 2″×4″ scantling on which 1-inch boards 6′ long are to be nailed? Allow one-eighth of the material required for the boards as waste due to cutting. (The estimate given in this problem does not include posts for the fence.)

9. The floor in a store is made of 1-inch boards. How many board feet are used if the room is 75 ft. long and 32 ft. wide? Allow $33\frac{1}{3}\%$ additional for waste in matching. What will be the cost of the flooring at $96 per M?

10. A lumber dealer told a class that the following rule is used at the lumber yard to find the number of board feet in lumber one inch thick:

If 3 inches wide, take $\frac{1}{4}$ the length.
If 4 inches wide, take $\frac{1}{3}$ the length.
If 6 inches wide, take $\frac{1}{2}$ the length.
If 8 inches wide, subtract $\frac{2}{3}$ the length.
If 10 inches wide, subtract $\frac{5}{6}$ the length.

Show by examples whether these rules give correct results.

11. Make rules similar to those given in *Problem 10* for finding the amount of lumber when boards are of the following widths: 9 inches, 12 inches, 15 inches.

12. Find the number of board feet of lumber in 35 pieces, each of which is 2 in. thick, 10 in. wide, and 14 ft. long.

13. A boy stated that a piece of lumber 8 in. by 8 in. by 16 ft. contains $85\frac{1}{3}$ board feet. Is this correct?

To Find the Volume of a Prism

A solid that has two parallel bases in the form of polygons exactly alike, the other sides being parallelograms, is called a *prism.*

If the base of a prism is a triangle, the prism is called a *triangular prism.* (See Fig. 69.) If the base is a rectangle, the prism is called a *rectangular prism.* (See Fig. 68.)

What would you call a prism whose base is a pentagon? A hexagon? (See Fig. 70.) An octagon? Do you know of objects that have any of these forms?

Recall the definition of a polygon. Is a square a polygon? Is a cube a prism? If so, what kind of prism is it?

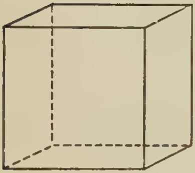


Fig. 68

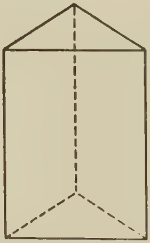


Fig. 69

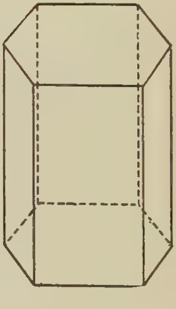


Fig. 70

You will recall that we find the volume of a rectangular solid by multiplying the length by the breadth by the height. Multiplying the length by the breadth gives the area of the base. We may say, then, that the volume of a rectangular solid equals the area of the base times the height. A rectangular solid is a kind of prism, and it is proved in geometry that the same method may be used to find the volume of any other kind of prism.

The volume of any prism equals the area of the base multiplied by the altitude.

How do you find the area of a right triangle? Of an isosceles triangle? Of a hexagon? If you know how to find the areas of different kinds of polygons, what help will it be to you in finding the volumes of prisms?

Exercise 78

Find the volumes of prisms with bases and altitudes as stated:

Base	Altitude	Base	Altitude
1. 48 sq. in.	18 in.	**4.** 465 sq. yd.	15.6 yd.
2. 540.4 sq. in.	16.5 in.	**5.** 4.8 sq. in.	2.5 in.
3. 43.8 sq. ft.	2.5 ft.	**6.** 74.9 sq. yd.	9.5 yd.

7. A monument whose height is 18 ft. is in the form of a rectangular prism. The length of its base is 12 ft., and the width is 8 ft. What is the volume of the monument?

8. The base of a triangular prism has an area of 24 sq. in. The altitude of the prism is 14 in. What is the volume of the prism?

9. What is the volume of a triangular prism whose base contains 148 sq. in., and whose altitude is 3.5 ft.?

10. Find the capacity in gallons of each of the following rectangular tanks, allowing 231 cu. in. to the gallon:

1. 6′×5′×10′
2. 8′×4.5′×12′
3. 3.5′×2.4′×6′
4. 15′×12′×12′
5. 12.5′×9.4′×15′
6. 20′×12′×36′

11. What is the capacity in gallons of a rectangular oil can whose interior dimensions are 5″×8″×12.5″?

12. A bushel contains 2150.4 cubic inches. If a box is 14 in. square, how deep must it be to have a capacity of one bushel?

13. The silo on the farm where Sam lives is in the form of a hexagonal prism. The height of the silo is 28 ft., and each side of the base measures 6 ft. in length. Sam's father explains that the formula for finding the area of a hexagon whose side is s is: $a=6\times\frac{s^2}{4}\times 1.73$. He then asks Sam to find the capacity of this silo. See whether you can do this.

To Find the Lateral Area of a Prism

The *lateral area* of any solid is the surface area, not including the area of the bases. As the lateral surface of a prism is made up of parallelograms, you can find the lateral area of a prism by finding the sum of the areas of these parallelograms.

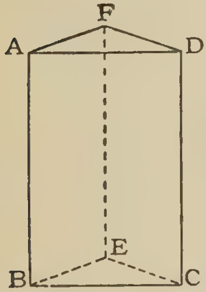


Fig. 71

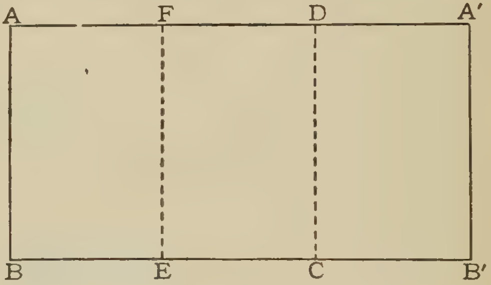


Fig. 72

For example: Fig. 71 shows a right prism whose base is a triangle and whose lateral faces are the rectangles *ABCD*, *DCEF*, and *FEBA*. If you will think of a sheet of paper cut to fit exactly around the prism and then unfolded, you will see that the paper will be in the form shown in Fig. 72. The rectangle *ABEF* in Fig. 72 corresponds to the rectangle *ABEF* of Fig. 71. The rectangle *FECD* of Fig. 72 corresponds to the rectangle *FECD* of Fig. 71. The rectangle *DCB′A′* (read "*B* prime *A* prime") corresponds to the rectangle *DCBA*. Now imagine the rectangle *ABB′A′* shown in Fig. 72 wrapped around the prism shown in Fig. 71 so that line *AB* of Fig. 72 will fall on line *AB* of Fig. 71. On what line does *FE* of Fig. 72 fall? On what line does *DC* of Fig. 72 fall? On what line does *A′B′* of Fig. 72 fall? It is plain that the area of the large rectangle *ABB′A′* is the same as the sum of the three rectangles shown in Fig. 71.

The base, *BB′*, of Fig. 72 is the same in length as the sum of the bases of the three rectangles of Fig. 71. The width of the large rectangle in Fig. 72 is the same as the height of the prism shown in Fig. 71. The area of the rectangle shown in Fig. 72 is equal to its base times its height.

The *lateral area* of any right prism is found by multiplying the perimeter of the base by the altitude of the prism.

Measure this book and find its lateral area.

Exercise 79

1. Find the lateral areas of the following prisms.

1. A rectangular prism, base 4′×3′, altitude 8′.
2. A rectangular prism, base 12.5′×6.5′, altitude 20′
3. A hexagonal prism, each edge of the base 5″, altitude 15″.
4. A prism, edges of base 6′, 8′, and 10′, altitude 22″.

2. A can 12″ high has a rectangular base 5″×8″. What is the area of a paper that just covers the lateral area?

3. A porch roof is supported by three hexagonal columns, each 14 ft. high. Each edge of the base of each column measures 6 in. What will it cost to paint these columns at $.54 a square yard?

4. Mary bought colored paper to cover the outside of a box that has a rectangular base 14 in. long and 9 in. wide, and a height of 8 in. How many square inches of paper are required to cover the box?

5. Fred wants to cover the sides and top of a box with gilt paper. The bottom of the box measures 9 in. by 6 in., and its height is 4 in. How many square inches of gilt paper are required to cover the box?

6. The large beam of a derrick is 32 ft. long. The end is a square, each side of which measures 1 ft. How much will it cost to paint this beam at $.45 a square yard?

7. Measure a chalk box or some other prism in your schoolroom, then find the lateral area.

8. Estimate the lateral area of some prism, then check your estimate by making the proper measurements and computing the area.

9. How many lateral faces has a hexagonal prism? What is the shape of each face?

10. How many lateral faces has a triangular prism? What is the shape of each face?

To Find the Volume of a Cylinder

As shown by Fig. 73, a *cylinder* is a solid body having two equal parallel circular faces called *bases*. The distance between the bases is the *altitude*.

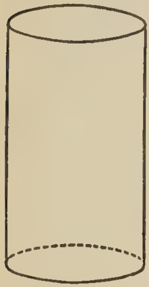


Fig. 73

What objects having the form of a cylinder have you seen?

You have learned that the volume of a right prism may be found by finding the area of the base and multiplying that by the altitude. The cylinder, like the right prism, is a solid body having bases that are alike in shape and size. You can find its volume just as you found the volume of the right prism.

You know how to find the area of a circle. You should now review the method of doing that, so that you may be able to determine the area of the base of any cylinder. The use of the following rule will enable you to find the volume of any cylinder.

The volume of a cylinder equals the product of its base and altitude.

This statement means that the number of cubic units in the volume equals the product of the number of square units in the base and the number of linear units in the altitude.

Exercise 80

The dimensions given in the following problems are inside measurements.

1. A cylindrical water tank with a diameter of 24 ft. is 36 ft. high. What is its capacity in cubic feet?

2. A circular pipe 12 ft. long is 4 in. in diameter. How many gallons of water will this pipe hold? (1 gal. = 231 cu. in.)

3. A hot water tank is 16 in. in diameter, and 60 in. high. How many gallons of water will it hold?

4. A standpipe in a certain town is 60 ft. high, and has a diameter of 20 ft. How many gallons of water will it hold?

5. A farmer has a cylindrical silo 12 ft. in diameter, and 32 ft. high. Allowing 42.8 lb. of ensilage to the cubic foot, how many tons of ensilage will it hold?

6. Mr. Carson has a silo 14 ft. in diameter filled to a depth of 32 ft. Counting 38 lb. to the cubic foot, what is the total weight of the ensilage?

7. How many days will the ensilage mentioned in *Problem 6* supply a herd of 24 cows, allowing each cow 30 pounds of ensilage a day? (Find answer to the nearest day.)

8. In a large city there is a gas tank 90 ft. in diameter, and 60 ft. in height. What is its capacity in cubic feet?

9. A gasoline tank is 24 in. in diameter, and 34 in. long. What is its capacity in gallons?

10. A water tank 9 ft. in diameter and 12 ft. long is placed on the roof of a building. What is the weight of the water which this tank will hold? (1 cu. ft. of water weighs 62.5 lb.)

11. An oil tank on a railroad car is 30 ft. long, and has a diameter of 5 ft. What is the capacity of the tank in gallons?

12. A sprinkling wagon has a cylindrical tank 4 ft. in diameter, and 12 ft. long. How many gallons will this tank hold?

13. A water main has a diameter of 15 in. How many barrels of water will a mile of the water main hold? (1 bbl. $=31\frac{1}{2}$ gal.)

14. Water runs through a 3-foot sewer pipe at the rate of 72 ft. per second. How many gallons can the pipe discharge per hour?

15. George finds that the water in a 6-foot square cistern at his home is 7 ft. 6 in. deep. How many gallons of water are in the cistern?

16. Measure three cylindrical objects at your home and find the volume of each.

17. Name four objects that are cylinders.

18. Underground gasoline tanks of various dimensions are used by gasoline filling stations. Find the capacity in gallons of the following cylindrical tanks.

	Diameter in Inches	Length in Feet
1.	36	6
2.	42	8
3.	48	12

19. An oil tank on a car is 30 ft. long and 5 ft. in diameter. Allowing 4.25 cu. ft. to the barrel, compute the capacity of this car in barrels.

To Find the Lateral Area of a Cylinder

You have learned how to find the lateral surface of a prism. Your observation of the *cylinder* will show you that it has a base and an altitude just as a prism has. If, now, you roll a rectangular piece of cardboard around a cylinder, allowing nothing for overlapping, then allow it to unroll, you will form a figure like that shown in Fig. 74. How does the circumference of the cylinder compare with the length of the rectangular piece of cardboard? How does the altitude of the cylinder compare with the width of the piece of cardboard?

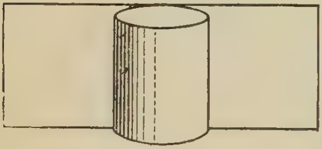


Fig. 74

The *lateral area* of a cylinder equals the product of its circumference and altitude.

Exercise 81

1. A roll of wrapping paper has a diameter of 15 in. The paper is 30 in. wide. What is the area of the outside layer of the paper?

2. A gas tank has a diameter of 72 ft., and a height of 60 ft. What will it cost to paint the lateral area of the tank at 25¢ a sq. yd.?

3. The roller of a rolling pin is 12 in. long, and has a diameter of 3 in. What is the area that may be rolled by one rotation of this pin?

4. A standpipe with a diameter of 24 ft. is 90 ft. high. What will it cost to paint the lateral area at 25¢ a square yard?

5. What is the heating surface of a 3-inch steam pipe 24 ft. long?

6. A large room is heated by 220 ft. of steam pipe 2 in. in diameter. What is the area of the surface that radiates the heat?

7. A circular smoke stack 60 ft. high has an outside diameter of 8 ft. What is its lateral area?

8. What is the lateral area of a cylindrical gasoline tank 4 ft. long, and 18 in. in diameter?

9. A power plant has a cylindrical tank 30 ft. deep and 24 ft. in diameter. What is its lateral area? What is its capacity in gallons? (Allow $7\frac{1}{2}$ gallons to the cubic foot.)

10. How many square feet of asbestos must be ordered to wrap a furnace 60 in. high and 52 in. in diameter? Asbestos for this purpose comes in rolls 36 in. wide, each roll containing 100 square feet.

11. The cylindrical tank on an automobile sprinkler is 12 ft. long. It has a diameter of 6 ft. What will it cost to paint the tank at 32¢ a square yard?

12. Frances has a cylindrical hat box 22 in. in diameter and 14 in. high. How many square inches of fancy paper are required to cover the lateral area of the box?

13. How many square feet of tin are needed to make a pipe 6 ft. long and 7 in. in diameter, allowing 1 in. for overlapping? (Use $\pi = \frac{22}{7}$.)

14. If A represents lateral area, R the radius of the base, and H the height of a cylinder, then $A = 2\pi RH$. State this formula in words.

To Find the Lateral Area of a Pyramid

A solid whose base is a polygon and whose lateral faces are triangles, as shown in Fig. 75, is called a *pyramid*. The point *V* at which the triangular faces meet is called the *vertex*. The perpendicular distance from the vertex to the base is called the *altitude* of the pyramid.

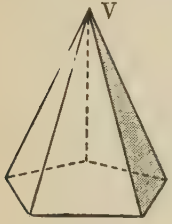


Fig. 75

If a perpendicular is dropped from the vertex of the pyramid in Fig. 75, at what point in the base will it touch?

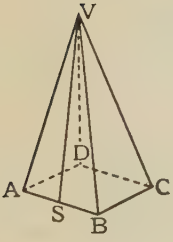


Fig. 76

The lateral area of a pyramid equals the sum of the areas of all the faces. A line drawn from the vertex of a regular pyramid to the middle of the base line of one of the faces is called the *slant height*. A pyramid whose faces are equal isosceles triangles, and whose base is a regular polygon, is called a *regular pyramid*.

What kind of triangle is each face of the pyramid shown in Fig. 76? How do you find the area of a triangle? How does the slant height of a pyramid compare with the altitude of each of its faces?

The lateral area of a pyramid equals the product of one-half the slant height and the perimeter of the base.

Exercise 82

1. Find the lateral areas of pyramids with the following perimeters and slant heights.

	Perimeter	Slant Height		Perimeter	Slant Height
1.	48 in.	30 in.	*6.*	14 ft.	18 ft.
2.	3 ft., 6 in.	1 ft., 6 in.	*7.*	5 ft., 2 in.	24 in.
3.	12 ft.	4.5 ft.	*8.*	60 ft., 8 in.	32 ft.
4.	8 ft., 4 in.	12 ft.	*9.*	194 ft., 6 in.	82 ft.
5.	24 ft., 8 in.	8 ft.	*10.*	84 ft., 2 in.	46 ft.

2. The Great Pyramid of Cheops, in Egypt, had a slant height of 616 ft., and square base measuring 768 ft. on each side. Find its lateral area.

3. A church steeple in the form of a pyramid has a slant height of 36 ft. The perimeter of its base is 42 ft. What is the lateral area of the steeple?

4. A monument whose slant height is 14 ft. is in the form of a pyramid with a square base that measures $3\frac{1}{2}$ ft. on a side. What is the lateral area of the monument?

To Find the Lateral Area of a Cone

As shown by Fig. 77, the *cone* resembles the pyramid in some respects. You may think of a cone as being formed by turning the right triangle *VOA* about the line *VO*. The lateral area of the cone is its curved surface. The side *VA* is called the *slant height* of the cone. *VO* is the altitude.

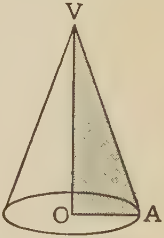


Fig. 77

If you take a cone like the one shown in Fig. 77, slit it from the point *V* to the point *A*, and flatten it out, you will have a surface like that shown in Fig. 79. This is a *sector* of a circle.

To what line in the cone (Fig. 77) does the line *VA* in Fig. 79 correspond? To what line in the cone (Fig. 77) does the line *AB* in Fig. 79 correspond?

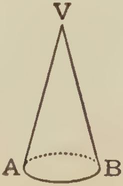


Fig. 78

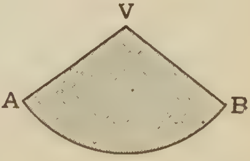


Fig. 79

Since the area of the sector corresponds to the lateral area of the cone, then a rule for finding the area of a sector will also be a rule for finding the lateral area of a cone whose dimensions correspond to similar lines of the sector. The area of the sector equals the product of its arc *AB* and half its radius *VA*. Since the radius of the sector is the same as the slant height of the cone, and the arc of the sector equals the circumference of the cone, the following is the rule for finding the lateral area of any cone.

The lateral area of a cone equals the product of one-half the slant height and the circumference of the base.

Exercise 83

1. How many square yards of canvas are required to make a conical tent 20 ft. in diameter at the base, and having a slant height of 18 ft.?

2. A cylindrical silo has a conical roof as shown in Fig. 80. What is the total surface area of the silo, if the base is 16 ft. in diameter and the altitude of the cylindrical part is 36 ft., the base of the conical top is 19 ft. in diameter, and the slant height is 10.8 ft.? What will it cost to paint this silo at $.45 a square yard?

Fig. 80

3. A conical funnel is 12 in. in diameter at the base, and has a slant height of 14 in. What is its lateral area?

4. A conical church spire has a slant height of 42 ft. The diameter of the base is 12 ft. What will it cost to paint the spire at $.45 a square yard?

5. Find the number of square yards of canvas required for a conical tent whose altitude is 8 ft., the radius of its base being 6 ft. Allow 1 square yard extra for overlapping.

6. Which has the larger lateral area, and by how many square inches: A cone having a slant height of 14 ft., the circumference of its base being 28 ft., or a cylinder having a height of 12 ft., the circumference of its base being 22 ft.?

7. Make the proper measurements for finding the lateral area of some cone.

To Find the Volume of a Pyramid or of a Cone

If you fill a pyramid with water, and then pour the water into a rectangular prism with equal base and equal altitude, as shown in Fig. 81, you will find that the pyramid holds just one-third as much as the prism. In the same way you can show that a cone holds just one-third as much as a cylinder of equal base and equal altitude, as illustrated by Fig. 82.

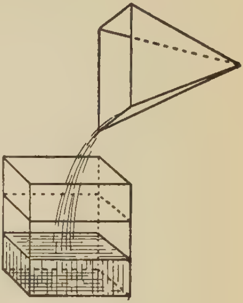


Fig. 81

The volume of a pyramid or of a cone is equal to one-third the product of its base and altitude.

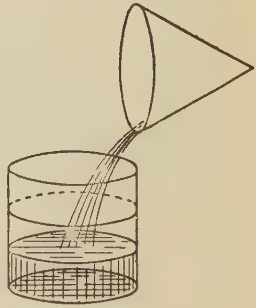


Fig. 82

Exercise 84

1. Find the volumes of pyramids or cones with bases and altitudes as follows:

	Base	Altitude
1.	225 sq. ft.	12.4 ft.
2.	1000 sq. yd.	36 yd.
3.	420.16 sq. in.	12.4 in.
4.	94.9 sq. ft.	6.5 ft.
5.	378.4 sq. in.	7.8 in.
6.	194.02 sq. ft.	11.4 ft.

2. The Great Pyramid of Cheops, in Egypt, was originally 482 ft. high and had a square base 768 ft. on a side. What was its volume? If the material averaged 168 lb. per cubic foot in weight, what was its original weight?

3. How many cubic feet of air space in a conical tent whose diameter is 16 ft., and whose height is 12 ft.?

4. In a corner of a bin is a pile of wheat forming one-fourth of a cone. The height of the pile is 3.6 ft., and the radius of the cone is 4.8 ft. How many bushels of wheat are in the pile? (Allow $1\frac{1}{4}$ cu. ft. to a bushel.)

5. Does a cone which measures 4 in. in height and 2 in. in diameter (inside measurements) have a capacity of more than half a pint? (1 gallon = 231 cu. in.)

6. How many cubic feet of sand in a conical pile with a diameter of 12.8 ft., and a height of 5.6 ft.?

7. If a prism weighs 42 lb., what is the weight of a pyramid made of the same kind of material if its base is equal to that of the prism, and its altitude is six times the altitude of the prism?

8. A monument 16 ft. high is in the form of a pyramid. It has a square base 3 ft. on a side. If a cubic foot of the material from which the monument is made weighs 185 pounds, what is the weight of the monument?

9. Find the lateral area and volume of a cone whose altitude is 15 in., the diameter of the base being 12 in.

10. A conical vessel is 24 in. high. The radius of its base measures 14 in. How many gallons will it hold?

11. The base of a pyramid is a right triangle, one side of which measures 8 in., the other 6 in. The altitude of the pyramid is 27 in. Find the volume.

12. How many square yards of canvas are required to make a tent with a square base 10 ft. on a side and a slant height of 14 ft.? Allow $2\frac{1}{2}$ sq. yd. extra for flaps and overlapping.

13. How many square yards of canvas are required to make a conical tent 18 ft. in diameter with a slant height of 15 ft.? Allow $2\frac{1}{2}$ sq. yd. extra for flaps and overlapping.

14. Helen and Clara are making 12 conical caps for a Halloween party. How many square yards of fancy paper will be required to cover the caps, if each cap is 22 in. in circumference and has a slant height of 16 in.? Allow $1\frac{1}{2}$ sq. in. waste for each cap.

15. If you know of any church spire in your vicinity having the shape of a pyramid or cone, estimate its dimensions and compute its approximate lateral surface.

How to Find the Surface and Volume of a Sphere

A *sphere* is a solid bounded by a surface, every part of which is equally distant from a point within called the *center*. Fig. 83 shows a sphere. A sphere has a radius and a diameter just as a circle has.

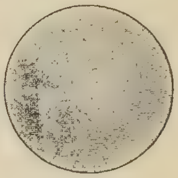


Fig. 83

If you wind a cord about half the surface of a sphere, as shown in Fig. 84, and then use the same cord to wind about a cylinder whose radius equals the radius of the sphere, and whose height equals the diameter of the sphere, you will find that the cord covers just half the cylinder.

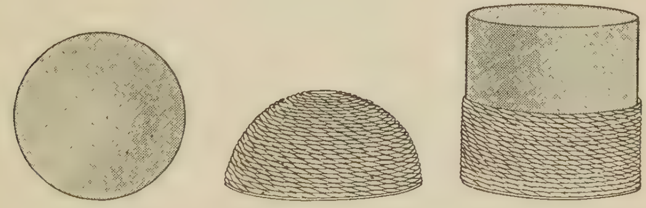


Fig. 84

The surface of a sphere is equal to the lateral surface of a cylinder of the same radius and height.

Since the lateral surface of the cylinder is $\frac{22}{7}\times$diameter$\times$ height, and since the height of the cylinder is $2\times$radius of the sphere, therefore the area of the surface of a sphere

$$=\tfrac{22}{7}\times 2\times \text{radius}\times 2\times \text{radius}$$
$$=4\times\tfrac{22}{7}\times \text{square of radius}$$
$$=4\pi R^2$$

This formula may be stated in words as follows:

The surface of a sphere is equal to the square of the radius multiplied by 4π.

It can be shown by geometry that the volume of a sphere is equal to $\frac{4}{3}\times\frac{22}{7}\times$the cube of the radius.

The volume of a sphere is equal to the cube of the radius multiplied by $\frac{4}{3}\pi$.

Find the area of the surface and the volume of a ball 8 inches in diameter.

Solution

$4\pi R^2 = 4\times\frac{22}{7}\times 16 = \frac{1408}{7} = 201\frac{1}{7}$.

Therefore: The area $= 201\frac{1}{7}$ square inches.

$\frac{4}{3}\pi R^3 = \frac{4}{3}\times\frac{22}{7}\times 64 = \frac{5632}{21} = 268\frac{4}{21}$ cubic inches.

Therefore: The volume $= 268\frac{4}{21}$ cubic inches.

Exercise 85

1. Find the area of the surface and the volume of a sphere whose radius is 2 ft.

2. Find the area of the surface of a sphere whose radius is 3 ft.

3. Find the volume of a sphere whose radius is 12 in.

4. Find the surface of a tennis ball with a diameter of $2\frac{3}{8}$ in.

5. A ball 18 ft. 6 in. in diameter is to be gilded and placed on a tower. What is the area of the surface of the ball?

6. One sphere has a radius of 5 in., and another sphere has a radius of 10 in. Compare their surfaces and their volumes.

7. A cubic foot of a certain kind of marble weighs 2.7 times as much as a cubic foot of water. What is the weight of a marble sphere 4 ft. in diameter? (1 cu. ft. of water weighs 1000 oz.)

8. A cube has an edge 3 in. long. The largest possible sphere is cut from this cube. What part of the volume of the cube is the volume of the sphere?

9. An iron ball 5 in. in diameter is used on an engine. What is the weight of this ball, if the iron weighs 480 lb. per cu. ft.?

10. Measure the diameter or circumference of a baseball, and find how many square inches of leather are needed to cover it.

REVIEW: MENSURATION

1. A surveyor finds that a rectangular field is 2460 ft. long and 1680 ft. wide. How many acres does it contain?

2. If you know the area and the width of a rectangle, how can you find the length?

3. If you know the area and the length of the base of a triangle, how can you find the altitude?

4. A locomotive drive wheel is 6 ft. in diameter. How many times does it turn in going a mile?

5. The diameter of an automobile tire is 37 in. How far does it go in making one turn?

6. At $1.20 per cubic yard, what is the cost of making an excavation $48' \times 32' \times 7'$?

7. The water from an artesian well flows through a 2-inch pipe at the rate of 2 feet per second. How many barrels of water flow out in an hour? (Allow $31\frac{1}{2}$ gal. to a bbl.)

8. At $1.60 a square foot, what is the cost of gilding a sphere 4 ft. in diameter?

9. If cast iron weighs 450 pounds to the cubic foot, what will be the weight of a solid ball of cast iron 8 in. in diameter?

10. The water in a cylindrical cistern 6 ft. in diameter is 12 ft. deep. How many gallons of water are in the cistern?

11. Estimate the number of board feet of lumber needed, and the cost at present prices, to build a board fence 6 feet high around some vacant lot of which you know.

12. A rectangular bin 12 ft. by 9 ft. is filled with grain to a depth of 5 ft. How many bushels of grain are in the bin, allowing $1\frac{1}{4}$ cubic feet to the bushel?

13. How many rails will be required to lay 10 miles of track, and what will be the weight of the rails, if each rail is 33 ft. long and the average weight is 90 lb. to the yard?

14. The inside diameter of a cylindrical silo is 14 ft. How many square yards of concrete must be laid to make the floor?

15. A baseball diamond is 90 ft. square. What is the distance around the bases? What is the shortest distance from first base to third base?

16. A cylindrical gasoline tank is 16 ft. long and 7½ ft. in diameter. What is its capacity in gallons, allowing 7½ gallons to the cubic foot?

17. What will it cost to plaster a room 12 ft. 6 in. long, 10 ft. wide, and 8 ft. high above the base-board, at $.50 a square yard? The room contains three windows, each 2 ft. 6 in. by 6 ft., and two doors, each 3½ ft. by 7 ft. Deduct one-half for openings.

18. If you know the diameter of the base, and the slant height of a cone, how can you find its lateral area?

19. If you know the area of the base of a pyramid, and the altitude, how can you find the volume?

20. What must be the length of a guy wire to brace a pole 36 ft. high, if one end of the wire is attached 3 ft. from the top of the pole, and the other end is fastened to a stake 12 ft. from the foot of the pole? Allow 5 ft. for fastening the wire.

21. An acre contains 43,560 square feet. What is the diameter of a circle whose area is 1 acre? (Give answer to nearest 0.1 ft.)

22. What is the speed in inches per minute of a point on the rim of an emery wheel 15 inches in diameter which is turning at the rate of 1500 revolutions per minute?

23. A bin is 14 ft. long, 6 ft. wide, and 5 ft. deep. What is its capacity in bushels?

24. A cylindrical hot water tank is 24 in. in diameter and 36 in. high. What is its capacity in gallons?

25. The piston of a pump is 6 in. in diameter; it moves 18 in. at one stroke. If the piston makes 20 complete strokes in 1 minute, how many gallons will it pump in an hour?

RATIO AND PROPORTION

The Meaning of Ratio

The ratio of one number to another is the quotient obtained by dividing the first number by the second.

For example: The ratio of 5 to 10 is $5 \div 10$ or $\frac{1}{2}$. The ratio of 8 to 6 is $8 \div 6$ or $\frac{4}{3}$. The ratio of 13 to 17 is $13 \div 17$ or $\frac{13}{17}$.

The ratio of one amount to an amount expressed in terms of the same unit is the number of units in the first divided by the number of units in the second.

For example: The ratio of \$4 to \$8 is $\frac{4}{8}$ or $\frac{1}{2}$. The ratio of 3 ft. to 12 ft. is $\frac{3}{12}$ or $\frac{1}{4}$.

How to Write Ratios

You have learned that the ratio of 5 to 10 may be written as the fraction $\frac{5}{10}$. This ratio may also be written 5:10. We may also use the colon to express a ratio such as 3 yd.: 12 yd.

Since any ratio may be expressed as a fraction, it is evident that the principles that apply to fractions apply to ratios.

The two numbers compared are called the *terms* of the ratio.

For example: 4 is the first term of the ratio $\frac{4}{8}$, and 8 is the second term.

Exercise 86

1. What is the ratio of 5 to 10? Of 4 to 12? Of 3 to 18? Of 12 to 9? Of 75 to 30? Of 12 to 8?

2. What is the ratio of $\frac{1}{2}$ to 2? Of 5 to 15? Of 15 to 5? Of $\frac{1}{3}$ to 1? Of 18 to 9? Of 25 to 20?

3. What is the ratio of 0.5 to 0.05? Of $4\frac{1}{2}$ to 9? Of .03 to .003? Of 2.5 to .5? Of $2\frac{1}{2}$ to 10?

4. What is the ratio of 8 lb. to 12 lb.? Of 6 oz. to 1 lb.? Of 8 qt. to 1 gal.? Of 8 hr. to 1 day?

5. What is the ratio of each number in row A to the number under it in row B?

(*A*)	7	$3\frac{1}{2}$	14	$\frac{1}{2}$	12	0.4	$\frac{1}{4}$	$\frac{1}{2}$
(*B*)	14	7	21	3	18	4	$\frac{3}{4}$	$\frac{1}{3}$

6. What is the ratio of each number in row *B* in *Example 5* to the number above it in row *A?*

7. State three other ratios that have the same value as the ratio of 4 to 8.

8. State three other ratios, each of which has the same value as the ratio of 4 to 5.

9. In the following problems the unknown number is represented by n. Find the value of n.

	a	*b*	*c*
1.	$\frac{n}{2}=6$	$\frac{n}{6}=\frac{1}{2}$	$\frac{6}{n}=\frac{1}{2}$
2.	$\frac{n}{4}=10$	$\frac{n}{3}=\frac{1}{4}$	$\frac{4}{n}=\frac{1}{3}$
3.	$\frac{n}{3}=2$	$\frac{10}{n}=5$	$\frac{20}{n}=4$

10. What is the ratio of the number of boys in your class to the number of girls? What is the ratio of each group to the total number of pupils?

11. What is the ratio of the width of your desk to its length?

12. What is the ratio of one side of a square to the perimeter?

13. What is the ratio of one side of an equilateral triangle to the perimeter?

14. What is the ratio of the circumference of a circle to the diameter?

15. One square has a side of 3 in., and a second square has a side of 9 in. What is the ratio of the area of the first square to that of the second?

How to Divide a Number Into Parts That Have a Given Ratio

If you divide 42 into two portions that have the ratio of 2 to 4, it is plain that, since there are to be 2 parts in one portion and 4 parts in the other, there are to be 6 parts in all. In one portion there are to be 2 of the 6 parts, that is, $\frac{2}{6}$ of 42, or 14. In the other portion there are to be 4 of the 6 parts, that is $\frac{4}{6}$ of 42, or 28.

Exercise 87

1. A woman's club voted to distribute $150 in prizes between two rural schools in the ratio of the number of pupils enrolled. The first school had an enrollment of 28 pupils, the second had an enrollment of 22. How much should each school receive?

2. George was to receive $10 for spading a garden for Mr. Smith and one for Mr. Carlson. Mr. Smith's garden was three times as large as Mr. Carlson's. What amount of the total should each pay?

3. Sam and Harry have a load of 78 lb. to carry to a boy scout camp. They agree to divide the load in the ratio of their ages. How much should each carry if Sam's age is 14 years, and George's is 12 years?

4. In making a certain kind of solder, 2 parts of tin are used to 3 parts of lead. How many pounds of each metal are used to make 12 pounds of such solder?

5. Two farmers shipped potatoes to market. The first shipped 14,850 lb., the second shipped 12,850 lb. The total freight bill was $46. How much of this bill should each farmer pay?

6. A club voted to divide $5 between two eighth-grade classes, in the ratio of the number of tickets each class should sell for an entertainment. One class sold 36 tickets, the other 24. How much should each class receive?

7. A man promised to divide $20 between his two children in the ratio of the amounts which they saved. One child saved $32; the other, $48. How much of the $20 should each receive?

Proportion

The ratio $\frac{3}{6}$ equals $\frac{1}{2}$; the ratio $\frac{4}{8}$ also equals $\frac{1}{2}$. Since each of these ratios is equal to $\frac{1}{2}$, the ratios must be equal to each other. We have, therefore, $\frac{3}{6}=\frac{4}{8}$. This statement means that the two ratios are equal.

A proportion is an expression of equality between two ratios.

For example: $\frac{2}{6}=\frac{5}{15}$ $\frac{4}{20}=\frac{3}{15}$ $\frac{6}{7}=\frac{42}{49}$

The above proportions may also be written in this way:

$2:6=5:15$ $4:20=3:15$ $6:7=42:49$

The first of these proportions is read: *2 is to 6 as 5 is to 15.* Read the other two.

The first and last terms of a proportion are called the *extremes;* the second and third terms are called the *means.*

Find the product of the means and the product of the extremes in each of the three proportions above. How do the products compare in each case? Your solution of these problems shows the truth of this principle:

In any proportion, the product of the *means* equals the product of the *extremes.*

Three terms are usually given in a problem that involves proportion. It is then necessary to find the fourth term.

For example: In the proportion $\frac{x}{3}=\frac{4}{6}$, x represents the unknown term. From the principle given above it follows that $6x=12$, and therefore $x=2$.

In the problem $\frac{5}{x}=\frac{3}{12}$, $3x=60$, and therefore $x=20$.

If one of the extremes in a proportion is not known, it can be found by dividing the product of the means by the extreme that is known. If one of the means is not known, it can be found by dividing the product of the extremes by the mean that is known.

For example: If $\frac{x}{7}=\frac{8}{28}$, find x.

$x=\frac{7\times 8}{28}$ Canceling, we have $x=2$. To check, see if the product of the means is equal to the product of the extremes.

If $\frac{9}{8}=\frac{18}{x}$, $x=\frac{8\times 18}{9}$ $x=16$ Check this.

If $\frac{5}{x}=\frac{7}{4}$, $x=\frac{5\times 4}{7}$ $x=2\frac{6}{7}$ Check this.

Exercise 88

1. Find the missing term in each of the following proportions. Check each example.

	a	*b*	*c*	*d*
1.	$\frac{x}{4}=\frac{5}{10}$	$\frac{4}{5}=\frac{x}{20}$	$\frac{x}{8}=\frac{7}{2}$	$5:7=3:x$
2.	$\frac{x}{3}=\frac{6}{9}$	$\frac{9}{4}=\frac{x}{5}$	$\frac{5}{6}=\frac{3}{x}$	$x:15.7=0.4:4$
3.	$\frac{7}{x}=\frac{4}{12}$	$\frac{x}{6}=\frac{2}{3}$	$\frac{7}{x}=\frac{5}{4}$	$15:13=x:65$
4.	$\frac{3}{7}=\frac{4}{x}$	$\frac{3}{25}=\frac{x}{100}$	$\frac{9}{13}=\frac{2}{x}$	$3.7:x=5.6:22.4$

2. I am thinking of a number which has to 6 the ratio 4:8. What is the number?

3. What number has to 12 the ratio 4:6?

4. When a tree 32 ft. high casts a shadow 14 ft. long, how long is the shadow cast by a nearby tree 64 ft. high?

5. How many gallons of gasoline would an automobile use on a trip of 169 mi., if it uses 7 gallons on a trip of 91 mi.? $\left(\frac{x}{7}=\frac{169}{91}\right)$.

6. A picture 4″ by 9″ is enlarged so that the length of the enlarged picture is 12″. What is its width?

7. If $\frac{3}{4}$ of an inch on a certain map represents 18 mi., what distance does $2\frac{1}{2}$ in. represent on the same map?

Similar Figures

Figures that have exactly the same shape are said to be *similar.* Fig. 85 shows three pairs of similar figures.

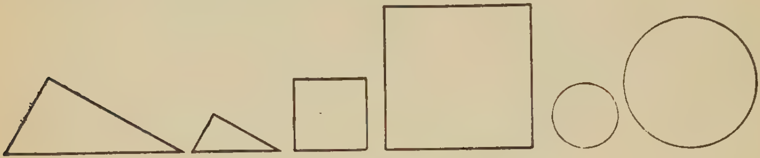


Fig. 85

A familiar example of similar figures is a map, which is always similar in shape to the state itself.

In similar figures lines that are similarly placed are called *corresponding lines.* The bases of two similar rectangles are corresponding lines. The diameters of two circles are corresponding lines.

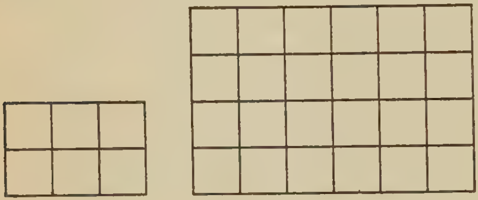


Fig. 86

In similar figures the ratio of any pair of corresponding lines is equal to the ratio of any other pair of corresponding lines. This is sometimes briefly stated as follows.

In similar figures corresponding lines are proportional.

For example: In the similar rectangles shown in Fig. 86, it is evident that the $\frac{\text{length of the smaller rectangle}}{\text{length of the larger rectangle}} = \frac{\text{width of the smaller rectangle}}{\text{width of the larger rectangle}}$.

A rectangle is 3 inches long and 4 inches wide. A similar rectangle is 6 inches long. How wide is it?

Using the equality of ratios as stated above, we have $\frac{3}{6}=\frac{4}{x}$, $3x=24$, $x=8$. Therefore the second rectangle is 8 inches wide.

Similar triangles are frequently used in finding heights and distances which cannot conveniently be measured directly.

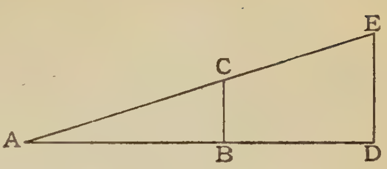


Fig. 87

For example: In Fig. 87, triangle ABC is similar to triangle ADE. Therefore

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

Draw two similar triangles in which the sides of one are twice as long as the sides of the other. State the equal ratios as above.

The heights of such objects as trees, flagstaffs, and buildings may be measured by the following method:

A boy scout made a right triangle with side AB equal to BC. He holds AB level, and walks towards a tree until he just sees the top along AC. Then since $AB = BC$, and since $\frac{AB}{BC} = \frac{AD}{DE}$, the height of the tree above D will equal AD. The boy then measures the distance AD, and the distance from D to the ground.

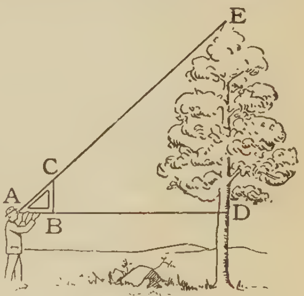


Fig. 88

The method of finding distances without actually measuring them, which is extensively used by geographers, engineers, surveyors, and astronomers, is called *indirect measurement*. By using extremely accurate instruments for making their observations, then computing the distances by methods like that we have explained, it is possible to measure the heights of mountains which men cannot climb, the distances of the planets, and the areas of large tracts of land that have not been explored.

Similar figures are also extensively used in many other practical ways. Maps, plans and drawings of architects, engineers, and mechanics, enlarged photographs, and patterns for garment making are all practical applications of the principles of similar figures.

Exercise 89

Estimate the heights and distances before making the measurements called for in *Problems 5* and *6*.

1. What is the height of the tree shown in Fig. 88, if AD is 34 ft. 6 in., and the triangle is 5 ft. from the ground?

2. Make a right triangle of wood or cardboard, and measure the height of several objects by the method explained above.

3. A boy whose eye is 5 ft. above the ground sights over an 8 ft. pole and just sees the top of a tree. The boy is 6 ft. from the pole, and 38 ft. from the tree. Draw an outline similar to Fig. 88 and find the height of the tree.

4. A tree casts a shadow 84 ft. long at the same time that a nearby post 6 ft. high casts a shadow 8 ft. long. What is the height of the tree?

5. By using the method of *Problem 4*, find the height of several objects near your school.

6. By measurement of their shadows, find the height of a water tower, flag pole, or building in your vicinity.

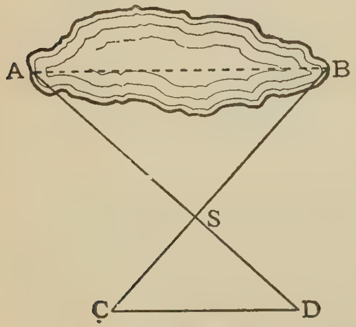


Fig. 89

7. Some boy scouts wished to find the length of a lake. They placed a stake at S as shown in Fig. 89. The line BS was then continued to C so that CS was 180 ft. long. BS was measured, and found to be 360 ft. long. By sighting from A, the angle BAS was marked off on a piece of cardboard, and the angle SDC was made equal to it. The point D was located by continuing the line AS until it met the line from C. The two triangles are similar. By measurement, the line CD was found to be 160 ft. long. What is the length of the lake?

8. Find the length or width of a real or imaginary lake near your school.

9. Some pupils who wished to find the width of a river made the following measurements. A point *C* was so chosen that the line *BC* made a right angle with the line *AB*. *BC* was then continued to a point *D*. A line was then drawn from *D* at right angles to *DC*. The pupils then sighted from *C* to *A*, and continued the line through *C* to cut the line drawn from *D* at *E*. By measurement, *DC* was found to be 180 ft. long, *CB* 220 ft. long, and *DE* 140 ft. long. The triangles are similar. What is the width of the river?

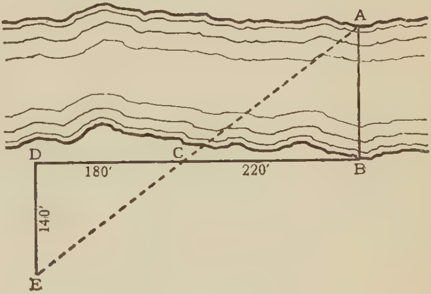


Fig. 90

10. A boy tied his kite string to a peg in the ground at *B*, and held an 8-foot pole upright at *C* where its top just touched the string, which was stretched tight. By measuring, he found the line *BC* to be 16 ft. long. The length of the kite string was 460 ft. How high was the kite?

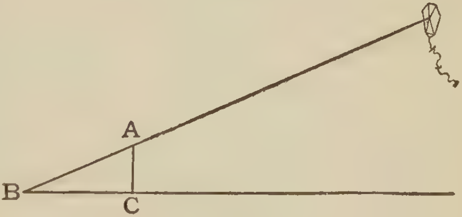


Fig. 91

11. If 1.5 inches on a map represents a distance of 150 miles, how many inches on this map represents a distance of 360 miles?

12. If $2\frac{1}{2}$ inches on a map represents a distance of 450 miles, how many miles will $1\frac{7}{8}$ inches represent?

13. Draw a plan of the floor of your schoolroom, representing 1 foot by $\frac{1}{2}$ inch.

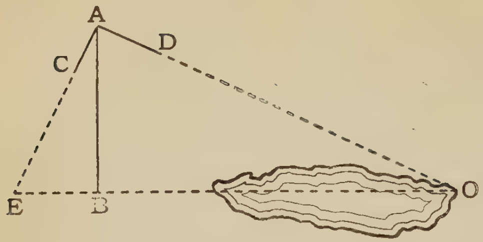


Fig. 92

14. Some boy scouts learned that a device like the one shown in Fig. 92 can be used to find the distance to an inaccessible point. *AB* is a pole of known length. At one end of the pole is a moveable frame *DAC* with sides *AC* and *AD* forming a right angle. To find the distance from *B* to an object at *O*, place the pole so that *AD* points to the object, and then note where the line *AC* extended meets the ground. Call this point *E*. The distance *EB* can be measured. The length of *AB* is known, and the triangles *ABE* and *ABO* are similar. What proportion may be used in finding *BO?* Find *BO* when *AB* is 5 ft. and *BE* is 3 ft.

15. After reading in a geometry that a line parallel to one side of a triangle divides the other two sides proportionally, Fred and Henry worked out the following method for finding the distance across a lake. They measured from *B* to *C* in line with *AB* extended. They drew a line from *C* at right angles to *BC*, and then a line from *A* cutting the line from *C* at *D*. At *B*, they drew a line at right angles to *BC*, meeting *AD* at *E*. This made the line *BE* parallel to *CD*. Therefore $\frac{AB}{BC}=\frac{AE}{ED}$.

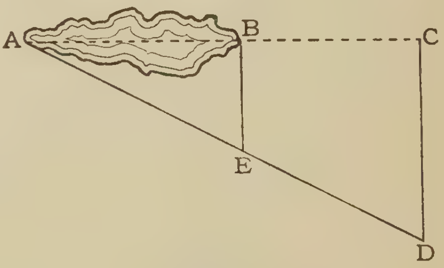


Fig. 93

Find *AB* when *ED* is 230 ft., *AE* is 210 ft., and *BC* is 190 ft.

16. Use the method explained in *Problems 14* and *15* to measure the distance of some inaccessible object.

To Represent the Relationships of Numbers by Means of Graphs

You have learned how to represent your progress in drill exercises by means of a broken line called a *graph*. This method of representing number facts or number relationships, and many other similar methods, are now extensively used by those who wish to present such facts in a simple, clear, and convincing manner. Most official reports, and many books, magazines, newspaper articles, and advertisements contain graphs of various kinds. Some of the more common forms are explained on this and the following pages.

In order to make a graph we must have a series of number facts which can be compared. We may show the growth in population of a state or city, the increase or decrease in crop production over a period of years, the changes in price of a given article, or something else that offers numbers for comparison. In making graphs, *round numbers* are commonly used.

The Broken-Line Graph

Fig. 94 shows a *broken-line* graph similar to those you have used to show your progress in arithmetic in earlier exercises. This graph shows the temperature in Chicago at noon on seven consecutive days.

How much does each vertical division on this graph represent? On what day was the temperature highest? On what day was it lowest?

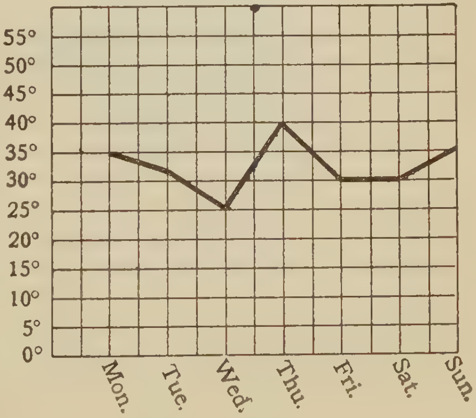


Fig. 94

Exercise 90

Use a broken-line graph to express the number facts in these problems.

1. John's record in spelling tests for the week was as follows: Monday, 80%, Tuesday, 90%, Wednesday, 85%, Thursday, 95%, Friday, 90%.

2. Beginning with September, Helen's monthly averages in arithmetic were as follows: 85%, 90%, 90%, 80%, 95%, 90%, 85%, 90%, 85%.

3. The average monthly rainfall in a city was as follows: January, 3 in., February, 3.5 in., March, 4 in., April, 3.8 in., May, 2.5 in., June, 2.2 in., July, 1 in., August, 0.5 in., September, 1.5 in., October, 2.5 in., November, 2.5 in., December, 2.8 in.

4. Graph the monthly attendance record for your class or school.

5. Graph the temperature for a certain hour daily for a week.

6. Graph your daily, weekly, or monthly averages in some subject.

7. Show by a graph the growth of population in your state. Use the population figures given by the census for each of the following years: 1880, 1890, 1900, 1910, and 1920.

8. If you can obtain the correct facts, graph the total of the two principal crops raised in your state during each of the last five years.

9. Select some common article of food, such as eggs, potatoes, or butter, and show by a graph the daily variation in the retail price over a period of two weeks.

10. Graph the weights and the heights of those in your class. Use two different graphs to show these number facts.

NOTE TO PUPIL: Graphs that consist of broken lines or of bars may be most conveniently made on *cross-ruled* or *squared* paper, which may be obtained from booksellers and school supply houses. Squared paper is commonly ruled in tenths of an inch, although other rulings may be obtained.

The Bar Graph

The *bar graph*, one of the most common forms of graphic representation, is extensively used because it is easy to construct and to interpret. Since the graph represents the size of a number by the length of a line, only one dimension is to be considered.

The bar graph in Fig. 95 shows the population of the United States in each census year from 1880 to 1920 inclusive, and so enables one to note at a glance the increase in population which took place during the period indicated. Each horizontal division on the squared paper represents 5 million. As is usual, round numbers are used.

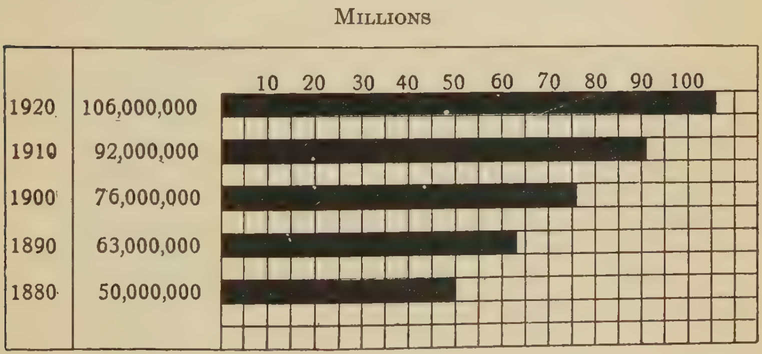


Fig. 95

As shown by the graph, what was the population for each census year? What does the graph tell about the growth of population from 1880 to 1920? How does each ten years' increase compare with the increase of the preceding ten years? What other facts about the growth of our population does this graph show?

Before constructing a bar graph, ask yourself the following questions.

1. What is the largest number to be represented?
2. How many spaces are available to represent this number?
3. What value can each space most conveniently represent?

Often a bar graph is constructed on a single bar, as shown by Fig. 96. In such cases, each division of the graph is either given a different color, or is cross-hatched, as shown here. Fig. 96 shows the sales made during one month by each department of a large department store; the accompanying graph expresses the number facts on a single bar.

Department	Sales	Per Cent of Total
A	$45,000	22.5
B	22,500	11.25
C	25,000	12.5
D	100,000	50
E	7,500	3.75
Total	200,000	100

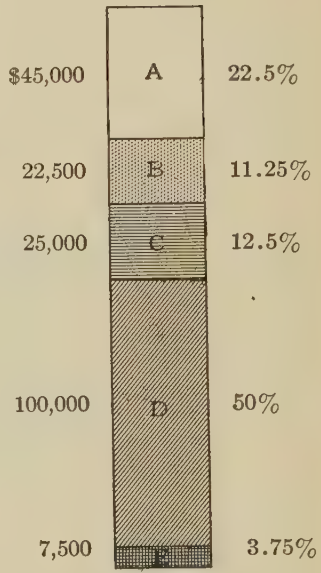


Fig. 96

Exercise 91

Use the bar graph to represent the following facts.

1. During one year, six of the world's leading crops were as follows: Corn, 3,710,000,000 bu.; wheat, 3,035,000,000 bu.; oats, 2,955,000,000 bu.; barley, 969,000,000 bu.; rye, 783,000,000 bu.; potatoes, 3,303,000,000 bu. (The yield of each crop is stated to the nearest million bushels.)

2. The number of bales of cotton produced in the United States during three years was as follows: 11,421,000, 13,440,000, and 8,340,000. Graph, expressing each to the nearest half-million.

3. The heights of several of the highest buildings in New York City are as follows: Woolworth, 792 ft., Metropolitan Life, 700 ft., Singer, 612 ft., Bankers' Trust, 539 ft., New York World, 335 ft.

4. The areas of the continents, to the nearest hundred thousand square miles, are as follows: Africa, 11,600,000, North America, 8,600,000, South America, 7,600,000, Asia, 17,200,000, Australasia, 3,300,000.

5. The total land and water area of the United States, including possessions, expressed to the nearest thousand square miles, is indicated below:

Year	Area	Year	Area
1790	892,000	1870	3,618,000
1810	1,720,000	1890	3,618,000
1830	1,792,000	1910	3,743,000
1850	2,997,000	1920	3,744,000

6. The total land and water areas of several states are given below. Include your state if it is not in the list. Follow the plan used in *Problem 5* when making your graph.

State	Area	State	Area
Texas	265,896	Missouri	69,420
California	158,297	Georgia	59,265
Montana	146,997	Illinois	56,665
New Mexico	122,634	Alabama	51,998
Colorado	103,948	New York	49,204
Minnesota	84,682	Louisiana	48,506
Kansas	82,158	Massachusetts	8,266
South Dakota	77,615	Rhode Island	1,248

7. When next you see an illustration in which a bar graph is used, bring it to class.

8. Does the bar graph shown in Fig. 95 seem to have a better form than that shown in Fig. 96? Explain your answer.

The Circle Graph

The *circle graph*, like the more common single-bar graph (Fig. 96), is used to show the relationship of parts to a whole. Each part is represented by a sector of the circle. Sometimes different colors or hatchings are applied to the sectors to make the contrasts among them more striking.

The circle graph is not easily constructed, because it is difficult accurately to measure or estimate the size of the angle or the length of the arc of the sectors. When well made, however, this graph is an excellent method of showing relationships.

In your geography, science book, or a magazine, find examples of the circle graph.

The circle graph shown in Fig. 97, which shows the amount of cotton produced in one year by the leading cotton-producing countries of the world, was constructed as follows:

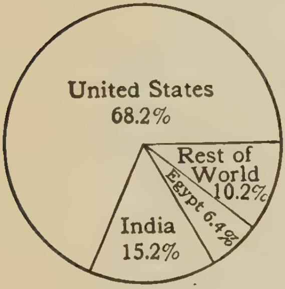


Fig. 97

The total amount of cotton produced was 19,595,000 bales. Of this amount the United States produced 13,366,000 bales, India, 2,976,000 bales, Egypt, 1,251,000 bales, and the rest of the world, 2,002,000 bales. Of the total, the United States produced 68.2%, India, 15.2%, Egypt, 6.4% and the rest of the world, 10.2%.

Since there are 360 degrees in a circle, we lay off 68.2% of 360 degrees, or 245.5 degrees, to represent the amount produced by the United States. In like manner we lay off 15.2% of 360 degrees, or 54.7 degrees for India, 6.4% of 360 degrees, or 23 degrees for Egypt, and 10.2% of 360 degrees, or 36.7 degrees for the rest of the world.

Having indicated the respective points on the circle by means of a protractor, draw a radius from each point to the center of the circle. The resulting sectors will indicate graphically the number facts under consideration, as shown by Fig. 97.

Exercise 92

1. In a certain elementary school, 10% of the pupils are in the first grade, 16% in the second grade, 17% in the third grade, 15% in the fourth grade, 14% in the fifth grade, 11% in the sixth grade, 10% in the seventh grade, and 7% in the eighth grade. Represent these facts by a circle graph.

2. During a recent year the immigrants who entered the United States, and the emigrants who left the United States, were classified as follows. Show these facts by means of circle graphs.

IMMIGRANTS	
No occupations	42.3%
Miscellaneous occupations	21.
Skilled laborers	16.7
Laborers	10.6
Farm laborers	5.9
Professional workers	3.5

EMIGRANTS	
Laborers	50.3%
No occupations	27.7
Skilled laborers	9.
Miscellaneous occupations	7.4
Farm laborers	3.9
Professional workers	1.7

3. The Eclipse Manufacturing Company uses 60% of its income for wages, 9% for office expenses, 7% for advertising, 10% for selling expenses, 6% for dividends, 2% for interest on bonds, and the remainder for miscellaneous expenses. Represent these facts by a circle graph and by a bar graph.

4. Obtain necessary number facts from your geography, and show the following by means of a circle graph or a bar graph. Each graph should clearly show the relationship of the facts. Place the facts where they can conveniently be read.

1. The area of the continents and the part which each is of the total area.

2. The area of each of the groups of states of the United States and the part which the area of each group is of the total area.

3. The population of each of the groups of states of the United States, and the part which the population of each group is of the total population of the country.

4. Select other facts from your geography and represent them by means of graphs.

The Pictorial Graph

The pictorial graph uses pictures of the objects to be compared. The respective pictures vary in size as the number facts vary. Although such graphs are difficult to make, they are very effective for strikingly presenting certain facts. The value of pictorial graphs is greatly increased when the facts on which they are based accompany the pictures.

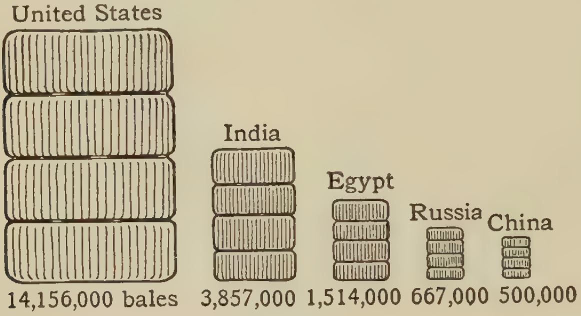


Fig. 98

Fig. 98 is a pictorial graph showing the relative amounts of cotton produced by the United States, India, Egypt, Russia, and China during a recent year.

Suggest how you might use a pictorial graph to represent the number of automobiles used in two states.

Suggest a way to use the pictorial graph for representing the number of pupils in each class in your school building.

Suggest a pictorial graph to represent the following:

1. The sizes of several ocean vessels.
2. The corn crops of two or more states.
3. The number of fires in a city during each of two years.

Find in magazines and newspapers other examples of the pictorial graph. Whatever graphs you find should be brought to school and interpreted to your classmates.

THE EQUATION AND ITS USE

All the formulas that you have studied and used are equations. As you have learned, equations are extensively used in solving problems in mathematics. You will remember that an *equation* is a mathematical statement that two expressions are equal.

For example: $A = bh$ $4n = 12$

If you are told that when 6 is added to a certain number the sum is 14, you may represent the unknown number by n. The relation stated by the problem may then be expressed as $n + 6 = 14$. This is an equation. Since 8 is the number that will produce 14 when 6 is added, then n must represent 8 in this equation.

The expression on each side of the sign of equality (=) is called a *member* of the equation.

For example: In the equation mentioned above, $n + 6$ is the first member, 14 is the second member.

How to Solve an Equation by Addition or Subtraction

You may illustrate the equality between two members of an equation by means of a pair of balances, as shown by Fig. 99. If the weight in either scalepan is changed after the two sides are balanced, you must make a corresponding change in the weight in the other scalepan if the sides are again to balance. That is, if 2 pounds are taken from or added to one scalepan, exactly the same amount must be taken from or added to the other in order to bring the scales into balance again.

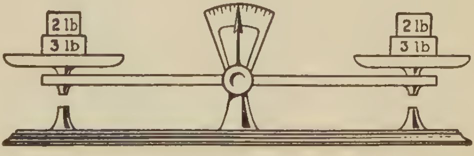


Fig. 99

To solve an equation you must find the value of the unknown number. The unknown number may be represented by n, or x, or by any other letter.

For example: To solve the equation $n - 5 = 14$, you must find the number that gives 14 when 5 is taken from it. In this case you may think of the first member as representing the weight in one scalepan of a balance, and of the second member as representing the weight in the other scalepan. Since the scales remain in balance if the same amount is added to each scalepan, so this equation will remain an equality if the same amount is added to each member.

If we add 5 to the first member of the equation, we shall have $n-5+5$. The 5's may then be canceled, leaving only n in the first member. However, since 5 has been added to the first member, 5 must also be added to the second member in order to keep the equality. The second member then becomes $14+5$. The equation then becomes $n-5+5=14+5$. Simplifying this by canceling the $+5$ and the -5, the equation reads $n=19$.

The principle illustrated by the above example may be stated as follows:

If equals are added to equals the sums are equal.

If you have to solve the equation $n+8=12$, you may think of the first member as representing the weight in one scalepan of a balance, and you may think of the second member as representing the weight in the other scalepan. Since the scales remain in balance if the same amount is removed from each scalepan, so this equation will remain an equality if the same number is subtracted from each member. By subtracting 8 from each member of this equation, it reads $n+8-8=12-8$. If, now, you take 8 from 8 in the first member, you have only n left. Similarly, if you take 8 from 12 in the second member, you have 4 left. The equation has now become $n=4$.

The principle illustrated by the above example may be stated as follows:

If equals are subtracted from equals the results are equal.

You should check the solution of an equation by substituting for the letter the value found.

For example: $n+4=10$. In this equation the value found for n is 6. Substitute 6 for n and the equation then reads $6+4=10$. Since the two members are now equal, the substitution shows that the solution checks.

What must be subtracted from both members of the equation $n+7=16$ so that only n will remain in the first member? What is the value of n in this equation? Check your solution.

Exercise 93

1. If 4 is added to a certain number, the sum is 8. What is the number?

2. If 6 is added to a certain number, the sum is 17. What is the number?

3. Find the value of n in each of the following equations.

	a	*b*	*c*	*d*
1.	$n+3=10$	$n-2=6$	$n+8=15$	$n-5=3$
2.	$n-4=5$	$7+n=12$	$n+4=20$	$n-3=12$
3.	$n+5=12$	$n-8=14$	$n-6=30$	$n+3=7$
4.	$n-4\frac{1}{2}=1\frac{1}{2}$	$n+9=18$	$n+14=20$	$n+2\frac{1}{2}=7\frac{1}{2}$
5.	$n-14.5=1.5$	$n+7\frac{2}{3}=8\frac{2}{3}$	$n-1\frac{1}{2}=3$	$n-4\frac{3}{4}=1\frac{1}{4}$
6.	$n+8.2=10.4$	$n+2\frac{5}{6}=4$	$3\frac{1}{2}+n=8$	$n+11=32$

How to Solve an Equation by Division or Multiplication

When solving problems you will often get such an equation as $5x=15$. If this occurs, you may simplify the equation by dividing both members as shown below.

For example: $\frac{5x}{5}=\frac{15}{5}$, or $x=3$. Similarly, if $4x=24$, then $\frac{4x}{4}=\frac{24}{4}$, or $x=6$.

The principle illustrated may be stated as follows:

If equals are divided by equals the quotients are equal.

The solution of problems often produces such an equation as $\frac{n}{5}=4$. This is the same as saying, "One-fifth of what number is 4?" In this case the answer is 20. In such cases the answer may be obtained by multiplying each member of the equation by the same number, as shown below.

For example: $\frac{n}{5}=4$. Multiplying both members by 5, you get $\cancel{5}\times\frac{n}{\cancel{5}}=5\times4$. Multiplying as indicated, the result is $n=5\times4$, or $n=20$.

The principle may be stated as follows:

If equals are multiplied by equals, the results are equal.

When it is necessary to solve such an equation as $5x+4=14$, first subtract 4 from each member of the equation. The resulting equation is then $5x=10$. This may be solved by dividing both members by 5 as explained on page 253.

When it is necessary to solve such an equation as $\frac{n}{2}+3=8$, first subtract 3 from each member of the equation. The resulting equation is then $\frac{n}{2}=5$. This may then be solved by multiplying both members by 2, as explained on page 253.

Exercise 94

Solve the following equations and check each.

	a	b	c
1.	$4x=20$	$6x=48$	$4n=32$
2.	$5x=35$	$5x=450$	$8a=24$
3.	$7x=42$	$9x=180$	$3t=18$
4.	$3x=8$	$2x=17$	$7r=21$
5.	$9x=14$	$8x=50$	$6m=10$
6.	$5x+9=29$	$4n-3=21$	$3x+2=7$
7.	$3x+5=20$	$5n-8=32$	$5t-4=28$
8.	$7x+6=55$	$6a+11=47$	$7n-5=20$
9.	$\frac{n}{3}=4$	$\frac{x}{5}=6$	$\frac{x}{4}=3.5$
10.	$\frac{n}{2}=5$	$\frac{a}{10}=3$	$\frac{n}{5}=4.5$
11.	$\frac{x}{4}=6$	$\frac{n}{2}=\frac{2}{5}$	$\frac{b}{6}=3.4$
12.	$\frac{a}{7}=2$	$\frac{a}{6}=\frac{2}{3}$	$\frac{a}{5}=4.2$
13.	$\frac{n}{4}+2=7$	$\frac{b}{3}-4=10$	$\frac{4}{5}n+6=8$
14.	$\frac{a}{6}+5=8$	$\frac{x}{6}+5=9$	$2n-3.5=4.5$
15.	$\frac{a}{4}-2=9$	$\frac{x}{4}-2=2$	$\frac{8x}{3}-2=6$
16.	$\frac{n}{4}-\frac{2}{3}=1\frac{1}{3}$	$4x+7=22$	$\frac{1}{2}x=4\frac{1}{2}$
17.	$\frac{a}{4}+7=21$	$\frac{3}{4}x=6$	$4n-8=\frac{2}{3}$
18.	$6r-8=30$	$5n-2=8$	$x-3\frac{1}{2}=6$
19.	$\frac{x}{5}=\frac{4}{5}$	$\frac{x}{6}=15$	$5t+6=16$

The Formula as an Equation

You will find that your knowledge of the equation will be of particular value to you when you are working with formulas. As you know, the formula for the area of a rectangle is $A = lw$. If you divide each member of this equation by l, you get $\frac{A}{l} = w$. That is, from the formula $A = lw$ you get a formula for the width of the rectangle in terms of its area and length.

For example: If the area of a rectangle is 480 square feet, and its length is 40 feet, the equation $\frac{A}{l} = w$ will read $\frac{480}{40} = w$. Solving this equation, you find that $w = 12$. The width is, therefore, 12 feet.

From the formula $A = lw$ make a formula for l.

The formula for the area of a triangle is $A = \frac{bh}{2}$. If you multiply both members by 2, you get $2A = bh$. If you then divide both members by b, you get $\frac{2A}{b} = h$. You now have a formula for finding the height of a triangle in terms of the area and the base.

From the formula $A = \frac{bh}{2}$ make a formula for finding b.

Exercise 95

1. The formula for finding the volume of a rectangular solid is $V = lwh$, in which V represents the volume, l the length, w the width, and h the height. Using this formula, make a formula for finding l. Make a formula for finding w. Make a formula for finding h.

2. From the formula $i = prt$, make a formula for finding r. State the formula in words.

3. Using the formula stated in *Problem 2*, find r when $i = 360$, $p = 3000$, and $t = 2$.

4. In the equation $S = \frac{a+b+c}{2}$, what is the value of S when $a = 12$, $b = 13$, and $c = 15$?

5. In the formula $S = 16t^2$, find S when $t = 4$. Find S when $t = 6$.

6. In the formula $A=\frac{h}{2}(a+b)$, what is the value of A, when $h=12$, $a=20$, $b=18$?

7. If $c=\frac{22d}{7}$, make a formula for d.

8. If $a=\frac{22r^2}{7}$, make a formula for r.

Problems to be Solved by Using Equations

Solve each of the following problems and check.

1. Four times a certain number is 36. What is the number? Represent the number by a letter, then state the equation, solve, and check.

2. Five times Henry's age is 55 years. How old is Henry?

3. The length of a rectangle diminished by 5 in. is 16 in. What is the length?

4. Clara's weight increased by 6 lb. is 94 lb. What is her weight?

5. Henry sold 5 more tickets to a school entertainment than George. Together they sold 37. How many did each sell? Let n represent the number George sold. What will then represent the number Henry sold?

6. In a class of 36 pupils, there are 4 more girls than boys. How many of each are there? If n represents the number of boys, what will represent the number of girls?

7. A rectangular lot is 50 ft. longer than it is wide. The distance around it is 300 ft. Find its length and width.

8. The sum of two consecutive numbers is 41. Find the numbers. If n represents the smaller, how will you represent the larger?

9. Give the class a problem like the one stated in *Problem 8*.

10. Mr. Arnold's house cost five times as much as his lot. If the total cost of both was $10,000, what was the cost of each?

11. In a class of 36 pupils there are $\frac{4}{5}$ as many boys as girls. How many are there of each?

12. Sam is 2 years older than Henry. The sum of their ages is 24 years. How old is Henry?

13. Ethel paid \$2.50 for an arithmetic and a geography. The geography cost \$.60 more than the arithmetic. How much did each cost?

14. John said: "I am thinking of a certain number, which, when multiplied by 2 and then increased by 14, equals 38." Of what number was he thinking?

15. The perimeter of a rectangular field 60 rd. wide is 264 rd. What is the length of the field?

16. Louise and Marian together sold 192 Red Cross seals. Louise sold twice as many as Marian. How many did each sell?

17. Using the formula $s = 16t^2$, find s when $t = 5$.

18. I am thinking of a number, which, when multiplied by 6, is equal to 4 more than 68. Of what number am I thinking?

19. The perimeter of a rectangle is 200 ft., and the length is 20 ft. more than the width. Find the length, the width, and the area.

20. George paid \$4.50 for a tennis racket and two balls. The racket cost \$2.50 more than the two balls. How much did the racket cost?

21. Represent the area of a rectangle whose base is 14 inches and whose altitude is h inches.

22. Represent the distance that an automobile will travel in h hours at the rate of x miles an hour.

23. Fred gave this puzzle problem to a class, and Helen and George got the answer in less than two minutes. Can you do so? "I am thinking of a number, which, if multiplied by 6 and then increased by 4, is equal to 52. What is the number?"

THE METRIC SYSTEM

The *metric system* of weights and measures is a decimal system in general use in nearly every civilized country of the world, except the United States and the possessions of the British Empire. Even in those countries the system is now generally used for all kinds of scientific measurements.

All units of the metric system are based on the *meter,* a unit of length that equals 39.37 inches. Since the metric system is based on the decimal scale, the only multiplier or divisor used for changing from one unit to another is 10 or a power of 10. As in the case of United States money, which is also based on the decimal scale, reductions are made by merely changing the position of the decimal point.

The unit of capacity in the metric system is the *liter* (pronounced *lee'ter*), whose volume equals that of a cube having an edge 0.1 meter in length. The unit of weight is the *gram.* It equals the weight of the water contained in a cube having an edge .01 meter in length.

Multiples of the metric units are expressed by means of the Greek prefixes, *deka-* meaning 10, *hekto-* meaning 100, *kilo-* meaning 1000, and *myria-* meaning 10,000.

For example: 1 *dekameter* is equal to 10 meters. 1 *hektometer* is equal to 100 meters. 1 *kilometer* is equal to 1000 meters. 1 *myriameter* is equal to 10,000 meters

From this it is plain that 10 dekameters equal 1 hektometer, 10 hektometers equal 1 kilometer, 10 kilometers equal 1 myriameter.

These prefixes are also attached to the names of the other units, *gram* and *liter,* in similar manner and with like meaning. Thus, one speaks of the *dekagram, hektogram,* and *kilogram,* and of the *dekaliter, hektoliter,* and *kiloliter.*

Fractional parts of the metric units are expressed by means of the Latin prefixes, *deci-* meaning 0.1, *centi-* meaning 0.01, and *milli-* meaning .001.

For example: 1 *decimeter* is equal to 0.1 meter. 1 *centimeter* is equal to .01 meter. 1 *millimeter* is equal to .001 meter.

These prefixes are also attached to the names of the other units, *gram* and *liter*, in similar manner and with like meaning.

What is a *centigram?* A *milligram?* A *deciliter?* A *centiliter?*

Since 10 units of any denomination make 1 unit of the next higher denomination, reductions from any denomination to the next higher or next lower are made by moving the decimal point one place to the left or to the right, as the case requires.

For example: .478 meters = 4.78 decimeters = 47.8 centimeters = 478 millimeters.

2765 millimeters = 276.5 centimeters = 27.65 decimeters = 2.765 meters.

Measures of Length

The meter is used for making measurements that we would express as yards, feet, and inches. The kilometer is used for expressing measurements that we should express as miles.

In the table below, and in the other tables of metric measurements that follow, the units more commonly used are printed in *italics*.

10 millimeters (mm.) = 1 *centimeter* (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 *meter* (m.)
10 meters = 1 dekameter (Dm.)
10 dekameters = 1 hektometer (Hm.)
10 hektometers = 1 *kilometer* (Km.)
10 kilometers = 1 myriameter (Mm.)

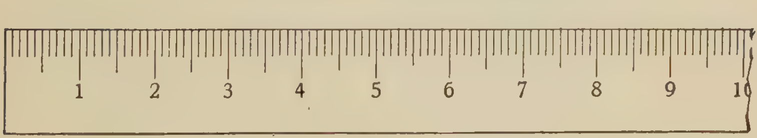


Fig. 100

The numerals mark the centimeter divisions.

Fig. 100 shows a portion of a *meter stick.* How many decimeters long is this piece? How many centimeters long is it? Are the millimeters shown? What would you measure by means of millimeters? Use Fig. 100 as a guide to measure the length and width of this book. Use Fig. 100 as a guide to construct a meter stick. Use the meter stick that you make to measure your height in meters.

Exercise 96

1. Read as meters: 36cm., 480Km., 7500mm., 26cm.
2. Read as kilometers: 7400mm., 10,000cm., 48.5m., 2.75cm.
3. Read as centimeters: 4m., 7cm.3mm., 980mm., 3m.5cm.
4. Write as meters: 5m.48cm., 798cm., 5.46Km., 342Km.
5. Write as centimeters: 5cm.6mm., 3m.8cm., 4m.8dm.
6. Write as kilometers: 72,840m., 870,000cm.

Square Measure

Metric units of square measure are formed by squaring the units of length, as in the system we use. For measuring land, the *are* (pronounced *âr*), a square 10 meters long is used as the unit. The *are* equals about 119.6 square yards, or .025 acres. In referring to the size of a farm or other large area, one usually speaks of it in terms of *hectares*. A hectare (pronounced *hĕk′târ*) equals about 2.471 acres. It is 100 ares.

100 square millimeters (sq. mm.) = 1 *square centimeter* (sq. cm.)
100 square centimeters = 1 square decimeter (sq. dm.)
100 square decimeters = 1 *square meter* (sq. m.)
100 square meters = 1 square dekameter (sq. Dm.)
100 square dekameters = 1 square hektometer (sq. Hm.)
100 square hektometers = 1 *square kilometer* (sq. Km.)

Instead of square centimeter we often use cm^2, and instead of square meter we often use m^2.

The method of reducing units of square measure to those of higher or lower denomination is illustrated below.

For example: 485 sq. cm. = 4.85 sq. dm. = .0485 sq. m.
35 sq. m. = 3500 sq. dm. = 350,000 sq. cm.

Exercise 97

1. Write 84 sq. m. as sq. cm. As sq. mm.

2. Write 485 sq. cm. as sq. m.

3. Draw a square 1 decimeter long. Divide it into square centimeters. How many square centimeters does it contain?

4. A field is 280m. long and 60m. wide. What is its area in hectares?

5. A field is 40m. long and 28m. wide. How many hectares does it contain?

6. Draw on paper a square which you believe contains just 100 sq. dm. Check the accuracy of your estimate.

Measures of Volume and Capacity

As in our own system, the metric measures of volume are the cubes of the units of measures of length.

1000 cubic millimeters (cu.mm.) = 1 *cubic centimeter* (cu.cm.)
1000 cubic centimeters = 1 cubic decimeter (cu.dm.)
1000 cubic decimeters = 1 *cubic meter* (cu.m.)

Make a rule for reducing units of measurement of volume to those of higher or lower denomination. Give examples.

The unit of capacity is the *liter*, which is the capacity of a cube whose edge measures 1 decimeter. The liter equals 1.0567 liquid quarts, or 0.908 dry quarts.

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 *liter* (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 *hektoliter* (Hl.)
10 hektoliters = 1 kiloliter (Kl.)

Make a rule for reducing units of measurement of capacity to those of higher or lower denominations. Give examples.

Exercise 98

1. Read as liters: 42Hl. 7.4Hl. 842cl.

2. Read as hektoliters: 9240cl. 860l. 47Kl.

3. A tank has a capacity of 74Hl. How many liters does it hold?

4. A dealer orders 6Hl. of olive oil. About how many quarts is that amount?

5. A merchant puts 6.5Kl. of olive oil into bottles that hold 0.5l. each. How many bottles does he use?

6. The milliliter, commonly used in scientific measurements, is equal to a cubic centimeter. What part of a liter is it?

Measures of Weight

Metric measurements of weight are quite generally used by our scientists, and among physicians and druggists the system is gradually coming into more general use for indicating the amounts of ingredients from which medicines are compounded.

The unit of weight is the *gram*, which equals approximately the weight of 1 cubic centimeter of pure water.

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 *gram* (g.)
10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 *hektogram* (Hg.)
10 hektograms = 1 *kilogram* (Kg.)

The *kilogram* and the *half-kilo* (500g.) are commonly used in trade in those countries where the metric system prevails. Sugar, coffee, tea, and other commodities are bought by the kilogram, the half-kilo, and the fractional parts of those units of weight. The gram and the milligram are used in making measurements for scientific purposes.

Make a rule for reducing units of weight to those of higher or lower denominations. Give examples.

Exercise 99

1. Read as grams: 14Kg. 7.42Kg. 340cg. 18Hg.

2. Read as kilograms: 74g. 4620g. 38,472cg.

3. Alcohol weighs 0.792 times as much as an equal volume of water. What is the weight of a liter of alcohol?

4. Milk weighs 1.03 times as much as an equal volume of water. What is the weight of half a liter of milk?

5. Ice weighs 0.92 times as much as an equal volume of water. What is the weight of a block of ice 0.7m. long, 0.4m. wide, and 0.2m. thick?

Comparative Table of Weights and Measures

The following comparative table of the more common metric units will give you an idea of the values of metric units in terms of those used in the United States. When necessary, use this table for solving examples.

1 inch = 2.54001 cm.
1 foot = .304801 m.
1 yard = .914402 m.
1 mile = 1.60935 Km.
1 square inch = 6.452 sq. cm.
1 square foot = .0929 sq. m.
1 square yard = .8361 sq. m.
1 square mile = 2.59 sq. cm.
1 cubic inch = 16.3872 cu. cm.
1 cubic foot = .02832 cu. m.
1 acre = .4047 hectares

1 liquid quart = .94636 liter
1 dry quart = 1.1012 liter
1 ounce = 28.3495 grams
1 pound = .45359 Kg.
1 meter = 39.37 in.
1 kilometer = .62137 mi.
1 hectare = 2.471 A.
1 liter = 1.05668 (liquid) qt.
1 liter = .9081 (dry) qt.
1 hectoliter = 2.83774 bu.
1 kilogram = 2.20462 lb.

Review: Metric Units

1. If a horse weighs 740Kg., how many pounds does it weigh?

2. A gun has a diameter of 40cm. What is the diameter in inches?

3. An irrigation canal is 5.4Km. long. Express this length in miles.

4. Helen's cousin visiting in Madrid told her that she walked 4.8Km. in one day. Express this distance in miles.

5. The Eiffel Tower in Paris is about 300m. in height. Express this height in feet.

6. A merchant wants 800 yards of French silk. How many meters of silk must he order?

7. Express your height in meters.

8. Express your weight in kilograms.

9. Draw a line a decimeter long. Check your work.

10. The distance between two cities in Brazil is 211Km. Express this distance in miles.

11. An Italian boat is 262m. long. Express this length in feet.

12. A Belgian aviator flew a distance of 88Km. in 1 hr. 4 min. What was the rate in miles per hour?

13. A merchant imported 4.5Kl. of olive oil. How many quarts is this?

14. An importer received 285Kg. of cheese from Switzerland. How many pounds did he receive?

15. A sewer in Rio de Janeiro is 1.4Km. long, 1.3m. wide, and averages 2.2m. in depth. How many cubic meters of earth were removed in digging it?

16. Tony's cousin in Italy writes that his father bought a farm containing 20 hectares. How many acres is this?

17. Does it require a larger number to express your weight in kilograms or in pounds? Why?

18. Does it require a larger number to express a distance in miles or in kilometers? Why?

19. Marie writes that milk sells in Paris for 11 cents a liter. Is this more or less than 12 cents a quart?

20. Carl's uncle in Argentina writes that he has 220 hectares of wheat planted. How many acres has he planted?

21. A man in Central America sent 4.5Kg. of coffee to a friend in Chicago. How many pounds of coffee did he send?

22. Charles said that during one year his lung capacity increased from 2800 cu. cm. to 3250 cu. cm. Express the increase in cubic inches.

23. During one school year Marie's weight increased from 38.5Kg. to 40.3Kg. Express the increase in pounds.

24. Fred told the class that while on a trip to France he one day walked 12.5Km. How many miles was that, to the nearest tenth?

25. A cubic centimeter of water weighs a gram. What is the weight of a liter of water?

26. A merchant in Italy bought a bolt of silk containing 25m. He sold 2.5m. to one customer, and 9.5dm. to another. How many meters were left on the bolt?

27. Marie has a cousin living in Belgium. The cousin writes that she weighs 44.5Kg. Marie weighs 93¼ pounds. Which girl weighs the more, and how much?

28. A French merchant ordered 100Hl. of rice from the United States. How many bushels are needed to fill the order?

29. Which is the longer, and by how much, an Italian ocean liner that measures 264m., or an American liner that measures 794.5 feet?

30. Explain the meaning of each of the following prefixes: deci-, kilo-, centi-, hecto-, and milli-.

31. When sound travels 332m. per second, how long will it take the report of a gun to travel 4Km.?

32. An American exporter received an order from a French miller asking for 750,000Kg. of wheat. A bushel of wheat weighs 60 pounds. How many bushels of wheat did the exporter use to fill this order?

33. A Belgian newspaper states that a certain fine horse weighs 740Kg. About how many pounds does the horse weigh?

GENERAL REVIEW

Our United States

1. There were 264,233 miles of railroads in the United States in a recent year. At the same time there were 730,988 miles of railroads in the world. What per cent of the railroads of the world were in the United States?

2. In 1900 the population of the United States was 75,994,575. Of those inhabitants, 45,197,390 were classed as rural. In 1920 the population was 105,710,620, of whom 51,406,017 were classed as rural. What was the per cent of the rural population in each of these census years? Did these figures show an increase or a decrease in the per cent of rural population?

3. It was recently estimated that the total stock of gold in the United States was $3,656,988,551. Ten years ago the total gold stock in this country was $1,812,856,241. What was the per cent of increase in the stock of gold during this period?

4. During one year the United States produced 3,208,584,000 bushels of corn. The total corn production of the world that year was 4,144,821,000 bushels. What per cent of the world's crop did the United States produce?

5. During the same year the world's production of wheat was 2,867,864,000 bushels, of which the United States produced 833,027,000 bushels. What per cent of the world's crop did the United States produce?

6. The United States produces annually about 40% of the world's production of iron and steel. If the United States produces 79,058,921 long tons of iron and steel in a year, what is the number of long tons produced in the world during the same year?

7. In one year the world produced 2,800,000,000 pounds of wool. Of that amount the United States produced 259,307,000 pounds. What per cent of the world's wool crop did the United States produce that year?

8. The United States produced 18,622,884,000 gallons of crude oil in one year. This was 60% of the world's supply. How many gallons of crude oil did the world produce that year?

9. Fifty-two per cent of the world's coal was produced in the United States when we produced 577,008,067 tons. How many tons did the world produce?

10. The United States' production of cotton in one year was 12,987,000 bales. This was 60% of the world's supply. What was the world's production of cotton?

11. In a year the United States produced 56,564,504 ounces of silver, which was 40% of the world's production. How much did the world produce?

12. About 60% of the copper in the world is produced in the United States. During one year our production was 1,209,061,040 pounds. What was the world's production on this basis?

13. The United States produces about 40% of the world's supply of lead annually. If we produce 476,849 tons in one year, what is the world's production?

14. In one year the United States produced about 51% of the world's zinc. Its production was 450,045 short tons. About what was the world's production? (See page 280 for an explanation of "short ton.")

15. In 1910 the population of the United States was 91,972,266. In 1920 it was 105,710,620. What was the per cent of gain?

16. In 1900 the population of the United States was 75,994,575. What was the per cent of gain in 20 years? (*See Problem 15 for the population in 1920.*)

17. In 1850 the population of the United States was 23,191,876. In 1900 it was 75,994,575. What was the per cent of gain?

18. The total continental area of the earth is about 51,000,000 square miles. The area of the United States is 3,026,789 square miles. What per cent of the earth's continental area is included in the area of the United States?

19. The area of North America is about 8,589,000 square miles. The area of the United States is 3,026,789 square miles. What per cent of the area of North America is included in the area of the United States? Make a graph to show this.

20. The area of Texas is 265,896 square miles. What per cent of the total area of the United States is included in Texas? (*See Problem 19 for the area of the United States.*)

21. The area of Rhode Island is 1248 square miles. What per cent of the area of the United States is included in Rhode Island?

22. In 1920 the population of New York City was 5,620,048. The population of the United States for the same year was 105,710,620. What per cent of the total population of the United States was in New York City?

23. In 1920 the population of the state of New York was 10,385,227. What per cent of the total population of New York state was in New York City? (*See Problem 22 for the population of New York City.*)

24. In 1920 the population of California was 3,426,861. The population of Los Angeles was 576,673, and of San Francisco 506,676. What per cent of the population of California was in the two cities at that time?

25. What per cent of the area of the United States is included in the area of your own state?

26. What per cent of the population of the United States is the population of your state?

27. In 1920 the population of the world was approximately 1,747,000,000. The population of the United States was 105,710,620. What per cent of the population of the world was the population of the United States?

28. In 1920 the population of North America was approximately 136,000,000. The population of the United States was 105,710,620. What per cent of the total population of North America was the population of the United States?

29. In 1920 the population of Canada was approximately 8,788,500. The population of the United States was 105,710,620. What per cent of the population of the United States was the population of Canada?

30. The area of Canada is 3,729,665 square miles. Compare it with the area of the United States on a per cent basis. (*See Problem 18.*)

31. The area of Mexico is 767,198 square miles. Compare it with the area of the United States on a per cent basis. (*See Problem 18.*)

32. The area of Alaska is 590,884 square miles. What per cent of the area of North America is Alaska? (*See Problem 19.*)

33. In 1920 the population of Alaska was 55,036. What per cent of the population of North America lived in Alaska at that time? (*See Problem 28.*)

34. What per cent of the continental land area of the earth is the area of North America? (*See Problems 18 and 19.*)

35. The table below shows the original area of our country, the dates of later additions, and the amount of each additional increase in area.

DATE	ADDITION	AREA IN SQUARE MILES	DATE	ADDITION	AREA IN SQUARE MILES
1790	Original Colonies....	892,135	1898	Hawaiian Islands....	6,449
1803	Louisiana Purchase .	827,987	1899	Porto Rico	3,435
1819	Treaty with Spain...	13,435	1899	Guam..............	210
1819	Florida.............	58,666	1899	Philippine Islands...	144,988
1845	Texas..............	389,166	1900	Samoa.............	58
1846	Oregon.............	286,541	1901	Additional Philippines	68
1848	Mexican Concession	529,189	1904	Panama Canal Zone	527
1853	Gadsden Purchase...	29,670	1917	Virgin Islands.......	132
1867	Alaska.............	590,884			

Find the present total area. Find the area that has been added since 1790. How many square miles have been added since the close of the Civil War? How many square miles have been added since the opening of the present century?

36. The total land area of the earth is approximately 57,255,000 square miles. The area of the surface of the earth is 196,940,000 square miles. What per cent of the earth's surface is water?

37. What per cent of the total land area of the earth is the area of the United States? (*See Problem 18.*)

Good Roads Problems

1. A farmer lives 1½ miles from a good road, on which his team can haul 72 bushels of wheat at a load. On the poor road, which runs from his farm to the good road, his team can haul only 40 bushels at a load. His total wheat crop is 1200 bushels. If the time of the farmer and his team is worth $5.50 a day, and he hauls two loads to market daily, find the hauling cost per bushel.

2. Compare the cost per bushel for hauling on the poor road with the cost as it would be if there were a good road all the way to market.

3. If the good road had extended to his farm, how much would this farmer have been saved in marketing the wheat?

4. Mr. Carson lives 7 miles from the market. The road was formerly so bad that his team could haul only 30 bushels of wheat at a load, and the round trip required a day's time. At $5 a day for man and team, what was the hauling cost per bushel?

5. After the road was improved, Mr. Carson hauled 70 bushels to the load and made the round trip in half a day. What was the hauling cost per bushel then?

6. It costs a farmer 36¢ a ton to haul his produce one mile on a certain dirt road, and 20¢ a ton to haul it on an improved road. How much will he save annually because of the improved road, if he hauls 64 tons of produce a distance of 6½ miles each year?

7. How many cubic yards of gravel are required for each mile of road 26 feet wide, if the average thickness of the layer of gravel is 8 inches?

8. On a certain bad road a farmer can haul 3 loads of 1500 pounds each daily. On a good road he can haul 4 loads of 2400 pounds each. In hauling his wheat crop of 1800 bushels to market, how much time will he lose because of the poor road? (A bushel of wheat weighs 60 lb.)

9. It is estimated that good roads will increase the value of farm land in a certain county $7.40 an acre. At this rate, what are good roads worth to a farmer who owns a quarter section?

10. Bring to class some illustration of the money value of good roads in your vicinity.

11. Make some practical problems about the value of good roads.

Miscellaneous Problems

1. If both terms of a common fraction are multiplied by the same number, what is the effect on the value of the fraction? Give an example.

2. Write from memory the table of avoirdupois weight. Of surface measure. Of linear measure.

3. One store advertises 5½ lb. of sugar for 35¢, and another 3½ lb. for 25¢. Which is the cheaper price? How much?

4. Find the number of tons of soft coal in a rectangular bin 16 ft. long and 6½ ft. wide, if the coal is 6 ft. deep. Allow 38 cu. ft. to the ton.

5. The population of six of the largest cities of the United States at the time of the last government census was as follows: New York City 5,620,048, Philadelphia, 1,823,779, Cleveland 796,841, Chicago 2,701,705, Detroit 993,678, St. Louis 772,897. Show the relative sizes by a bar graph.

6. Mr. Carter bought property for $6500, which he rents for $65 per month. Each year he pays $82.50 for taxes, $60.50 for repairs, and allows $120 for depreciation in value. How much more or less net profit does he receive than he would have received if he had invested his money at 6% simple interest?

7. Write a check for \$18.50, drawn against the First National Bank, as if written by J. W. Sanders to be given to C. M. Moore. Who indorses the check? How?

8. How does the amount of water delivered in a given time by a 2-inch pipe compare with the amount delivered by a $\frac{1}{2}$-inch pipe?

9. At \$2.10 per 100 sq. ft., what will it cost to paint the cylindrical surface of a silo 42 ft. high and 16 ft. in diameter?

10. The time required for an object to fall a given distance may be expressed by the formula $t=\sqrt{\frac{2s}{g}}$, in which t = the time in seconds, s = the distance in feet that the body is to fall, and g = the force of gravity, which is 32. State the formula in words. How long does it take a stone to fall 64 feet?

11. A bag of sand is dropped from a balloon at a height of 6400 feet. How long will it take the bag to reach the earth?

12. The total area of the forty-eight states is 3,026,789 square miles. What per cent of the total area is the area of your state?

13. The population of the United States at the time of the last general census was 105,710,620. What per cent of the total population was the population of Texas (4,663,228)?

14. A certain road has an average rise of $3\frac{1}{2}\%$, that is $3\frac{1}{2}$ feet in 100 feet of length. What is the rise in $\frac{1}{2}$ mile of the road?

15. A farmer raised 480 tons of sugar beets. If 13.6% of the weight of the beets was sugar, how many pounds of sugar were obtained from the beets?

16. At the beginning of a season Mr. Carter's automobile was worth \$1850. During the season he drove 6250 miles. The automobile decreased \$350 in value. He wore out 4 tires that cost \$28.50 each. The engine required a quart of oil every 75 miles, and a gallon of gasoline every 15 miles. The oil cost 55¢ a gallon, and the gasoline 14¢ a gallon. He paid \$30 for his license, \$24.50 for repairs, and \$15.50 for insurance. Find the total expense for the season, and the average cost per mile.

17. How many pounds of sugar beets containing 13.8% sugar are required to make 100 lb. of sugar?

18. Find the cost of painting a hemispherical dome 80 feet in diameter at 50¢ a square yard.

19. Use the following facts and make out a bill: R. M. Carpenter & Company sold Mrs. Henry Meyers of Omaha, Nebraska, one dining-room table for $60, six dining-room chairs for $32.50, and one buffet for $90.

20. A retail dealer bought goods at 20% and 10% off. What was the list price of an article for which he paid $1.44?

21. A cylindrical silo is 36 ft. high and 16 ft. in diameter, inside measurements. How many tons of ensilage does it hold, allowing 40 cu. ft. to the ton?

22. Write a promissory note for $350, with interest at 6%, dated today, payable in 90 days. Make Henry Curtis the payee and yourself the maker. When does the note mature? What amount will be due when the note matures?

23. A writer states that $\frac{1}{2}$ pt. of milk has the same food value as 2 eggs. How much milk has the food value of a dozen eggs?

24. In a class of 39 there are 5 more girls than boys. How many girls are in the class?

25. Find the value of A in the equation $A = lw$, when $l = 6$ and $w = 8$. State the formula as a rule.

26. Find the value of A in the equation $A = \pi r^2$, when $r = 14$. State the formula as a rule

27. Find the value of A in the equation $A = 4\pi r^2$, when $r = 7$. State the formula as a rule.

28. Two boys wished to find the distance across a lake. They found the distance from a certain point on the land to one end of the lake to be 360 ft., and the distance from this point to the other end of the lake to be 300 ft. The lines drawn from this point to the ends of the lake make an angle of 75°. Make a drawing to the scale 1 in. = 60 ft. Find the length of the lake.

29. A rectangular room is 18 ft. long and 16 ft. wide. Draw the room to scale, and from your drawing determine the length of the diagonal of the room. Find the length of the diagonal by using the right triangle. How do the two results compare?

30. A boy standing 60 ft. from the foot of a tree found that the angle which a line drawn to the top of the tree makes with a horizontal line is 50°. Draw a sketch of this to scale. Find the height of the tree.

31. When a flag pole which is 120 ft. high casts a shadow 300 ft. long, how long a shadow will a boy 5 ft. 2 in. tall cast?

32. A wholesaler sold some goods to a retailer at a profit of 20% on the cost, and the retailer sold them to a customer at a profit of 20% on what the goods cost him. What per cent of the wholesaler's cost did the customer pay for the goods?

33. How can you find the volume of a cylindrical gasoline tank?

34. Each of the 19 pupils in a cooking class needs $3\frac{1}{2}$ cups of water for a certain lesson. Allowing 4 cups to the quart, how many quarts are needed to supply the class?

35. A man bought stock at 85, including brokerage. What rate of income does he receive on his investment, if the stock pays a 6% dividend?

36. Carson & Company sold the following to Mrs. Charles Peterson: $4\frac{3}{4}$ yd. of silk at $3.40 a yard, $3\frac{1}{4}$ yd. of silk at $2.80 a yard, $2\frac{1}{2}$ yd. of velvet at $3.30 a yard. Make out the bill.

37. How many acres in a square field of which the perimeter is 380 ft.?

38. When sound travels 1130 ft. a second, and the thunder is heard $9\frac{3}{4}$ seconds after the flash of lightning is seen, how far away was the flash?

39. What is the area of a watch crystal if its diameter is 1.75″?

40. A man receives a salary of $2000 a year, plus 6% on all sales in excess of a total of $25,000. How much is his income if his annual sales are $36,500?

41. A 90-day note for \$1500 at 5% is discounted at a bank at 6% on the day the note is made. Find the proceeds.

42. Make and receipt a bill for the following items bought of Hamilton & Company by William Evans: 12 bu. of potatoes at 90¢ per bu., 6 lb. of coffee at 32¢ per lb., 24 lb. of lard at 21¢ per lb.

43. A grocer bought potatoes at 90¢ a hundred pounds, and sold them at 25¢ a peck. What per cent of the cost did he gain?

44. An airplane traveled 90 miles in 42 minutes. What was the rate per hour?

45. An oil stove holding 2½ qt. of oil is filled from a can containing 3½ gal. How much oil is left in the can?

46. Mr. Lenox owns a farm assessed at \$9600. His tax rate on the farm is 5½ mills. How much is his tax?

47. A merchant bought gloves at \$5.40 a dozen pairs, and sold them at 75¢ a pair. What per cent of the cost did he gain?

48. Clara's father bought a lawn mower listed at \$14.50, and was allowed discounts of 20% and 10%. What price did he pay?

49. A company cuts ice 18 inches thick from an area of 2 acres. If the ice is .92 as heavy as an equal volume of water, what is the weight of the ice? (1 cu. ft. of water weighs 62.5 lb.)

50. Multiply each of the following numbers by 25, using a short method: 480 360 21 18 426 984

51. Multiply each of the above numbers by 33⅓. By 75.

52. How many barrels of water fall on an acre when the rainfall is 1 inch? (Allow 31.5 gal. to the bbl.)

53. A merchant bought some goods at 20% below the wholesale list price, and sold them at the wholesale list price. What per cent of the cost did he gain?

54. Each of a herd of 18 dairy cows averages 24 lb. of milk per day. The milk tests 3.9% butter fat, and the butter fat sells for 32¢ per lb. What is the daily income from the herd?

55. A dealer sold goods at what would have been a gain of 40% on the cost, but was able to collect only 75% of the sale price. What per cent of the cost did he gain or lose?

56. How many years, months, and days have passed since the birth of George Washington?

57. When the wind pressure is .08 pound per square foot, what is the total wind pressure against the side of a rectangular wall 86 ft. long and 48 ft. high?

58. What will a postal money order for $23.40 cost?

59. A flywheel on an engine has a diameter of 6 ft., and makes 100 revolutions a minute. At what rate does a point on the rim travel? (Find answer to the nearest foot.)

60. An architect told Carl that the window area of a schoolroom should be about 20% of the floor area. Carl's schoolroom is 29 ft. by 22 ft. What should the window area be?

61. Mr. Mitchell bought a house for $4500. He spent $900 for painting and repairs, then sold the house for $6500. What per cent did he gain?

62. A roller is 8 ft. long and 3 ft. in diameter. How large an area will it roll over in one revolution?

63. How many barrels of water will a 5 ft. cylindrical cistern hold if the cistern is 11 ft. deep? (Allow 4.2 cu. ft. to the barrel.)

64. A picture, 14″×9″, is to be framed with 1″ molding. How many feet of molding are required? Make a drawing to show the frame.

65. A grocer buys apples at $3.19 per barrel of $2\frac{3}{4}$ bushels. At what price per peck must he sell them to gain 20% of the cost?

66. A swimming pool 80 ft. long and 28 ft. wide is filled to an average depth of 4.5 ft. How many gallons of water does it contain? (1 cu. ft. = 7.5 gal.)

67. A Maypole, 12 ft. high, is at the center of a circle whose radius is 10 ft. How long must the streamers be just to reach the circle?

68. A $4\frac{1}{4}$-lb. beef roast weighed $3\frac{1}{2}$ lb. when roasted. What per cent of the weight was lost in roasting?

69. An emery wheel has a diameter of 10 inches, and makes 2000 revolutions a minute. What is the speed of a point on the rim?

70. If $\frac{7}{8}$ in. on a map represents an actual distance of 35 mi., what distance on the map represents 250 mi.?

71. How high is a chimney that casts a shadow 90 ft. long at the same time that the shadow of a boy 4 ft. 8 in. tall is 5 ft. long?

72. Two flag poles stand on level ground 150 ft. apart. One pole is 60 ft. high, and the other is 110 ft. high. What is the distance between their tops?

73. Sarah's mother made a bedspread 84 in. long and 72 in. wide. How much did she pay for hemstitching at 10¢ a yard?

74. At $1.25 per sq. yd., how much will it cost to build a six-foot walk across the front and along one side of a lot which measures 60 ft. by 150 ft.?

75. A rectangular lot is 75 ft. by 150 ft. What distance is saved by using a diagonal path across the lot instead of going round it?

76. A room 18 ft. by 24 ft. has a 3-foot wainscoting. What will it cost to varnish this wainscoting at 45¢ a square yard?

77. Each pupil who sells 8 tickets to a school entertainment is given one ticket. What rate of commission does the pupil receive?

78. Three men living in different cities were comparing their tax rates. A's rate was $2.74, B's was 28 mills, and C's was 2.7%. Which had the highest rate? How much tax would each pay on an assessed valuation of $6000?

79. The world's sugar production in a recent year was 39,177,600,000 lb., of which the United States produced 1,802,234,000 lb. What per cent of the total did this country produce?

80. A contractor receives $860 for making an excavation 30 ft. by 45 ft. by 6 ft. The labor costs him $265, and teaming costs an average of $1.40 per cu. yd. How much does the contract net him?

81. A gas burner consumes 75 cu. ft. of gas per hour. It burns an average of 8 hours a day for 30 days. At 45¢ per M for the first 10,000 cu. ft., and 40¢ per M for the second 10,000 cu. ft., what is the cost of the gas used?

82. A piece of land $\frac{1}{8}$ mi. long and 32 rd. wide produced 1440 bushels of potatoes. Another plot 40 rd. by 60 rd. produced 2400 bushels. Which plot had the better yield?

83. It is frequently necessary to check numbers accurately. From among the pairs of numbers below make a list of the pairs that are not exactly alike.

(*a*)	46382	46382	(*g*)	30984	30498
(*b*)	39408	39048	(*h*)	93827	93287
(*c*)	94087	94087	(*i*)	78943	79834
(*d*)	17904	17904	(*j*)	74206	74206
(*e*)	807623	807632	(*k*)	469236	469236
(*f*)	306902	306902	(*l*)	45325	45235

84. A recent investigation shows that the following distribution of incomes of various sizes as reasonable in families of four or five persons.

INCOME	FOOD	RENT	OPERATING EXPENSES	CLOTHES	OTHER EXPENSES AND SAVINGS
$ 500 to $ 800	45%	15%	10%	10%	20%
$ 800 to $1000	30%	20%	10%	15%	25%
$1000 to $2000	25%	20%	15%	20%	20%
$2000 to $4000	25%	20%	15%	20%	20%

From a salary of $1800 what amount may reasonably be used for each item? How much may be used from a salary of $2400?

Problems Without Numbers

1. If you know the circumference of an automobile tire, how can you find the number of turns the tire will make in going a mile?

2. What must you know and what must you do to find the area of a rectangle? Of a triangle? Of a trapezoid? Of a circle?

3. If you know the assessed valuation and the amount to be raised by taxation, how can you find the tax rate?

4. If you are told the cost price and the rate of gain on the cost, how can you find the selling price?

5. If you are told the selling price and the loss, how can you find the per cent of the cost that was lost?

6. If you know only the perimeter of a square, how can you find the area?

7. If you know the perimeter of a rectangle, and the base, how can you find the area?

8. If you know the price paid for stock and the income received from it, how can you find the rate of income on the investment?

9. What must you know and what must you do to find the capacity of a cylindrical tank?

10. What must you know and what must you do to compute the weight of the water which a cylindrical sprinkler will hold?

11. If you know the height of a cylinder and the diameter of the base, how can you find its lateral area?

12. If you know the radius of a sphere, how can you find the area of its surface? How can you find its volume?

13. If you know the length of the side of a square, how can you find the length of the diagonal?

14. If you know the length and the width of a rectangle, how can you find the length of the diagonal?

Tables of Denominate Numbers

A thorough knowledge of the tables of denominate numbers will be of value to you in the solution of many practical problems. Review the following tables, and refer to this page whenever necessary.

Avoirdupois Weight

16 oz. = 1 lb.
100 lb. = 1 cwt.
2000 lb. = 1 short ton (T.)
2240 lb. = 1 long ton

Linear Measure

12 in. or ″ = 1 ft. or ′
3 ft. = 1 yd.
5½ yd. or 16½ ft. = 1 rd.
320 rd. or 5280 ft. = 1 mi.

Square Measure

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
30¼ sq. yd. = 1 sq. rd.
160 sq. rd. = 1 A.
640 A. = 1 sq. mi.

Liquid Measure

4 gi. = 1 pt.
2 pt. = 1 qt.
4 qt. = 1 gal.
31½ gal. = 1 bbl.
63 gal. = 1 hhd.
1 gal. = 231 cu. in.

Dry Measure

2 pt. = 1 qt.
8 qt. = 1 pk.
4 pk. = 1 bu.

Cubic Measure

1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.
1 cu. yd. = a *load*
128 cu. ft. = 1 cord

Time Measure

60 sec. = 1 min.
60 min. = 1 hr.
24 hr. = 1 da.
7 da. = 1 wk.
30 da. = 1 mo.
12 mo. = 1 yr.
365 da. = 1 yr.
366 da. = 1 leap year
100 yr. = 1 century

Miscellaneous Measure

12 things = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 things = 1 score

Weights of Grain by the Bushel

Corn on cob	70 lb.
Barley	48 lb.
Oats	32 lb.
Rye	56 lb.
Wheat	60 lb.
Clover seed	60 lb.
Timothy seed	45 lb.

Weights of Common Things

A bbl. of flour = 196 lb.
A bbl. of salt = 280 lb.
A cu. ft. of water = 62.5 lb.
400 cu. ft. of hay = 1 T.
1 bu. of white beans = 60 lb.
1 bu. of bran = 20 lb.
1 bu. of shelled corn = 56 lb.

Table of Apothecaries' Weight

Apothecaries' weight is used by doctors and druggists in writing and filling prescriptions for medicines. Note that there are only 12 ounces in the apothecaries' pound.

20 grains (gr.) = 1 scruple (℈)
3 scruples = 1 dram (ʒ)
8 drams = 1 ounce (℥)
12 ounces = 1 pound (℔)

Table of Troy Weight

Troy weight is used by jewelers and others whose business requires them to handle platinum, gold, silver, and other precious and semi-precious metals.

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (℔)

The troy pound contains 5760 grains. The avoirdupois pound contains 7000 grains.

Some Common Units of Measure

The unit of weight for diamonds and other precious and semi-precious stones is the *carat*, which equals about 3.02 troy grains.

The word *carat* is also used to denote the fineness of gold. Pure gold is 24 carats fine.

Gold that is 18 carats fine is $\frac{18}{24}$ pure gold.

The *hand*, a measure commonly used for measuring the height of horses, equals 4 inches. That distance is supposed to be the width of an average hand measured across the palm.

The *furlong* = 220 yards = 40 rods = $\frac{1}{8}$ of a mile. It is a unit of measure commonly used in laying out race courses.

The *league* is often used by sailors in estimating distances at sea. It varies in length from 2.4 to 4.6 miles. The English league is usually estimated as 3 miles.

The *fathom* is a unit of measurement used to measure the depth of water. It is reckoned as 6 feet.

Table of Squares and Square Roots

The following table gives the *square* and the *square root* of each number from 1 to 100 inclusive.

NUMBER	SQUARE	SQUARE ROOT	NUMBER	SQUARE	SQUARE ROOT
1	1	1.000	**51**	2601	7.141
2	4	1.414	**52**	2704	7.211
3	9	1.732	**53**	2809	7.280
4	16	2.000	**54**	2916	7.348
5	25	2.236	**55**	3025	7.416
6	36	2.449	**56**	3136	7.483
7	49	2.646	**57**	3249	7.550
8	64	2.828	**58**	3364	7.616
9	81	3.000	**59**	3481	7.681
10	100	3.162	**60**	3600	7.746
11	121	3.317	**61**	3721	7.810
12	144	3.464	**62**	3844	7.874
13	169	3.606	**63**	3969	7.937
14	196	3.742	**64**	4096	8.000
15	225	3.873	**65**	4225	8.062
16	256	4.000	**66**	4356	8.124
17	289	4.123	**67**	4489	8.185
18	324	4.243	**68**	4624	8.246
19	361	4.359	**69**	4761	8.307
20	400	4.472	**70**	4900	8.367
21	441	4.583	**71**	5041	8.426
22	484	4.690	**72**	5184	8.485
23	529	4.796	**73**	5329	8.544
24	576	4.899	**74**	5476	8.602
25	625	5.000	**75**	5625	8.660
26	676	5.099	**76**	5776	8.718
27	729	5.196	**77**	5929	8.775
28	784	5.291	**78**	6084	8.832
29	841	5.385	**79**	6241	8.888
30	900	5.477	**80**	6400	8.944
31	961	5.568	**81**	6561	9.000
32	1024	5.657	**82**	6724	9.055
33	1089	5.745	**83**	6889	9.110
34	1156	5.831	**84**	7056	9.165
35	1225	5.916	**85**	7225	9.220
36	1296	6.000	**86**	7396	9.274
37	1369	6.083	**87**	7569	9.327
38	1444	6.164	**88**	7744	9.381
39	1521	6.245	**89**	7921	9.434
40	1600	6.325	**90**	8100	9.487
41	1681	6.403	**91**	8281	9.539
42	1764	6.481	**92**	8464	9.592
43	1849	6.557	**93**	8649	9.644
44	1936	6.633	**94**	8836	9.695
45	2025	6.708	**95**	9025	9.747
46	2116	6.782	**96**	9216	9.798
47	2209	6.856	**97**	9409	9.849
48	2304	6.928	**98**	9604	9.899
49	2401	7.000	**99**	9801	9.950
50	2500	7.071	**100**	10000	10.000

INTERESTING QUESTIONS AND NUMBER FACTS

1. What is the difference, if any, between six dozen dozen and half a dozen dozen?

2. What is the difference, if any, between twice twenty-five and twice five and twenty?

3. If you multiply 37 by each of the following, you will get some interesting results. 3 6 9 12 15 18 21 24 27

4. This is called a "Magic Square." Add the numbers in each row, column, and diagonal.

25	6	7	24	3
4	10	17	12	22
5	15	13	11	21
8	14	9	16	18
23	20	19	2	1

5. Test these results and complete the work.

1×9+1=10
1 2×9+2=110
1 2 3×9+3=1110
1 2 3 4×9+4=11110
1 2 3 4 5×9+5=111110
1 2 3 4 5 6×9+6=
1 2 3 4 5 6 7×9+7=
1 2 3 4 5 6 7 8×9+8=
1 2 3 4 5 6 7 8 9×9+9=

6. Test these results and complete the work.

1×9+ 2=11
1 2×9+ 3=111
1 2 3×9+ 4=1111
1 2 3 4×9+ 5=11111
1 2 3 4 5×9+ 6=
1 2 3 4 5 6×9+ 7=
1 2 3 4 5 6 7×9+ 8=
1 2 3 4 5 6 7 8×9+ 9=
1 2 3 4 5 6 7 8 9×9+10=

7. Test these results and complete the work.

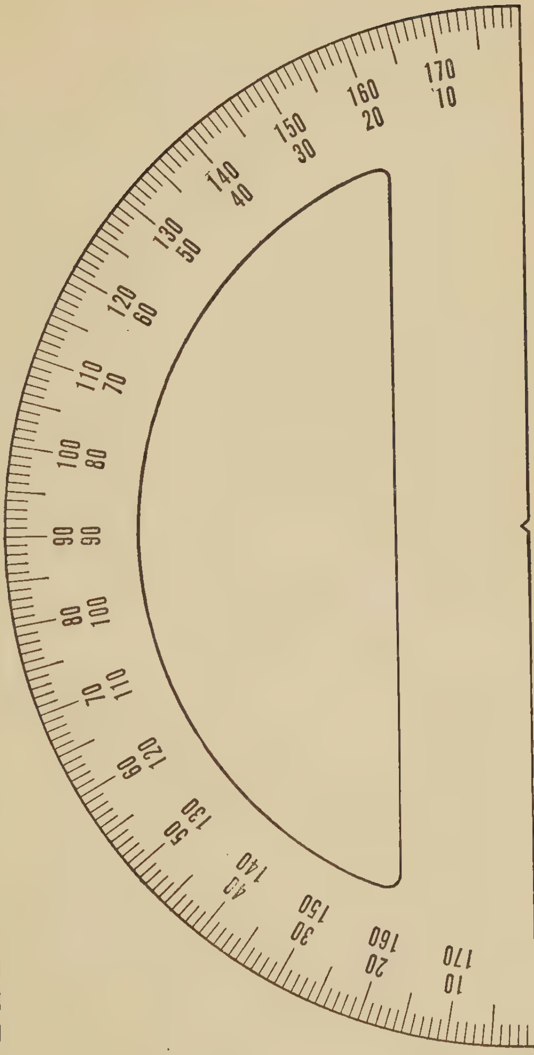
1×8+1=9
1 2×8+2=98
1 2 3×8+3=987
1 2 3 4×8+4=9876
1 2 3 4 5×8+5=98765
1 2 3 4 5 6×8+6=
1 2 3 4 5 6 7×8+7=
1 2 3 4 5 6 7 8×8+8=
1 2 3 4 5 6 7 8 9×8+9=

8. Ask a friend to write down the numbers 1, 2, 3, 4, 5, 6, 7, 9, omitting the number 8. Now tell him to select any number from the list, and multiply it by 9. Now tell him to multiply the number 12,345,679 by this result. He will find that the final product is interesting.

9. Multiply 73 by each of the following numbers: 3, 6, 9, 12, 15, 18, 21, 24, and 27. Now note what the final figure is in each product taken in order.

10. See if you can make a "Magic Square" containing the numbers 1 to 16, inclusive, so arranged that the sum of the numbers in each row, column, and diagonal will be 34.

How to Prepare This Protractor for Use: (1) Remove this sheet from the book by cutting carefully along the dotted line. (2) Paste the sheet on a piece of cardboard about the thickness of a heavy visiting card. Use thin paste and be careful to rub out any air bubbles that may form. (3) Lay the cardboard under a pile of books until it is thoroughly dry. (4) Use scissors to trim away the paper from the outside of the protractor; be careful to cut just outside the printed outside lines. (5) Use a sharp pocketknife to remove the inner white area. (6) See page 192 for instructions as to how to use this protractor in your work.



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ANSWERS

ANSWERS

Page 3, Ex. 1: **6.** 1,702,230,000 **7.** 21,732,340 **8.** 13,439,603 bales 3,179,313,336 lb. **9.** $181,882,615

Page 4, Ex. 2: **1.** 476,000,000 mi. **2.** 2,348,000; 940,000; 1,381,000; 968,000; 2,896,000; 6,485,000; 2,404,000; 3,852,000; 3,404,000; 5,759,000; 8,720,000; 4,663,000; 449,000; 1,357,000 **3.** $35,152,211,000; 23,596,912,000; 14,113,979,000 **4.** 7,954,000 bales; 3,339,113,000 lb. **5.** 36,827,900,000 lb.

Page 21, Ex. 3: **3.** $9 **4.** $37.50 **5.** $120 **6.** $4.70 **7.** $1.80 **8.** $2.75 **9.** $900 **10.** $400 **11.** $600 **12.** $32 **13.** $34

Page 22, Ex. 4:

1.	*a*	*b*	*c*	*d*	*e*
1.	5800	6000	19,500	12,100	24,300
2.	4800	10,200	105,000	9700	15,600
3.	120,900	8100	91,000	5100	10,800
4.	60,000	5900	198,000	19,000	28,800
5.	401	4500	1100	1,980,800	31,200
6.	53,000	710	245,000	1240	17,400

2. $3 **3.** $32.50; $56.25; $90; $18 **4.** $4.20; $6.20; $18; $32 **5.** $4; $6; $15; $30

Page 23, Ex. 4: **6.** $8 **7.** $900 **8.** $33 **9.** $6 **10.** $15 **11.** $106.25 **12.** $6

Page 23, Ex. 5: **2.** 716 **3.** $7.25 **4.** 424 lb.

Page 25, Ex. 6: **2.** 9; 48 **3.** 160 **4.** 29; 150; 75; 289 **5.** 252; 1872; 2700 **6.** 18; 27 **7.** 24; 56; 88; 64 **11.** Multiply the dividend by 8 and divide by 300.

Page 28, Ex. 7: **1.** \$10,125 **2.** 55° **3.** 56 lb.

Page 29, Ex. 7: **4.** \$12.70 **5.** 240 **6.** 22,000 ft. **7.** 15,260 ft. **8.** 13 sec. **9.** 92,777,400 mi. **10.** 6,719,801,500 lb. **11.** \$326.10 **12.** 10 wk. **13.** 16 **14.** 103,680 **15.** 42 bu. **16.** 6 pr.

Page 31, Ex. 8: **5.** Greatest is $\frac{1}{2}$; least is $\frac{1}{5}$. **6.** $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{6}$, $\frac{2}{8}$, $\frac{2}{15}$ **7.** $\frac{1}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$ **8.** The fraction with the smaller denominator. **9.** The fraction with the larger numerator.

Page 33, Ex. 9: **1.** $\frac{5}{10}=\frac{1}{2}$; $\frac{12}{15}=\frac{3}{5}$; $\frac{8}{12}=\frac{2}{3}$; $\frac{9}{15}=\frac{3}{5}$; $\frac{6}{10}=\frac{3}{5}$; $\frac{16}{32}=\frac{1}{2}$ **2.** $\frac{1}{4}$; $\frac{3}{16}$; $\frac{1}{3}$; $\frac{4}{5}$; $\frac{1}{5}$; $\frac{3}{4}$; $\frac{3}{5}$ **3.** $\frac{4}{12}$; $\frac{2}{12}$; $\frac{3}{12}$; $\frac{8}{12}$; $\frac{30}{12}$; $\frac{9}{12}$; $\frac{28}{12}$; $\frac{6}{12}$; $\frac{20}{12}$; $\frac{8}{24}$; $\frac{4}{24}$; $\frac{6}{24}$; $\frac{16}{24}$; $\frac{60}{24}$; $\frac{18}{24}$; $\frac{56}{24}$; $\frac{12}{24}$; $\frac{40}{24}$ **4.** $\frac{10}{20}$; $\frac{5}{20}$; $\frac{4}{20}$; $\frac{6}{20}$; $\frac{15}{20}$; $\frac{16}{20}$; $\frac{14}{20}$; $\frac{12}{20}$; $\frac{18}{20}$; $\frac{28}{20}$

Page 34, Ex. 10:

1.		*a*	*b*	*c*	*d*	*e*
	1.	$\frac{5}{2}$	$\frac{10}{3}$	$\frac{11}{2}$	$\frac{14}{3}$	$\frac{25}{2}$
	2.	$\frac{200}{3}$	$\frac{125}{2}$	$\frac{46}{5}$	$\frac{47}{8}$	$\frac{50}{3}$
	3.	$\frac{25}{4}$	$\frac{75}{2}$	$\frac{100}{3}$	$\frac{37}{5}$	$\frac{35}{4}$
2.						
	1.	$1\frac{2}{3}$	$3\frac{1}{2}$	$4\frac{2}{3}$	4	$2\frac{1}{4}$
	2.	3	$9\frac{1}{10}$	8	$8\frac{1}{6}$	$12\frac{1}{3}$
	3.	$9\frac{2}{5}$	$3\frac{1}{5}$	$2\frac{3}{5}$	$1\frac{1}{5}$	4

5. 3 integral and 2 fractional; 4 integral and 1 fractional; 7 integral and 3 fractional.

Page 35, Ex. 11: **5.** Not possible.

Page 35, Ex. 12:

	a	**b**	**c**	**d**
1.	$\frac{5}{6}$	$\frac{19}{90}$	$1\frac{7}{60}$	$1\frac{2}{5}$
2.	$\frac{11}{30}$	$\frac{7}{24}$	$1\frac{7}{12}$	$1\frac{7}{10}$

Page 35, Ex. 12: (*Continued*)

	a	b	c	d
3.	$\frac{13}{40}$	$\frac{13}{15}$	$\frac{9}{16}$	$1\frac{5}{12}$
4.	$\frac{7}{10}$	$\frac{7}{8}$	$1\frac{19}{30}$	$1\frac{9}{32}$
5.	$\frac{5}{12}$	$\frac{7}{8}$	$1\frac{7}{24}$	$1\frac{5}{6}$
6.	$\frac{9}{40}$	$1\frac{3}{20}$	$1\frac{3}{8}$	$1\frac{11}{12}$
7.	$\frac{5}{12}$	$1\frac{1}{24}$	$1\frac{11}{20}$	$2\frac{1}{8}$

Page 36, Ex. 12:

	a	b	c	d	e
8.	$13\frac{1}{24}$	$17\frac{5}{16}$	$18\frac{7}{30}$	$20\frac{7}{16}$	19
9.	$26\frac{1}{2}$	$72\frac{14}{15}$	$31\frac{1}{3}$	$118\frac{17}{24}$	$23\frac{5}{6}$

10. $1\frac{7}{8}$ yd. **11.** $1\frac{1}{8}$ acres **12.** $61\frac{7}{8}$ in. **13.** $26\frac{7}{8}$ mi. **14.** $27\frac{2}{3}$ hr. **15.** $303\frac{5}{8}$ bu. **16.** $25\frac{5}{8}$ in. **17.** Yes, greater.

Page 37, Ex. 13:

3.	*a*	*b*	*c*	*d*	*e*
1.	$\frac{1}{6}$	$\frac{1}{24}$	$\frac{1}{15}$	$\frac{3}{20}$	$\frac{5}{12}$
2.	$\frac{3}{40}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{3}{40}$	$\frac{2}{15}$
3.	$\frac{1}{15}$	$\frac{1}{40}$	$\frac{5}{12}$	$\frac{1}{24}$	$\frac{1}{16}$
4.	$\frac{1}{40}$	$\frac{1}{16}$	$\frac{5}{24}$	$\frac{1}{20}$	$\frac{7}{16}$
5.	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{15}{56}$	$\frac{1}{16}$	$\frac{11}{80}$
6.	$\frac{7}{20}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{1}{20}$	$\frac{3}{5}$
4.					
1.	$5\frac{4}{15}$	$4\frac{5}{12}$	$14\frac{1}{6}$	$15\frac{1}{16}$	$5\frac{11}{12}$
2.	$4\frac{3}{5}$	$13\frac{1}{12}$	$51\frac{5}{6}$	$21\frac{3}{4}$	$43\frac{5}{24}$
3.	$23\frac{1}{3}$	$20\frac{3}{8}$	$19\frac{1}{2}$	$24\frac{39}{40}$	$16\frac{19}{24}$

Page 38, Ex. 14:

	a	b	c	d	e
1.	$3\frac{1}{3}$	$\frac{21}{32}$	$\frac{5}{16}$	$\frac{7}{30}$	$\frac{7}{16}$
2.	$3\frac{1}{3}$	$\frac{5}{8}$	24	$\frac{4}{9}$	$\frac{7}{10}$

Page 38, Ex. 14: (*Continued*)

	a	b	c	d	e
3.	6	$\frac{1}{2}$	$8\frac{1}{3}$	$\frac{8}{15}$	$\frac{5}{18}$
4.	$\frac{1}{4}$	$1\frac{1}{4}$	$\frac{5}{48}$	$\frac{7}{20}$	$\frac{4}{9}$
5.	1	$\frac{7}{12}$	$2\frac{1}{4}$	$1\frac{11}{16}$	2
6.	$\frac{7}{12}$	55	12	$2\frac{1}{2}$	$4\frac{19}{32}$
7.	$\frac{3}{20}$	$\frac{1}{30}$	12	$2\frac{3}{16}$	$\frac{3}{20}$
8.	$\frac{2}{3}$	$\frac{21}{32}$	$4\frac{1}{5}$	2	$\frac{3}{10}$
9.	$\frac{15}{32}$	$\frac{2}{3}$	$8\frac{3}{4}$	$\frac{9}{16}$	$\frac{7}{12}$
10.	$\frac{5}{12}$	$\frac{3}{20}$	6	$\frac{3}{2}$	1
11.	$\frac{7}{12}$	$\frac{7}{20}$	10	$3\frac{1}{2}$	$\frac{7}{20}$
12.	$\frac{1}{2}$	$\frac{2}{9}$	$3\frac{1}{3}$	$\frac{7}{10}$	$\frac{3}{20}$

Page 40, Ex. 15:

	a	b	c	d	e
1.	$1\frac{1}{5}$	16	$2\frac{1}{3}$	$\frac{1}{10}$	27
2.	$1\frac{1}{7}$	42	20	$\frac{1}{6}$	$1\frac{1}{4}$
3.	$\frac{5}{8}$	$3\frac{9}{10}$	$7\frac{17}{25}$	2	$\frac{1}{6}$
4.	$1\frac{1}{20}$	$1\frac{1}{2}$	$\frac{9}{10}$	$1\frac{1}{2}$	6
5.	$\frac{16}{21}$	$15\frac{3}{4}$	$1\frac{9}{16}$	$2\frac{2}{9}$	9
6.	$\frac{3}{10}$	5	$2\frac{2}{49}$	6	8

Page 40, Review:

1.	*a*	*b*	*c*	*d*	*e*
1.	1	$\frac{5}{6}$	$\frac{13}{40}$	$\frac{5}{8}$	$\frac{35}{48}$
2.	$1\frac{11}{12}$	$1\frac{4}{15}$	1	$\frac{7}{12}$	$\frac{1}{12}$
3.	$1\frac{1}{24}$	1	$78\frac{1}{8}$	12	$\frac{9}{80}$
4.	$\frac{11}{30}$	8	$\frac{3}{5}$	38	$\frac{1}{20}$
5.	$\frac{2}{3}$	16	$781\frac{1}{4}$	6	$1\frac{1}{4}$
6.	$\frac{7}{24}$	$1\frac{7}{12}$	$1\frac{1}{8}$	$\frac{9}{16}$	$\frac{1}{4}$

2. $23\frac{11}{12}$ yd. **3.** $91\frac{1}{4}$ lb. **4.** $40\frac{1}{3}$ ft. **5.** $8\frac{7}{8}$ lb. **6.** $56\frac{1}{2}$ ft.

Page 41, Review: **7.** $51\frac{3}{4}$ ft. **8.** $4\frac{7}{8}$ in. **9.** $1\frac{1}{4}$ in. **10.** 56 sacks **11.** $\frac{3}{5}$ **12.** $16\frac{2}{3}$ bu. **13.** $.17 **14.** $.36 **15.** $7\frac{19}{33}$ yd. **16.** $1\frac{3}{8}$ in. **17.** $6.90 **18.** (*1*) $1.52 (*2*) $6.75 (*3*) $.88 (*4*) $5.61 (*5*) $2.76 **19.** $1.20 **20.** $\frac{3}{16}$

Page 42, Review: **21.** $73\frac{11}{12}$ ft. **22.** $83\frac{1}{4}$ ft. **23.** $2\frac{9}{10}$ sec. **24.** $9\frac{6}{7}$ bu. **25.** $\frac{3}{4}$¢ **26.** Remnant $.08 a yd. **27.** $.20 **28.** $3.22 **29.** $78\frac{1}{2}$ ft. **30.** 8

Page 44, Ex. 16: **13.** $4.1\dot{7}$ **14.** 500.015 **15.** 2000.002 **16.** $.7\frac{2}{3}$ **17.** $5.016\frac{1}{3}$

Page 45, Ex. 16: **18.** .003224 **19.** 1500.015

	a	*b*	*c*	*d*
20.				
1.	.49	38.75	3.14	4.07
2.	.01	154.01	14.57 mi.	19.11
3.	52.20	38.28	984.75 ft.	.05
4.	7.09	42.75	985.03	.01
21.				
1.	24.191	14.061	194.251	48.308
2.	.701	3.048	.365	3.142
3.	19.055	23.157	.841	.071
22.				
1.	.9	18.1	.5	3.1
2.	.8	342.0	14.0	25.1
3.	.1	75.6	.8	92.5
23.				
1.	.0786	19.7027	5.0805	14.7037
2.	7.3085	324.0092	13.2968	5.0407

Page 46, Ex. 17:

	a	b	c	d
1.	33.828	431.9753	27.382	32.672

2. 19.946 **3.** 47.906 **4.** 11.659 **5.** 57.7446 **6.** 21.55 **7.** .589 **8.** 23.8584 **9.** 962.9514 **10.** 85.97 mi. **11.** 745.6 mi. **12.** 377.32 mi. **13.** $1015 **14.** 183.125 tons

Page 47, Ex. 18:

1.

	a	*b*	*c*	*d*
1.	329.42	.1152	.120176	30.10892
2.	.000432	64.5215	8.79648	14.53725
3.	89.5856	3.0504	.166530	419.9374

Page 48, Ex. 18:

2.

1.	6.345	.01728	7.78815	.51175
2.	24.300	.9965285	14.97320	.0172664

4. Ten times in each case. **5.** Number is divided by 10; by 100; by 1000. **6.** Number is multiplied by 10; by 100; by 1000. **7.** Same values. **8.** Value is not changed.

11.

	a	*b*	*c*
1.	1.6751	.502656	9.2416
2.	.275724	2.7375	137.79948

Page 49, Ex. 18: **12.** 297.9 mi. **13.** 18.43 mi.; 21.30 mi.; 24.76 mi. **14.** 1822.59 lb. **15.** 83.4 mi. **16.** 506 ft. **17.** 389.2 bu. **18.** .747 lb. **19.** 984.7 lb.

Page 50, Ex. 19:

	a	b	c	d	e
1.	.07	.204	.375	.0473	5.401
2.	2.06	.12	.875	2.95	.0205
3.	.04	.005	.75	.0091	.0022
4.	.1429	.93	17.34	.212	.508
5.	4.027	73.2	.627		
6.	.0732	.0054	.705		
7.	.0043	.042	.00042		
8.	7.342	.0857	.782		
9.	9.032	9.073	1.073		
10.	3.84	.000378	46.02		

Page 51, Ex. 20:

1.	*a*	*b*	*c*
1.	10	1000	10
2.	100	100	10,000

Page 52, Ex. 20:

2.		
1.	3.2	1682.4
2.	1.5	32.0

	a	**b**	**c**
3.	.047	.048	140.00
4.	4736000	.0478	29.0
5.	1.9	73.21	6.25
6.	7.603	.13	4.032
7.	758000	7.04	.032

Page 53, Ex. 21:

1.	*a*	*b*	*c*	*d*	*e*	*f*
1.	.5	.75	$.62\frac{1}{2}$	.12	$.87\frac{1}{2}$	.2
2.	$.37\frac{1}{2}$	$.43\frac{3}{4}$	$.46\frac{7}{8}$	2.375	.35	.46

2.						
1.	$.333\frac{1}{3}$	$.142\frac{6}{7}$	$.111\frac{1}{9}$	$.090\frac{10}{11}$	$.666\frac{2}{3}$	$.416\frac{2}{3}$
2.	$.916\frac{2}{3}$	$.466\frac{2}{3}$	$.428\frac{4}{7}$	$.454\frac{6}{11}$	$.272\frac{8}{11}$	$.733\frac{1}{3}$

Page 54, Ex. 22:

	a	**b**	**c**
1.	193.484	.275	9.247
2.	105.352	114.374	.286
3.	9.565	208.4	2.19
4.	24.88	8077.026	880.274
5.	.47	1.4	.07
6.	325.77	2.42	64.93
7.	.82	.09	.43
8.	.47	55.86	76.49

9. 172.2 **10.** $983.52 **11.** 34.9 **12.** 12.3 **13.** $33\frac{1}{3}$

Page 55, Review: **1.** 4.78 in. **2.** 1213.9 lb. **3.** 24.4 bu. **4.** 1206.25 lb.; .698 lb. **5.** \$32.19 **6.** \$11.05 **7.** \$89.49 **8.** \$53.35 **9.** \$.28 **10.** \$1283.36 **11.** .1024; .09; .07984; .061; .03786 **12.** 69.2 mi. per hour.

Page 56, Review: **13.** 34.9 **14.** .005 **15.** 213.06 **16.** $.77\frac{7}{9}$; $.62\frac{1}{2}$; First better by $.15\frac{5}{18}$ **17.** $83\frac{1}{3}$ bu. **18.** 48+ **19.** 23.3 mi. **20.** \$33.38 **21.** \$$21.83\frac{1}{4}$

Page 59, Ex. 23: **1.** \$6.34 **2.** \$8.25

Page 60, Ex. 23: **3.** \$136 **4.** \$351.15 **5.** \$152.16

Page 64, Ex. 25: **1.** \$2.52 **2.** \$9.53 **3.** \$6.11 **4.** \$22.13

Page 66, Ex. 26: **1.** \$9.30 **2.** \$13.22 **3.** \$11.65 **4.** \$18.55 **5.** \$52.75 **6.** \$5.50 **7.** \$19.82

Page 67, Ex. 27:

	a	b	c	d	e	f	g	h
4.	.07	.18	.15	.20	.50	.75	.165	.0325
5.	.025	$.33\frac{1}{3}$	.375	.96	.625	.0625	.82	.875

Page 68, Ex. 28: **1.** 25% **2.** $8\frac{1}{3}\%$ **3.** $33\frac{1}{3}\%$ **4.** 50%; $12\frac{1}{2}\%$ **5.** 10%; 25%; 100% **6.** 50%; 10% **7.** 50%; 25%; 5%

Page 69, Ex. 28:

	a	b	c	d	e
8.	25%	$12\frac{1}{2}\%$	75%	20%	$16\frac{2}{3}\%$
9.	80%	$83\frac{1}{3}\%$	2%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
10.	$37\frac{1}{2}\%$	$62\frac{1}{2}\%$	10%	60%	$6\frac{1}{4}\%$

	a	b	c
11.	210 gal.	500	490
12.	\$18	\$30	\$17.50
13.	90 mi.	28 ft.	2 ft.
14.	40 lb.	70 men	34 acres

Page 69, Ex. 28: (*Continued*)

	a	**b**	**c**
15.	36¢	$38	13 in.
16.	21 in.	30 acres	6 mi.
17.	3.15	$36.10	$26.40
18.	108	121.60	288 mi.
19.	4.0	4607.5	3.616 lb.
20.	3.3	$20.25	6.3 ft.
21.	$160	$530	$124
22.	246 yd.	8 lb.	2883 tons
23.	600 acres	$24	250 lb.
24.	8 lb.	320 acres	480 acres
25.	$1350	$160	$62
26.	1760 ft.	660 ft.	336 mi.

Page 70, Ex. 29: **1.** 36 **2.** 102 lb. **3.** 300 **4.** $187.20 **5.** 12¢ **6.** $5.07 **7.** $2.71 **8.** $1.80 **9.** Helen **10.** 36 **11.** $42.50 **12.** $1050 **13.** 78 lb.

Page 71, Ex. 29: **14.** $11.50 **15.** $72 **16.** 72 lb. **17.** 162 **18.** $1680 **19.** $30; $2.00; $1.25; $5.00 **20.** $483 **21.** $576; $432; $504; $120; $432; $336 **22.** $139.33 **23.** 2111.4 lb. **24.** 360 **25.** 55,982

Page 72, Ex. 29: **26.** 45,000 lb. **27.** $5.94 gain **28.** $24.50 **29.** 600 **30.** 93 **31.** $36.60

Page 73, Ex. 30: **1.** 50% **2.** 20% **3.** 40% **4.** $66\frac{2}{3}\%$ **5.** $133\frac{1}{3}\%$ **6.** 80% **7.** 50% **8.** 25% **9.** $33\frac{1}{3}\%$ **10.** 90% **11.** 90% **12.** 20% **13.** 10% **14.** 80%; $3600

Page 74, Ex. 30: **15.** 25% **16.** 50% **17.** $14\frac{2}{7}\%$ **18.** $12\frac{1}{2}\%$ **19.** 55% **20.** 4% **21.** 75%; 80% **22.** 18% **24** $93\frac{1}{3}\%$ **25.** 9.82% **27.** 7% **28.** B

Page 75, Ex. 30: **29.** $3\frac{1}{3}\%$ **30.** $269\frac{3}{13}\%$ **31.** $11\frac{1}{9}\%$ **32.** $21\frac{5}{8}\%$ **33.** 16% **34.** 40% **35.** $5\frac{5}{7}\%$ **37.** 180% **38.** 20% **39.** Calif., 44.13+%; Ill., 15.02%; Ohio, 20.81%; Tex., 19.68%

Page 76, Ex. 31:

	a	b	c
1.	500	10	400
2.	400	8	144
3.	300	4500	7
4.	500	200	52
5.	500	225	80

6. 300 **7.** 200 **8.** 400

Page 77, Ex. 31: **9.** 15 **10.** $24.24 **11.** 450 **12.** 4500 tons **13.** 160 cu. in. **14.** $160 **15.** 10 yd. **16.** $5750 **17.** $6 **18.** $6500 **19.** $36

Page 78, Review: **1.** $3.92 **2.** $12\frac{1}{2}\%$ **3.** $11\frac{1}{9}\%$ **4.** Cora **5.** 6% **6.** 49 **7.** $5.53 **8.** $44\frac{4}{9}\%$ **10.** $233\frac{1}{3}\%$ **11.** 25%; 400% **12.** $6\frac{1}{4}\%$; 1600% **13.** 8911.47 mi.

Page 79, Review: **14.** $2\frac{2}{49}\%$ **15.** 700 lb. **16.** $62\frac{1}{2}\%$ **17.** 42,350 **18.** $6 **19.** 85 lb. **20.** 25% **21.** 4% **22.** 158.4 lb. **23.** 1250 lb.

Page 80, Review: **25.** $11\frac{27}{43}\%$ **26.** $4.60 **27.** 5355.41%; 1.86% **28.** 650 **29.** 37.8%; 16%; 14.9%; 6%; 4.4%; 20.9% **30.** $700; $220; $500; $160; $200; $100; $120 **31.** $990; $450; $1125; $900; $225; $360; $450 **32.** Dallas, 72.6%; Detroit, 113.3%; Mobile, 18%; Omaha, 54.4%; Seattle, 32.9%

Page 81, Review: **33.** 250% **34.** Giants, 44%; Lions, 48%; Midgets, 64%; Tigers, 72%

Page 82, Ex. 32:

1.	*a*	*b*
1.	25% gain on cost. 20% gain on selling price.	25% gain on cost. 20% gain on selling price.

Page 82, Ex. 32: (*Continued*)

	a	*b*
2.	$33\frac{1}{3}\%$ gain on cost. 25% gain on selling price.	90% gain on cost. $47\frac{7}{19}\%$ gain on selling price.
3.	25% gain on cost. 20% gain on selling price.	$66\frac{2}{3}\%$ gain on cost. 40% gain on selling price.

Page 83, Ex. 32:

2.	*a*	*b*
1.	\$.02 gain. \$.22 selling price.	\$.02 loss. \$.18 selling price.
2.	\$.30 gain. \$2.10 selling price.	\$100 loss. \$200 selling price.
3.	\$126.67 gain. \$506.67 selling price.	\$.80 loss. \$4 selling price.
4.	\$.30 gain. \$1.80 selling price.	1¢ loss. 7¢ selling price.

3. 25% **4.** 20% **5.** 20% **6.** \$1.80 **7.** \$585 gain; 12% **8.** 20% of cost; $16\frac{2}{3}\%$ of selling price **9.** 30% **10.** $16\frac{2}{3}\%$ **11.** 20% **12.** $12\frac{1}{2}\%$ **13.** $9\frac{1}{11}\%$ **14.** 40%

Page 84, Ex. 32: **15.** $33\frac{1}{3}\%$ of cost; 25% of selling price **16.** $16\frac{2}{3}\%$ of cost; $14\frac{2}{7}\%$ of selling price **17.** $38\frac{8}{9}\%$ **18.** \$1.58 **19.** 7.9% **20.** \$7790 **21.** 34¢ **22.** 11.4+% **23.** $33\frac{1}{3}\%$ **24.** 50¢; $4\frac{1}{6}$¢ **25.** 65.6%

Page 85, Ex. 32: **26.** $28\frac{4}{7}\%$ **27.** $33\frac{1}{3}\%$ **28.** 49 mi. **29.** \$1.20

Page 86, Ex. 33: **1.** \$27 **2.** \$4 **3.** \$6.60 **4.** The first. **5.** \$.10 gain **6.** \$12; \$5.52; \$19.68; \$10.08; \$27.60; \$35.80 **7.** Yes

Page 87, Ex. 33:

8.	*a*	*b*	*c*
1.	\$436.50	\$37.80	\$243.75
2.	\$19.60	\$3958.20	\$756.00
3.	\$3570.00	\$540.50	\$.12
4.	\$7.98	\$2.64	\$3.60

9. (*1*) 20% (*2*) $33\frac{1}{3}\%$ (*3*) 40% (*4*) 20% **10.** $33\frac{1}{3}\%$ **11.** \$10 **12.** 92.16 **13.** \$1.44; 188% **14.** \$7614 after 8 da.; \$8121.60 after 28 da.

Page 89, Ex. 34: **1.** Total is \$934.20. **2.** Total is \$176.30. **3.** Total is \$200.70. **4.** (*1*) \$915.52 (*2*) \$172.77 (*3*) \$196.69

Page 90, Ex. 35: **1.** \$345.60 **2.** \$5.40 **3.** \$532.92 **4.** Second

Page 91, Ex. 35: **5.** 48.148% **6.** 23.46%

7.	*a*	*b*
1.	\$64.13	\$64.13
2.	\$64.13	\$64.13
3.	\$64.13	\$64.13

9. 44%; 19%; 11.8%; 24% **10.** \$1812.90 **11.** .25% and 5% **13.** \$167.20 **14.** 328.60

Page 92, Ex. 36: **1.** \$63 **2.** \$130 **3.** \$28.10 **4.** \$3097

Page 93, Ex. 36:

5.	*a*	*b*
1.	\$10; \$240	\$.90; \$44.10
2.	\$178.50; \$1606.50	\$322.50; \$6127.50
3.	\$25.60; \$1254.40	\$552; \$1104

6. \$541.50 **7.** \$416 **8.** \$42.50; \$325.95 **9.** \$760 lost **10.** \$323.43 **11.** $\$.67\frac{8}{25}$ **12.** \$51 **13.** \$543.13

Page 94, Ex. 37: **1.** \$12; \$10 **2.** \$10.80; \$12.60 **3.** \$22.50; \$17.50 **4.** \$12; \$6; \$36 **5.** \$27.75; \$83.25

Page 95, Ex. 37: **6.** $112.80 **7.** $21.60 **8.** $16.80 **9.** $9.45 **10.** $17.55 **11.** $29.23 **12.** $5 **13.** $20 **14.** $3.70 **15.** $2.25 **16.** $19 **17.** $76.50 **18.** $8.50 **19.** $5.25 **20.** $17.50 per yr.; $8.75 each six months **21.** $8.50; $6.38 **22.** $3 **23.** $17 **24.** Yes $2.25 **25.** $54 **26.** $255 **27.** $48; $24

Page 96, Ex. 38: **1.** $2 **2.** $1.20 **3.** $9.75 **4.** $7.33 **5.** $2.38 **6.** $2.83 **7.** $2.45 **8.** $44.95 **9.** $6.33 **10.** $.65

Page 97, Ex. 38: **11.** $33.68 **12.** $21.33; $17.78 **13.** $14.60; $12.17 **14.** $32.41; $35.36 **15.** Sept. 12; $12.75 **16.** $12.90 **17.** No

Page 99, Ex. 40: **1.** 245 da. **2.** 244 da. **3.** 123 da. **4.** 122 da. **5.** 92 da. **6.** 245 da. **7.** 356 da. **8.** 214 da. **9.** 225 da. **10.** 61 da. **11.** 137 da. **12.** 205 da. **13.** 311 da.

Page 100, Ex. 41: **1.** $26,301.37 **2.** $18,164.38 **3.** $3406.41 **4.** $22,705.48

Page 101, Ex. 42: **1.** $12.60 **2.** $81.60 **3.** $4.32 **4.** $54.34 **5.** $23.18 **6.** $36.22½ **7.** $705 **8.** $11.25 **9.** $5.20 **10.** $3.29 **11.** $142.76 **12.** $622.94

Page 102, Ex. 43: **1.** $16.88 **2.** $45 **3.** $7.40 **4.** $32 **5.** $150 **6.** $168 **7.** $144.38 **8.** $40.50 **9.** $22.58 **10.** $2.63 **11.** $198.45 **12.** $3.61 **13.** $132 **14.** $462 **15.** $58 **16.** $63.79 **17.** $472.50 **18.** $11.76

Page 103, Review: **1.** $28.80 **2.** $.81 **3.** $58.29 **4.** $8.54 **5.** $11.33 **6.** $289.16 **7.** $63 **8.** $71.25 **9.** $25.88 **10.** $29.09 **11.** $103.31 **12.** $97.94 **13.** $9.53 **14.** $.55 **15.** $.381 **16.** $4.432 **17.** $.51 **18.** $2.63 **19.** $45 **20.** $9.92 **21.** $22.08 **22.** $90 **23.** $360 **24.** $17.33

Page 104, Review: **25.** \$72.96 **26.** \$1,792,538.08; \$1,817,434.44 **27.** \$5200 **28.** \$.68 **29.** Howard; 8¢ **30.** \$20.25 **31.** \$38.80 **32.** \$10.63 **33.** \$36,000; \$.05 **34.** 36% **35.** \$392

Page 105, General Review: **1.** $15\frac{15}{19}\%$ **3.** Yes **4.** Yes **6.** $33\frac{1}{3}\%$ **7.** 20% **8.** .5; .1098; .0942; .0375; .0099; .00875; sum is .76015 **9.** $12\frac{1}{2}\%$ **13.** Single discount of 29%.

Page 106, General Review: **14.** 3.7% **15.** \$12.40 **16.** July 26 **17.** 15.79 **18.** \$2.19 **19.** 25%; \$4.80 **20.** 5500 lb.; $233\frac{1}{3}\%$ **21.** \$8.00 **22.** $66\frac{2}{3}\%$ **23.** 127,944

Page 107, General Review: **24.** $38\frac{8}{9}\%$ **28.** \$10,206 **29.** 24.2 mi. **30.** She is wrong. **31.** $\frac{1}{3}$; $\frac{1}{10}$; $\frac{2}{3}$; $\frac{1}{8}$; $\frac{1}{5}$; $\frac{3}{8}$; $\frac{1}{2}$ **32.** 40%; $33\frac{1}{3}\%$; 10%; $16\frac{2}{3}\%$; $12\frac{1}{2}\%$; 60%; 50%; $62\frac{1}{2}\%$; $87\frac{1}{2}\%$ **33.** 4914 lb.; 70,200 lb. **34.** 25% of cost; 20% of selling price. **35.** 17%

Page 108, General Review: **36.** First; 1¢ **39.** 1099 **40.** .999 **41.** 105.4% **42.** $82\frac{3}{4}$ lb. **43.** $\frac{5}{6}$ **44.** \$349.20 **45.** Total = \$5.57 **46.** 47.1% **47.** Total = \$2.90

Page 109, General Review: **48.** \$19.49 **49.** 28% **50.** 19.4% **51.** Lost 4% **52.** 1.6 mi.; $\frac{5}{8}$ min. **53.** $33\frac{1}{3}\%$

Page 110, General Review: **61.** 7 yr., 8 mo., 21 da.

Page 116, Ex. 44: **1.** 90°; 10°; 45°; 15°; 30° **2.** 2 **3.** Right angles. **4.** Acute angles; acute. **5.** Right angles; acute; acute.

Page 118, Ex. 45: **1.** 8 sq. in.; 5 rows; 40 sq. in. **2.** 240 sq. ft. **3.** 1000 sq. rd.; 6.25 acres **4.** \$100 **5.** .0644 acres **6.** More

Page 119, Ex. 45: **12.** \$26 **14.** \$1518.75 **15.** 36 sq. in. **17.** 10 acres **18.** \$49,280 **19.** 23,040 acres **20.** 6.16 squares **21.** No **22.** No **23.** Possible **24.** Not possible

Page 120, Ex. 46: **1.** 336 sq. in. **2.** $693
3. (*1*) 630 sq. in. (*3*) 432 sq. rd. (*5*) 3456 sq. rd.
(*2*) 408 sq. ft. (*4*) 3024 sq. ft. (*6*) 36.12 sq. ft.
8. First is 3 times as great. **9.** First is 9 times as great.

Page 121, Ex. 47: **1.** 1344 sq. ft. **2.** 3150 sq. ft. **3.** 2044 sq. rd.

Page 122, Ex. 47: **5.** (*1*) 609 sq. in. (*2*) 2067 sq. ft. (*3*) 1512 sq. yd. (*4*) 376,200 sq. yd.

Page 125, Ex. 48: **1.** 60 sq. in. **2.** 42 sq. ft. **3.** 340 sq. rd. **4.** 90.75 acres **5.** 2436 sq. ft. **6.** 3542 sq. rd.

Page 126, Ex. 48:
7.
(*1*) 48 sq. ft. (*5*) 2109 sq. rd.
(*2*) 83.95 sq. rd. (*6*) 19.35 sq. in.
(*3*) 60.06 sq. ft. (*7*) 86 sq. ft.
(*4*) 45,695 sq. ft. (*8*) 220.69 sq. ft.
9. 960 sq. in. **10.** $1199.11 **12.** No Yes **13.** Yes
14. 144 sq. ft.

Page 128, Ex. 49: **1.** 36 sq. mi.; 23,040 acres **2.** 1 sq. mi.; $\frac{1}{2}$ sq. mi.; $\frac{1}{4}$ sq. mi.; 320 acres; 160 acres **4.** (*a*) 80 acres (*b*) 80 acres (*c*) 40 acres (*d*) 80 acres (*e*) 80 acres (*f*) 80 acres

Page 129, Ex. 49: **5.** 480 rd. **6.** $35,200 **7.** $14,400 **8.** $19,200 **9.** $6800; 560 acres **10.** $23,200 **11.** 96 acres **12.** Not adjoining **13.** 320 acres **15.** 80 acres **16.** 480 rd. **18.** 10 acres

Page 132, Ex. 50: **8.** $21.60 **9.** $50.60 **13.** $14 **14.** $35.46 **16.** 36 sq. ft.

Page 136, Ex. 51: **1.** Total, $108.13 **2.** Total, $119.03 **3.** Total, $16.25 **4.** Balance, $66.83

Page 137, Ex. 51: **6.** Balance, $980.02 **7.** Balance, $24.40

Page 139, Ex. 52: **1.** 10¢; 5¢; 12¢; 12¢; 5¢; 15¢ **2.** Cost is the same. **3.** Postal money order, 3¢ **4.** 18¢; 8¢; 12¢; 18¢; 18¢; 7¢; 12¢; 18¢ **5.** 10¢ **6.** 5¢ **9.** 6¢; 8¢; 12¢; 15¢

Page 140, Ex. 52: **10.** 12¢ **11.** 5¢; 7¢; 12¢; 15¢ **13.** $17.20; $5.60; $32.20

Page 143, Ex. 53: **1.** $500 **2.** Maker, Charles Nelson; payee, Walter Andrews. **3.** Aug. 1 **4.** $507.50 **5.** Yes. It can be transferred. **6.** To Charles M. Cook by Walter Andrews. **9.** July 13

Page 144, Ex. 53: **11.** June 7; Sept. 7; June 5

14.	Face	Time	Rate	Amount Due
1.	$350	90 da.	6%	$355.25
2.	$605	60 da.	6%	$611.05
3.	$750			$750

15. $790.16 **16.** $267.02

Page 146, Ex. 54: **1.** $8; $792 **2.** $1.31; $73.69 **3.** $10.39; $1261.49 **4.** 90¢; $256.60 **5.** $361.99 **6.** $6806.58 **7.** $3642.70 **8.** $2364 **9.** (*1*) $802.35 (*2*) $495.56 (*3*) $789.59 (*4*) $1487.82

Page 151, Ex. 56: **1.** Dec. 1; Mar. 1 **2.** $5 and $1; Nov. 1 **3.** Jan. 1 **4.** $20; $5; $2; $1 **5.** $31.20 **6.** First of the following month. **9.** $1 **10.** 220

Page 152, Ex. 56: **11.** $4.10 **12.** 10 weeks; 5 **13.** $155.80 **14.** $3.80

Page 156, Ex. 57: **4.** $337.85; $37.85; $36 **5.** $175.75 **6.** $4\frac{1}{2}\%$ yields more. **7.** $116.05

Page 157, Ex. 57: **8.** The first; $10.90 **9.** $428.07 **10.** $640.04 **11.** $5218.62

Page 160, Ex. 58: **1.** \$286,000; \$22 **2.** \$717.50; \$35 **3.** \$812; \$34 **4.** \$1104; \$51 **7.** 5.24% **8.** \$106,232,939.75 **9.** 5.56%

Page 163, Ex. 59: **1.** Arnold, 250 shares; Peterson, 300 shares. 450 shares; 550 votes **2.** \$96

Page 164, Ex. 59: **3.** \$2000; 20 **4.** \$120 **5.** \$60,000 **6.** 400 **7.** \$198 **9.** \$160 **10.** \$49 **11.** \$140 **12.** \$1; \$1.25; 75¢; \$1.50; \$3; \$1.75 **13.** \$3443.63; \$8725.50; \$2275.50; \$5626.25

Page 165, Ex. 59: **14.** \$8828 **15.** \$1866 **16.** \$15,000; \$120 **17.** \$1375 **18.** \$245 **19.** \$200 **20.** 7% **21.** $8\frac{1}{3}\%$ **22.** 8% **23.** Was mistaken; error $\frac{2}{3}\%$ **24.** \$320; 6.4% **25.** \$150

Page 166, Review: **1.** \$4800; \$180 **2.** \$81.25 **3.** 1% **5.** \$28,837.50 **6.** 5% **7.** $12\frac{1}{2}\%$ **8.** $1\frac{2}{3}\%$ more **9.** \$$133\frac{1}{3}$ **10.** \$200 **11.** \$3600 **12.** \$106.25 **13.** 120

Page 167, Review: **14.** \$4400 **16.** \$15,360 **17.** \$2400; \$420 **18.** \$140 **19.** \$5000; \$5762.50 \$762.50

Page 168, Ex. 60: **1.** \$150 **2.** \$94.79 **3.** \$3600 **4.** \$60 **5.** \$47 **6.** \$3400 **7.** \$53.50 **8.** \$648, less

Page 169, Ex. 60: **9.** 14.6%

Page 172, Ex. 62: **1.** \$12.32 **2.** \$360 **3.** \$595 **4.** \$105 **5.** \$67.20 **6.** \$30,000

Page 173, Ex. 62: **7.** \$65.40 **8.** \$3022.50 **9.** \$2295 **10.** \$140 **11.** \$438.75
12. *(1)* \$12 *(3)* \$150 *(5)* \$36
(2) \$84 *(4)* \$31.50 *(6)* \$1012.50

Page 174, Ex. 63: **1.** 8.29%; \$135 **2.** \$10; \$500; \$333.33 **3.** \$7650 **4.** \$8100 **5.** \$7350

Page 175, Ex. 63: **6.** \$5000; $16\frac{2}{3}\%$ **7.** 10%

Page 177, Ex. 64: **1.** On \$1000: \$18.76; \$28.73. On \$5000: \$93.80; \$143.65 **2.** On \$1000: \$39.34; \$23.31. On \$5000: \$196.70; \$116.55 **3.** On \$1000: \$41.10; \$31.56. On \$5000: \$205.50; \$157.80 **4.** On \$1000: \$48.86; \$30.63. On \$5000: \$244.30; \$153.15 **5.** On \$1000: \$46.36; \$53.86. On \$5000: \$231.80; \$269.30 **8.** \$102.75 **9.** \$5381.44 **10.** \$85.95 less **11.** $\$12.11\frac{1}{4}$

Page 180, Ex. 65: **4.** \$59.20 **5.** 1.85% **6.** (*1*) \$55.20 (*2*) \$233.23 (*3*) \$90.83 (*4*) \$234.95 (*5*) \$73.50 (*6*) \$300.30

Page 181, Ex. 65: **8.** \$107.25 **9.** \$129.50 **14.** \$68 **15.** \$48,600 **16.** \$275.40 **17.** (*1*) \$125.78 (*2*) \$308.53 (*3*) \$81.70 (*4*) \$571.04 (*5*) \$266.60 (*6*) \$550.40 (*7*) \$320.35 (*8*) \$1263.99

Page 184, Ex. 66: **1.** \$300; specific **2.** \$1680; ad valorem **3.** \$11.20 **4.** \$14,910 **5.** \$720; specific **6.** \$5600 **7.** \$65 **8.** \$96 **9.** \$76; \$44 **10.** \$1480 **11.** \$300 **12.** \$202.50 **13.** \$2618

Page 186, Ex. 67:

	a	b	c	d	e
1.	$12d$	$20d$	$13n$	$15x$	$16m$
2.	$5d$	b	$40n$	$4h$	$2x$

		a	b	c	d
3.	*1.*	20	252	$1\frac{1}{6}$	38
	2.	37	33	1050	294
4.	*1.*	16	120	60	14
	2.	18	26	117	68

5. 9000 sq. ft.; 6500 sq. ft. **6.** *bh*; 18 sq. ft. **7.** *nd*

Page 187, Ex. 67: **16.** 12 **17.** \$72 **18.** 104 sq. in. **19.** 144.72

Page 189, Ex. 68: **1.** 259.8 sq. in. **2.** 1039.2 sq. in. **3.** 482.8 sq. in. **4.** 34.375 sq. ft.

Page 189, Practical Measurements: **2.** Right; obtuse; acute.

Page 190, Practical Measurements: **8.** Same areas. **12.** Obtuse.

Page 193, Ex. 69: **2.** 90° **3.** 180°; equal. **4.** 180°; equal. **5.** 90°; 45°; 180°

Page 194, Ex. 69: **7.** Amount of opening. **9.** Sum in each case = 360°

Page 196, Ex. 70: **1.** (*1*) $100\frac{4}{7}$ ft. (*2*) 66 yd. (*3*) 880 ft. (*4*) $18\frac{6}{7}$ in. **2.** 14 ft. **3.** (*1*) $8\frac{10}{11}$ in. (*2*) $17\frac{9}{11}$ ft. (*3*) $11\frac{3}{22}$ yd. (*4*) $13\frac{4}{11}$ in. **4.** 132 in. **5.** $31\frac{3}{7}$ in. **6.** $106\frac{6}{7}$ in. **7.** One is twice the other. **8.** One is twice the other. **9.** 252 **10.** More **11.** $25{,}142\frac{6}{7}$ miles **12.** Yes **13.** 245 ft.

Page 197, Ex. 70: **14.** 3600 revolutions **15.** 32.9 **16.** $62\frac{6}{7}$ in. **17.** 236.3 rotations **18.** 3300 ft. **19.** 88 ft. **20.** 2 places

Page 199, Ex. 71: **1.** (*1*) 154 sq. ft. (*2*) 9856 sq. in. (*3*) 2464 sq. in. (*4*) $314\frac{2}{7}$ sq. in. (*5*) 6.16 sq. in. (*6*) 38.5 sq. ft. (*7*) 5544 sq. in. (*8*) 63.6 sq. mi. **2.** (*1*) 154 sq. in. (*2*) 9856 sq. ft. (*3*) 1386 sq. yd. (*4*) 2464 sq. rd. **3.** 22 ft. **4.** $12\frac{4}{7}$ sq. in. **5.** 87.96 ft.; 615.75 sq. ft. **6.** $201\frac{1}{7}$ sq. ft. **7.** One is 4 times the other. **8.** $12\frac{4}{7}$ sq. in. **9.** 154 sq. in. **10.** $116.21 **11.** $5\frac{5}{14}$ sq. in. **12.** Oscar **13.** $28\frac{2}{7}$ sq. mi.

Page 200, Ex. 71: **14.** 19.635 sq. mi. **15.** $3\frac{1}{7}$ sq. mi.; $12\frac{4}{7}$ sq. mi.; 4 **16.** $4\frac{6}{7}$ yd. **17.** $1810\frac{2}{7}$ sq. ft. **18.** $11\frac{11}{224}$ sq. in. **19.** No **20.** $3394\frac{2}{7}$ lb. **21.** $2.89 **22.** $9\frac{5}{8}$ sq. ft. **23.** $1414\frac{2}{7}$ sq. in. **24.** $49\frac{1}{2}$ sq. in.

Page 202, Ex. 72:

1.	*a*	*b*	*c*	*d*	*e*	*f*
1.	81	196	900	2.56	.0025	145.2025
2.	.0016	.49	36.6025	3.24	.0324	.00000324

3.						
1.	55	18	36	72	42	32
2.	24	15	16	85	45	27

4.						
1.	9	6	14	18	13	11
2.	8	10	11	15	19	22

5. (*1*) 13 in. (*2*) 15 in. (*3*) 48 ft. (*4*) 55 ft. (*5*) 100 ft. (*6*) 72 yd. (*7*) 95 in. (*8*) 15 ft. (*9*) 12 ft. **6.** 85 ft. **9.** No **10.** 100; 9801

Page 203, Ex. 72: **11.** No **13.** 260 ft. **14.** 52 rd., garden; 128 rd., field. **15.** 160 rd.; $102.40

Page 204 Ex. 73:

1.	*a*	*b*	*c*	*d*
1.	87	39	59	61
2.	43	79	99	324

2. 79 ft. **3.** 7.4; 1.43; .132; 3.08 **4.** 1.41; 2.24; 3.16; 6.32; 2.65

Page 206, Ex. 74: **2.** 15 **3.** 13 **4.** 35

Page 207, Ex. 74: **5.** 40 ft. **7.** 49 ft. **8.** 29.7 ft. **9.** 12.8 in. **10.** 40 ft. **11.** 127.3 ft. **12.** 27.5 ft. **13.** 46.9 mi.

Page 209, Ex. 75: **1.** $\frac{1}{4}$; 26.2 ft. **3.** $22\frac{1}{5}$ ft. **5.** 41 ft. **6.** 8.9 ft. **7.** 2 mi. **8.** 41.73 ft. **9.** Yes

Page 211, Ex. 76:

1.	*a*	*b*	*c*
1.	24 cu. in.	30 cu. ft.	48 cu. in.
2.	180 cu. ft.	1536 cu. ft.	560 cu. ft.

Page 211, Ex. 76: (***Continued***)

	a	*b*	*c*
3.	84 cu. yd.	59.85 cu. yd.	648 cu. ft.
4.	60 cu. ft.	206.15 cu. in.	192 cu. ft.

2. 27 cu. in.; 125 cu. in.; 512 cu. in. **3.** 1728 cu. in.; 1728 cu. in. **4.** 27 cu. ft.; 27 cu. ft. **5.** $100.80 **6.** 65 cu. ft. **9.** 128 **10.** 12

Page 212, Ex. 76: **11.** 8 tons **12.** $7333\frac{1}{3}$ **13.** 6.3 in. **14.** 7.5 gal. **15.** 72,000 lb. **16.** 4787.5 gal. 40,000 lb. **18.** 24.75 cu. ft. **19.** $179.20 **20.** 4042.9 tons **21.** $1843\frac{1}{5}$ bu.

Page 213, Ex. 76: **22.** $78{,}643\frac{1}{5}$ bu. **23.** 44.1 gal. **24.** 240 cu. ft. **25.** 31,418.2 gal. **27.** 113.44 tons **28.** 625 cu. ft. **29.** $384

Page 214, Ex. 77: **1.** (*1*) $298\frac{2}{3}$ (*2*) 3072 (*3*) 1024 (*4*) 252 **2.** $32.26 **3.** $32.26 **4.** 840 **5.** $53.76

Page 215, Ex. 77: **6.** $420 **7.** $200.76 **8.** 3395 bd. ft. **9.** $307.20 **10.** They do. **12.** $816\frac{2}{3}$ bd. ft. **13.** Yes **14.** 640 bd. ft.

Page 217, Ex. 78: **1.** 864 cu. in. **2.** 8916.6 cu. in. **3.** 109.5 cu. ft. **4.** 7254 cu. yd. **5.** 12 cu. in. **6.** 711.55 cu. yd. **7.** 1728 cu. ft. **8.** 336 cu. in. **9.** 3.59 cu. ft. **10.** (*1*) 2244.1 gal. (*2*) 3231.5 gal. (*3*) 377 gal. (*4*) 16,157.9 gal. (*5*) 13,184.4 gal. (*6*) 64,631.7 gal. **11.** 2.16 gal. **12.** 10.97 in. **13.** 2615.76 cu. ft.

Page 219, Ex. 79: **1.** (*1*) 112 sq. ft. (*2*) 760 sq. ft. (*3*) 450 sq. ft. (*4*) 528 sq. ft. **2.** 312 sq. ft. **3.** $7.56 **4.** 368 sq. in. **5.** 174 sq. in. **6.** $6.40 **9.** 6; rectangle **10.** 3; rectangle

Page 220, Ex. 80: **1.** 16,292.57 cu. ft. **2.** 7.83 gal. **3.** 52.2 gal.

Page 221, Ex. 80: **4.** 141,061.2 gal. **5.** 77.48 tons **6.** 187,264 lb. **7.** 261 da. **8.** 381,857.1 cu. ft. **9.** 66.6 gal. **10.** 47,732.1 lb. **11.** 4408.2 gal. **12.** 1128.4 gal. **13.** 1539.5 bbl. **14.** 13,711,151 gal. **15.** 2019.74 gal.

Page 222, Ex. 80: **18.** (*1*) 317.3 gal. (*2*) 576 gal. (*3*) 1128 gal. **19.** 138.6 bbl.

Page 222, Ex. 81: **1.** $1414\frac{2}{7}$ sq. in. **2.** $377.15

Page 223, Ex. 81: **3.** 113.14 sq. in. **4.** $188.57 **5.** 18.85 sq. ft. **6.** 115.2 sq. ft. **7.** 1508.6 sq. ft. **8.** 18.85 sq. ft. **9.** 2262.9 sq. ft.; 101,828.57 **10.** 68.1 sq. ft. **11.** $8.05 **12.** 968 sq. in. **13.** 11.5 sq. ft.

Page 224, Ex. 82: **1.** (*1*) 720 sq. in. (*2*) 2.625 sq. ft. (*3*) 27 sq. ft. (*4*) 50 sq. ft. (*5*) $98\frac{2}{3}$ sq. ft. (*6*) 126 sq. ft. (*7*) 744 sq. in. (*8*) $970\frac{2}{3}$ sq. ft. (*9*) 7974.5 sq. ft. (*10*) 1935.8 sq. ft.

Page 225, Ex. 82: **2.** 946,176 sq. ft. **3.** 756 sq. ft. **4.** 98 sq. ft.

Page 226, Ex. 83: **1.** 62.85 sq. yd. **2.** 2132.7 sq. ft.; $106.63 **3.** 264 sq. in. **4.** $39.60 **5.** 21.9 sq. yd. **6.** Cylinder; 9792 sq. in.

Page 227, Ex. 84: **1.** (*1*) 930 cu. ft. (*2*) 12,000 cu. yd. (*3*) 1736.7 cu. in. (*4*) 205.6 cu. ft. (*5*) 983.8 cu. in. (*6*) 737.3 cu. ft. **2.** 94,765,056 cu. ft.; 7,960,264.7 tons **3.** 804.6 cu. ft. **4.** 17.4 bu.

Page 228, Ex. 84: **5.** No **6.** 240.3 cu. ft. **7.** 84 lb. **8.** 8880 lb. **9.** 304.5 sq. in.; 565.71 cu. in. **10.** 21.3 gal. **11.** 216 sq. in. **12.** $33\frac{11}{18}$ sq. yd. **13.** $49\frac{9}{14}$ sq. yd. **14.** 1.64 sq. yd.

Page 230, Ex. 85: **1.** 50.3 sq. ft.; 33.5 cu. ft. **2.** 113.1 sq. ft. **3.** 4.2 cu. ft. **4.** 17.7 sq. in. **5.** 1075.6 sq. ft. **6.** The second has 4 times the surface and 8 times the volume of the first. **7.** 5656.5 lb. **8.** $\frac{11}{21}$ **9.** 18.2 lb.

Page 231, Review: **1.** 94.9 acres **4.** 280 **5.** 9.7 ft. **6.** $477.86 **7.** 37.3 bbl. **8.** $80.45 **9.** 69.8 lb. **10.** 2539.1 gal. **12.** 432 bu. **13.** 3200 rails; 1584 tons

Page 232, Review: **14.** 17.1 sq. yd. **15.** 360 ft.; 127.3 ft. **16.** 5303.6 gal. **17.** $24.33 **20.** 40.1 ft. **21.** 235.5 ft. **22.** $70{,}714\frac{2}{7}$ in. **23.** 336 bu. **24.** 70.5 gal. **25.** 2644.9 gal.

Page 233, Ex. 86: **1.** $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{4}{3}$; $\frac{5}{2}$; $\frac{3}{2}$ **2.** $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{1}$; $\frac{1}{3}$; $\frac{2}{1}$; $\frac{5}{4}$ **3.** $\frac{10}{1}$; $\frac{1}{2}$; $\frac{10}{1}$; $\frac{5}{1}$; $\frac{1}{4}$ **4.** $\frac{2}{3}$; $\frac{3}{8}$; $\frac{2}{1}$; $\frac{1}{3}$

Page 234, Ex. 86: **5.** $\frac{1}{2}$; $\frac{1}{2}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{2}{3}$; $\frac{1}{10}$; $\frac{1}{3}$; $\frac{3}{2}$ **6.** $\frac{2}{1}$; $\frac{2}{1}$; $\frac{3}{2}$; $\frac{6}{1}$; $\frac{3}{2}$; $\frac{10}{1}$; $\frac{3}{1}$; $\frac{2}{3}$

9.	*a*	*b*	*c*
1.	12	3	12
2.	40	$\frac{3}{4}$	12
3.	6	2	5

12. $\frac{1}{4}$ **13.** $\frac{1}{3}$ **14.** $3\frac{1}{7}$ **15.** $\frac{1}{9}$

Page 235, Ex. 87: **1.** $84; $66 **2.** $2.50; $7.50 **3.** 42 lb.; 36 lb. **4.** $4\frac{4}{5}$ lb.; $7\frac{1}{5}$ lb. **5.** $24.66; $21.34 **6.** $3; $2 **7.** $8; $12

Page 237, Ex. 88:

1.	*a*	*b*	*c*	*d*
1.	2	16	28	$4\frac{1}{5}$
2.	2	$11\frac{1}{4}$	$3\frac{3}{5}$	1.57
3.	21	4	$5\frac{3}{5}$	75
4.	$9\frac{1}{3}$	12	$2\frac{8}{9}$	14.8

2. 3 **3.** 8 **4.** 28 ft. **5.** 13 gal. **6.** $5\frac{1}{3}$ gal. **7.** 60 mi.

Page 240, Ex. 89: **1.** 39 ft. 6 in. **3.** 24 ft. **4.** 63 ft. **7.** 320 ft.

Page 241, Ex. 89: **9.** $171\frac{1}{9}$ ft. **10.** 205.5 ft. **11.** 3.6 in. **12.** 337.5 mi.

Page 242, Ex. 89: **14.** $EB:AB=AB:BO$ **15.** $173\frac{11}{23}$ ft.

Page 253, Ex. 93: **1.** 4 **2.** 11

3.

	a	*b*	*c*	*d*
1.	$n=7$	$n=8$	$n=7$	$n=8$
2.	$n=9$	$n=5$	$n=16$	$n=15$
3.	$n=7$	$n=22$	$n=36$	$n=4$
4.	$n=6$	$n=9$	$n=6$	$n=5$
5.	$n=16$	$n=1$	$n=4\frac{1}{2}$	$n=6$
6.	$n=2.2$	$n=1\frac{1}{6}$	$n=4\frac{1}{2}$	$n=21$

Page 254, Ex. 94:

	a	b	c		a	b	c
1.	5	8	8	**11.**	24	$\frac{4}{5}$	20.4
2.	7	90	3	**12.**	14	4	21
3.	6	20	6	**13.**	20	42	$2\frac{1}{2}$
4.	$2\frac{2}{3}$	$8\frac{1}{2}$	3	**14.**	18	24	4
5.	$1\frac{5}{9}$	$6\frac{1}{4}$	$1\frac{2}{3}$	**15.**	44	16	3
6.	4	6	$1\frac{2}{3}$	**16.**	8	$3\frac{3}{4}$	9
7.	5	8	$6\frac{2}{5}$	**17.**	56	8	$2\frac{1}{6}$
8.	7	6	$3\frac{4}{7}$	**18.**	$6\frac{1}{3}$	2	$9\frac{1}{2}$
9.	12	30	14	**19.**	4	90	2
10.	10	30	$22\frac{1}{2}$				

Page 255, Ex. 95: **1.** $l=\frac{v}{hw}$; $w=\frac{v}{lh}$; $h=\frac{v}{lw}$ **2.** $r=\frac{i}{pt}$ **3.** $\frac{3}{50}$ **4.** 20 **5.** 256; 576

Page 256, Ex. 95: **6.** 228 **7.** $d=\frac{7c}{22}$ **8.** $r=\sqrt{\frac{7a}{22}}$

Page 256, Problems Involving Equations: **1.** 9 **2.** 11 yr. **3.** 21 in. **4.** 88 lb. **5.** 16; 21 **6.** 20; 16 **7.** 100; 50 **8.** 20; 21; $n+1$ **10.** \$1666$\frac{2}{3}$; \$8333$\frac{1}{3}$

Page 257, Problems Involving Equations: **11.** 16 boys; 20 girls **12.** 11 yr. **13.** Arithmetic, \$.95; geography, \$1.55 **14.** 12 **15.** 72 rd. **16.** Louise, 128; Marion, 64 **17.** 400 **18.** 12 **19.** 60 ft., length; 40 ft., width; 2400 sq. ft. **20.** \$3.50 **21.** 14 h **22.** hx **23.** 8

Page 260, Ex. 96: **1.** .36m.; 480,000m.; 7.5m.; .26m. **2.** .0074Km.; .1Km.; .0485Km.; .0000275Km. **3.** 400cm.; 7.3cm.; 98cm.; 305cm. **4.** 5.48m.; 7.98m.; 5460m.; 342,000m. **5.** 5.6cm.; 308cm.; 480cm. **6.** 72.84Km.; 8.7Km.

Page 261, Ex. 97: **1.** 840,000 sq. cm.; 84,000,000 sq. mm. **2.** .0485 sq. m. **3.** 100 sq. cm. **4.** 1.68 hectares. **5.** .112 hectares

Page 262, Ex. 98: **1.** 4200l.; 740l.; 8.42l. **2.** .924Hl.; 8.6Hl.; 470Hl. **3.** 7400l. **4.** 634 qt. **5.** 13,000

Page 263, Ex. 99: **1.** 14,000g.; 7420g.; 3.4g.; 1800g. **2.** .074Kg.; 4.62Kg.; .38472Kg. **3.** 792g. **4.** 515g. **5.** 51.52Kg.

Page 263, Review: **1.** 1631.4 lb. **2.** 15.75 in. **3.** 3.36 mi.

Page 264, Review: **4.** 2.98 mi. **5.** 984.3 ft. **6.** 731.5m. **10.** 131.1 mi. **11.** 859.6 ft. **12.** 51.3 mi. **13.** 4755.1 qt. **14.** 628.3 lb. **15.** 4004 cu. m. **16.** 49.42 acres **17.** Pounds **18.** Kilometers **19.** Less **20.** 543.6 acres

Page 265, Review: **21.** 9.9 lb. **22.** 27.5 cu. in. **23.** 3.96 lb. **24.** 7.76 mi. **25.** 1Kg. **26.** 21.55m. **27.** Belgian, 4.9 lb. **28.** 283.8 bu. **29.** Italian, 71.6 ft. **31.** 12.1 sec. **32.** 27,557.6 bu. **33.** 1631.4 lb.

Page 266, General Review: **1.** 36.1% **2.** 59.4%; 48.6%; decrease. **3.** 101.7% **4.** 77.4% **5.** 29% **6.** 197,647,302.5 long tons **7.** 9.2%

Page 267, General Review: **8.** 31,038,140,000 gal. **9.** 1,109,630,898.1 tons **10.** 21,645,000 bales **11.** 141,411,260 oz. **12.** 2,015,101,733 lb. **13.** 1,192,122.5 tons **14.** 882,441.2 tons **15.** 14.9% **16.** 39.1% **17.** 228.1% **18.** 5.9%

Page 268, General Review: **19.** 35.2% **20.** 8.7% **21.** .04% **22.** 5.3% **23.** 54.1% **24.** 31.6% **27.** 6.1% **28.** 77.7%

Page 269, General Review: **29.** 8.3% **30.** Area of Canada is 123.2% of that of United States. **31.** Area of Mexico is 25.4% of that of United States. **32.** 6.9% **33.** .04% **34.** 16.8% **35.** 3,773,540; 2,881,405; 746,751; 785

Page 270, General Review: **36.** 70.9% **37.** 5.3%

Page 270, Good Roads Problems: **1.** 6$\frac{7}{8}$¢ per bu. **2.** 3¢ cheaper per bu. **3.** $35.75 **4.** 16$\frac{2}{3}$¢ per bu. **5.** 3$\frac{4}{7}$¢ per bu. **6.** $66.56 **7.** 3389.6 cu. yd.

Page 271, Good Roads Problems: **8.** 12$\frac{3}{4}$ da. **9.** $1184

Page 271, Miscellaneous Problems: **3.** First, by $.008 per lb. **4.** 16.4 **6.** $127 more

Page 272, Miscellaneous Problems: **8.** 16 times **9.** $44.35 **10.** 2 sec. **11.** 20 sec. **13.** 4.4% **14.** 92.4 ft. **15.** 130,560 lb. **16.** $603.79; $.096

Page 273, Miscellaneous Problems: **17.** 724.6 lb. **18.** $558.73 **19.** Bill, $182.50 **20.** $2 **21.** 181 tons **23.** 3 pt. **24.** 22 **25.** 48 **26.** 616 **27.** 616

Page 274, Miscellaneous Problems: **29.** 24.1 ft. **31.** 12.9 ft. **32.** 144% **34.** 16$\frac{5}{8}$ qt. **35.** 7.1% **37.** .207 acres **38.** 11,017.5 ft. **39.** 2.41 sq. in. **40.** $2690

Page 275, Miscellaneous Problems: **41.** $1495.97 **43.** 85.2% **44.** 128$\frac{4}{7}$ mi. **45.** 11$\frac{1}{2}$ qt. **46.** $52.80 **47.** 66$\frac{2}{3}$% **48.** $10.44 **49.** 7,514,100 lb. **50.** 12,000; 9000; 525; 450; 10,650; 24,600 **51.** 16,000; 12,000; 700; 600; 14,200; 32,800; 36,000; 27,000; 1575; 1350; 31,950; 73,800 **52.** 862 bbl. **53.** 25% **54.** $5.39

Page 276, Miscellaneous Problems: **55.** 5% gain **57.** 330.24 lb. **58.** 15¢ **59.** 1886 ft. **60.** 127.6 ft. **61.** 20.4% **62.** 75.4 sq. ft. **63.** 51.4 bbl. **64.** 4$\frac{1}{6}$ ft. **65.** 35¢ **66.** 75,600 gal. **67.** 15.6 ft.

Page 277, Miscellaneous Problems: **68.** 17.7% **69.** 62,857$\frac{1}{7}$ in. **70.** 6$\frac{1}{4}$ in. **71.** 84 ft. **72.** 158.1 ft. **73.** 87¢ **74.** $170 **75.** 57.3 ft. **76.** $12.60 **77.** 12$\frac{1}{2}$% **78.** $164.40; $168; $162 **79.** 4.6%

Page 278, Miscellaneous Problems: **80.** $175 **81.** $7.70 **82.** First **84.** $450; $360; $270; $360; $360; $600; $480; $360; $480; $480

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